Fusion Reactions in Self-Colliding Orbits*†

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A principle for a small nonplasma fusion device is proposed. Energetic ions are injected through the center of a Precetron-type magnetic field. An organized non-Maxwellian mixture ("migma") of particles having a central high-density region results. The collisions are predominantly head-on and Coulomb scattering effects significantly reduced. It is shown that the device can become critical, although, in the examples considered here, only at microwatt levels.

In the course of the work of our study group on true π^+-π^- and μ^+-μ^- interactions based upon the idea of "self-colliding orbits"[1] ("Precetron"), we observed that the Precetron concept contains properties which suggest a principle for a *nonplasma* fusion device that can be made critical within the bounds of present technology, although only at microwatt levels in the cases considered here. In this Letter we discuss these features and outline the conditions required to achieve criticality for the $d+t$ fusion reaction.

The self-colliding particle orbits are quasicircles of radius a passing through the center of symmetry of a magnetic field of radius $R\sim 2a$, whose strength decreases with increasing radius and exerts both vertical and horizontal "weak" focusing. The field is of the form $B_z = B_0[1-k(r/R)^2+2k(z/R)^2]$, where the constant k is the "field index" and B_0 the field at the center. The orbits are stable both in z and r. The precession and revolution periods are related by $\tau_p=(4/k)\tau_R$. An example of an orbit configuration for $k=0.4$ is shown in Fig. 1.

We shall refer to the organized mixture of precessing, self-colliding orbits which has a high particle density at the center as a *migma* (a Greek word for mixture or jumble), and the device containing it as a *migmatron*. Any similarity between the migmatron and plasma fusion devices hitherto proposed is only a superficial one. The following properties of a migma are important:

(1) Injection is designed such that all particle orbits pass through the field center. The particle density is therefore sharply peaked at this point in the $r\theta$ plane (Fig. 2). In space, it is a column of height Δz. By contrast, the density of the plasma in a "mirror machine" is isotropic.

(2) Head-on collisions of particles of the same sign of charge are favored in a migma, in spite of the fact that their orbits precess in the same direction. All crossing angles of particle orbits, α, are allowed at the center [Fig. 1(a)]. The luminosity $L=\int v_{12}\rho_1\rho_2 d^3r$ is high only in the central zone where the particles have large relative velocities v_{12} and the density is high. Since in this region the collisions are predominantly head-on, the effective kinetic energy T_{12} and hence the fusion rate are increased while the Coulomb scattering effects are decreased. Typically, for a d-t migma 66% of the reactions occur in 8.5% of the radius (Fig. 2). The increase in the multiple Coulomb scattering cross section towards the periphery is more than compensated by the low luminosity.

(3) The energy distribution in the migma is far from Maxwellian and it can be "tailored." It is single valued in the laboratory system, but in the rest frame of a single particle 80% of interacting particle pairs have T_{12} greater than the injection

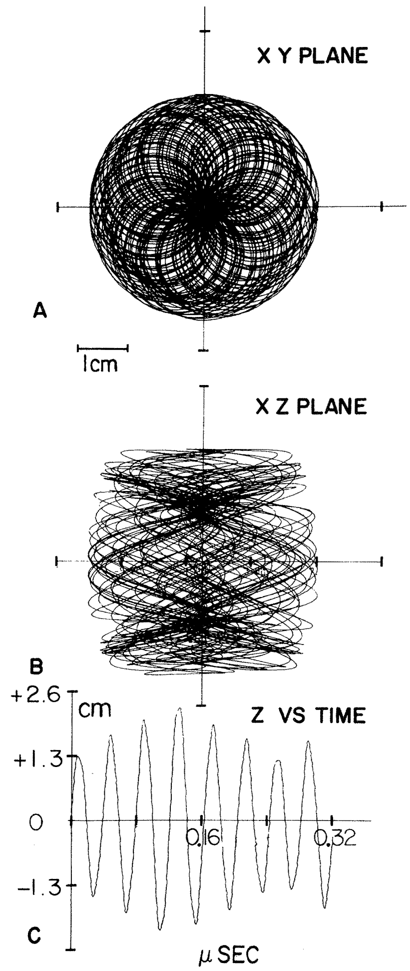

FIG. 1. (a) Migma. Computer plot of the precessing orbit of one 100-keV deuteron in a Precetron-type field with $k=0.4$ and $B_0=60$ kG; the gyroradius is ~1 cm. The particle was "injected" at the center at an angle of $\theta_z=20°$ to the horizontal plane. If continuous injection of particles had been maintained from $t=0$, the resulting configuration of the migma would also look like this. (b) The projection on the xz plane of the orbit traced in (a). It is seen that the maxima in z correspond to large radii. (c) Vertical oscillations of the first 17 precessions in (a) and (b).

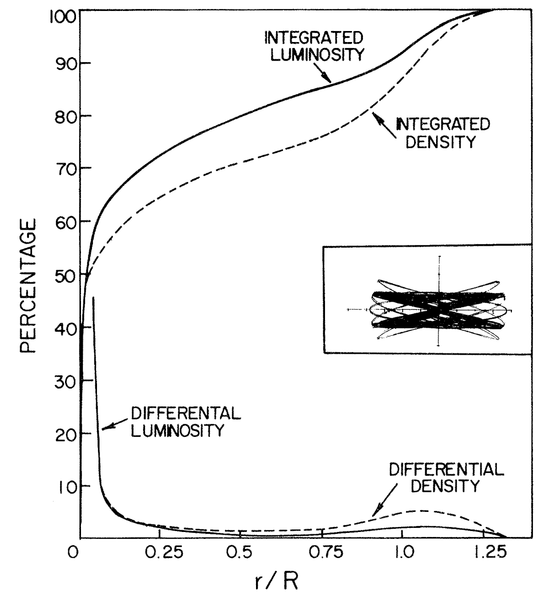

FIG. 2. Migma density ρ and luminosity as functions of the radius assuming an energy spread of 4%; R is the "radius" of the magnetic field. *Inset:* Migma profile (projection of orbits on xz plane) for two ions injected at the center at 10° and 20°, weighted by the relative Coulomb cross section.

energy T_1. For d and t of equal momentum, the effective kinetic energy of a deuteron with injection energy T_1 as seen by a triton is given by $T_{12} \simeq \frac{1}{9}T_1(13-12\cos\alpha)$, while for equal-velocity particles, $T_{12}=2T_1(1-\cos\alpha)$. Since α has a unique value at each r, T_{12} is a function of the position r.

(4) Instabilities that may develop in a migma will be more similar to those in accelerators than to those in plasmas. To avoid the negative-mass or other collective instabilities, it is believed that the spatial extent of the migma must be less than or of the order of the Debye length L of the ions[2]: $L=2.35\times10^4(T/\rho)^{1/2}$ cm, where T is the ion kinetic energy and ρ the ion density. For $T_1 \sim 100$ keV and $\rho \sim 10^{10}$ cm^{-3}, $L \simeq 2.35$ cm.

(5) Because of property (4), a migma need not be neutral. If neutralization is done, it is applied only locally [See Sect. (6)]. The maximum number of positively charged ions of mass m (amu) and kinetic energy T_1 (keV) that can be stored as a migