Induced-Torque Anisotropy in Potassium*

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We have measured the induced torque on a large number of spherical single crystals of potassium in the geometry employed in magnetoresistance investigations and have observed a large twofold anisotropy for all orientations at 4.2 K. The preferred direction required by this departure from cubic symmetry is a ⟨110⟩ axis; which particular ⟨110⟩ axis depends on the residual stress in the crystal.

We have measured the induced torque on a large number of single crystals of potassium in the geometry employed in magnetoresistance investigations[1-3] and have observed a large twofold anisotropy for all orientations at 4.2 K. The torque is sinusoidal for magnetic fields up to $H \sim 2$ kOe but at higher fields secondary minima develop, resulting in the four-peak form of twofold symmetry shown in Figs. 1, 2, and 3 for ⟨111⟩, ⟨100⟩, and ⟨110⟩ axes, respectively, parallel to the suspension. The occurrence of twofold symmetry for these orientations of a cubic crystal requires the presence of a preferred direction. Because the induced torque depends on resistivity in a plane rather than on that in a single direction, each run can only determine the plane in which the preferred axis lies. In all fifteen cases for which the orientation could be determined by x rays, the low-field torque minimum occurred when $\vec{\Omega}$, $\vec{H}$, and a ⟨110⟩ axis were coplanar, indicating that a ⟨110⟩ axis is the preferred direction in the crystal. ($\vec{\Omega}$ is the angular rotation frequency of the magnet.) Which particular ⟨110⟩ axis becomes the preferred direction in a given experiment appears to depend on the residual stress in a complicated way, and requires further investigation. The situation is similar to that for chromium before it was discovered that a preferred direction of the spin-density wave could be established by field cooling or by the application of a stress.[4-6] Although Penz and Bowers[7] have reported a strain-dependent anisotropy in the magnetoresistance of potassium, the range of orientations studied was insufficient to indicate that the symmetry was lower than cubic.

A well-annealed,[8] spherical, single crystal of potassium was suspended between the pole pieces of an A. D. Little 11-in. Bitter-type, iron-core

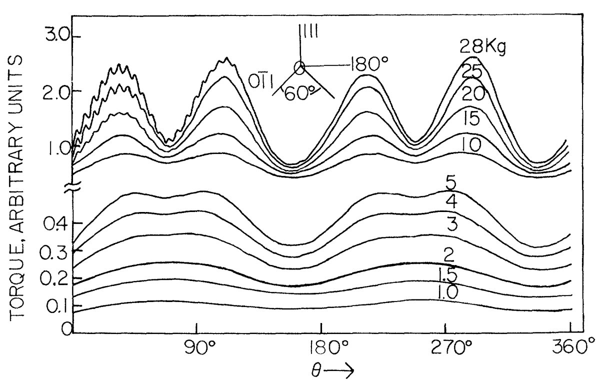

FIG. 1. Recorder trace of torque versus angle for sample No. 58 having the (111) axis parallel to the suspension, as shown in insert. Oscillations at the start of rotation at 0° are believed to be similar to those observed in aluminum by Delaney and Pippard (see Ref. 11).

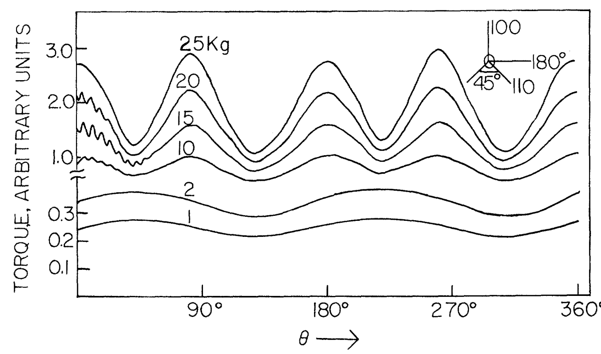

FIG. 2. Recorder trace of torque versus angle for sample No. 68 having the (100) axis parallel to the suspension, as shown in insert.

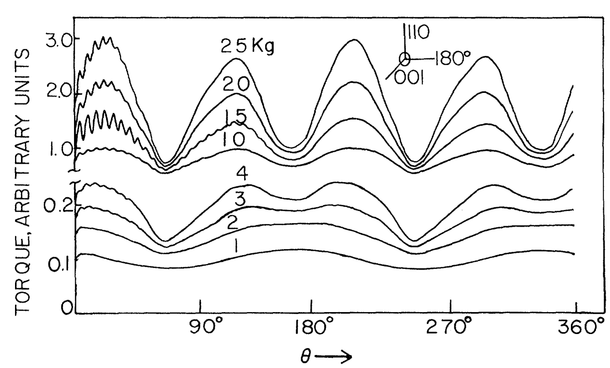

FIG. 3. Recorder trace of torque versus angle for sample No. 61 having the (110) axis parallel to the suspension, as shown in insert.

electromagnet, in a quartz cup attached to a 1-mm-diam quartz rod as shown in Fig. 4. A small amount of Vaseline or a drop of mineral oil held the sample in place. The sample was suspended inside an evacuated Pyrex tube into which a small amount of helium exchange gas was admitted.

The top end of the quartz rod was connected rigidly to the coil of a Condon torsion balance.[9] The induced torque was generated by rotating the magnet at constant field strength about the axis of suspension by means of an electric motor with a variable-speed transmission capable of any speed from 0 to 260 deg/min. The output signal from the Condon torsion balance was used to drive the y axis of an x-y recorder. A voltage proportional to the angular position of the magnetic field drove the x axis. The resulting rotation diagrams map the torque over a full 360°. By orienting the sphere with different axes parallel to the suspension, the torque could be investigated over 4π sr.

Considerable difficulty is encountered in orienting potassium crystals by x-ray methods because of thermal diffuse scattering and, as a result, the samples used initially were determined to be single crystals by optical means but did not have the crystallographic axes identified. Fifteen crystals were oriented using the transmission Laue x-ray method.[10] The samples used had diameters which ranged from 2 to 7 mm and were prepared by the method described by Lee[11] from stock of 99.95% purity obtained from MSA Research Corporation, Callery, Pa. Residual resistance ratios were measured by the eddy-current decay method[12] and varied from 2400 to 3800.

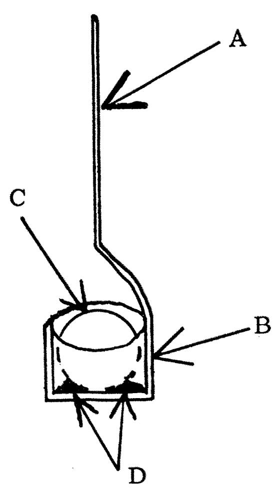

FIG. 4. Sample holder: *A*, 1-mm-diam quartz rod; *B*, quartz cup; *C*, spherical sample; *D*, Vaseline or mineral oil.

Figure 1 presents a rotation diagram for single crystal No. 58 with a ⟨111⟩ axis parallel to the suspension and a ⟨110⟩ axis in the horizontal plane at $\theta = 90°$ on the arbitrary magnet angle scale (insert). For this alignment there is a ⟨110⟩ axis in the horizontal plane at $\theta = 150°$ which is within 5° of the low-field minimum in the torque. In Fig. 2 for sample No. 68 there is a ⟨100⟩ axis parallel to the suspension and a ⟨110⟩ axis at $\theta = 135°$. The low-field minimum occurs at $\theta = 130°$. In Fig. 3, for sample No. 61, a ⟨110⟩ axis is parallel to the suspension and another ⟨110⟩ is coplanar with $\vec{\Omega}$ and $\vec{H}$ when $\vec{H}$ is along $\theta = 235°$. The low field minimum occurs at 245°.

In the course of some 200 runs on seventy different samples, seven runs gave little or no anisotropy. This can be attributed either to the preferred axis being parallel to the suspension axis or to the existence of different preferred directions in different parts of the crystal.

A plot of the dependence of the torque on field for the four angular positions corresponding to maxima and minima of Fig. 1 is shown in Fig. 5. The torque increases linearly with field above 4 kOe ($\omega_c\tau \geq 10$) for all four positions. The torque does not saturate at the highest fields available (29 kOe). The torque is also linear in H for the cases of low anisotropy.

When a sample was warmed to room temperature and then recooled to 4.2 K for another run without changing its orientation, the torque of the first run was reproduced exactly. If, however, the sample was rotated in the holder between runs, in general a new ⟨110⟩ preferred direction was established, with its location probably being determined by the stress due to the freezing of

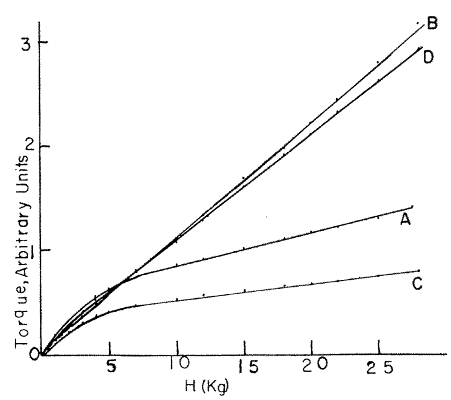

FIG. 5. Torque versus magnetic field for sample No. 58, for which torque versus angle is shown in Fig. 1: *A*, $\theta = 70°$; *B*, $\theta = 110°$; *C*, $\theta = 160°$; *D*, $\theta = 210°$.

the mineral oil or Vaseline.

Numerous checks were made in order to establish that the torque was not due to some experimental artifact. The torque was determined to be caused by eddy currents by its dependence on speed and direction of magnet rotation.[13] The magnitude of the torque had a linear dependence on rotation rate as predicted by the theory of the induced-torque method.[1] Point-by-point static torque gave only the anisotropy due to the quartz sample holder and was a factor of 50 to 100 smaller in magnitude than the eddy-current torque. Torque measurements on single crystals of gallium reproduced the results of Datars[14] without the presence of the twofold anisotropy observed in potassium. Experiments performed on nonspherical samples indicated that the anisotropy was not due to the departure of the sample shape from sphericity. We have also observed oscillations associated with the starting of the rotation of the magnet similar to those reported in aluminum by Delaney and Pippard.[15]

It should be noted that Lass and Pippard[16] performed a similar experiment using the same technique and did not report observing any anisotropy on the *one* single crystal they used. This could be due, in part, to the fact that they employed an averaging method in obtaining their data.[17] They also did not report observing the oscillations associated with the starting of the rotation,[15] which we see even in cases of no anisotropy. If the averaging method they used eliminated these oscillations, it may well have eliminated the anisotropy, too. In addition, since the mechanism producing the anisotropy is stress dependent, then the *one* single crystal for which Lass and Pippard[16] report data may have been strain-free or it may have been mounted so that the preferred ⟨110⟩ axis was parallel to the suspension as conjectured for the seven cases of zero anistropy observed in our investigations.

The most striking feature of these data is the presence of twofold anisotropy for all orientations of a crystal which is normally considered to possess cubic symmetry. This implies the existence of a preferred direction in the crystal.

The large anisotropy also implies a highly anisotropic resistivity tensor which means that the full tensor expression for the torque derived by Visscher and Falicov[1] must be used rather than the simplified expression obtained by assuming that there is no longitudinal-transverse mixing.[3] For this reason we have presented only plots of torque versus angle and of torque versus field, rather than of resistivity versus angle of field.

We are continuing our investigation using a sample rotator of the type described by Moss and Datars[13] which allows rotation of the sample with respect to the field at 4.2 K. This will permit determining the location of the preferred direction for different orientations of a given sample without changing its stress state. To determine whether the observed anisotropy is due to an inherent anisotropy of the Fermi surface or to an anisotropic scattering mechanism we plan to look at the symmetry of the de Haas–van Alphen effect using the torsion balance at 50 kOe. In addition, we have begun similar experiments on sodium, and preliminary results show that the same anisotropy occurs in sodium but higher values of the magnetic field are required to produce the four-peak form.

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Theory of Induced-Torque Anomalies in Potassium

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The bizarre torque anisotropies observed by Schaefer and Marcus in single-crystal spheres of potassium show that the Fermi surface is neither simply connected nor of cubic symmetry. We show further that a charge-density–wave structure provides a complete account of the observed behavior.

According to conventional views the induced torque exerted on a sphere of K by a rotating magnetic field H should be[1]

$$N_y = \frac{2\pi R^5 \Omega n^2 e^2 \rho_0}{15c^2}\,\frac{(\omega_c \tau)^2}{1+(\frac{1}{2}\omega_c \tau)^2}, \tag{1}$$

where R is the radius of the sphere; Ω, the rotation speed of the magnet; n, the electron density; ω_c, the cyclotron frequency; τ, the relaxation time; and ρ_0, the resistivity. This result is independent of the crystal orientation and approaches a limiting value at high fields. The extraordinary torque anisotropies observed by Schaefer and Marcus[2] in K (and Na) require attention.

It is possible to obtain torque anisotropies by postulating a parallel array of dislocations, which would cause anisotropy of the resistivity. The torque is then given by the general expression[3]

$$N_y = \frac{4\pi(15c^2)^{-1}R^5 H^2 \Omega \lambda}{\lambda(\rho_{yy}+\rho_{zz}) - (\rho_{xx}+\rho_{zz})\rho_{xz}\rho_{zx} - (\rho_{xx}+\rho_{yy})\rho_{xy}\rho_{yx} - \rho_{xy}\rho_{yz}\rho_{zx} - \rho_{xz}\rho_{zy}\rho_{yx}}, \tag{2}$$

where

$$\lambda \equiv (\rho_{xx}+\rho_{zz})(\rho_{xx}+\rho_{yy}) - \rho_{yz}\rho_{zy}.$$

$\{\rho_{ij}\}$ are the elements of the resistivity tensor (including the Hall terms). At low fields the torque would have a twofold anisotropy; but at high fields it would again saturate and the anisotropy would vanish. The latter behavior can be understood: In the high-field limit, induced currents circulate in a horizontal plane,[1] so the azimuthal angle of the tensor axis is irrelevant. This holds as long as all cyclotron orbits are closed.[3] Therefore a dislocation model cannot explain the high-field torque anisotropy. Even at low fields the observed anisotropy is several orders of magnitude larger than what one might reasonably estimate.

Only open orbits can cause a high-field torque which does not saturate.[3] Since a nonsaturating four-peaked pattern is observed[2] even when a threefold axis of K is vertical, one must conclude that the Fermi surface is neither simply connected nor of cubic symmetry.

A charge-density–wave (CDW) modulation[4] of the conduction electrons has provided successful explanations of other alkali-metal anomalies. These have been summarized.[5] CDW properties which have been previously supposed[5] and are pertinent here are the following: (a) The positive ions are displaced from their bcc lattice sites $\{\vec{L}\}$ by $\vec{A}\sin(\vec{Q}\cdot\vec{L})$. (b) These displacements cause a mixing of the CDW and reciprocal-lattice periodicities, and give rise to "heterodyne" gaps with periodicity $\vec{Q}' = 2\pi\vec{G} \pm \vec{Q}$. (c) The $\langle 110\rangle$ crystal axes are the preferred $\vec{Q}$ directions. (d) $\vec{Q}$ tends to align parallel to $\vec{H}$ at 4°K in a stress-free crystal. (e) Alignment of $\vec{Q}$ is inhibited by elastic stress. (f) There are *low-frequency* vibrational modes—"phasons"—which are describable as a phase modulation of the CDW, and which cause the *local* direction of $\vec{Q}$ to fluctuate slightly about the [110] axis chosen in each $\vec{Q}$ domain. (g) Phason and umklapp scattering between the two conical points of the distorted Fermi surface cause significant zero-field aniso-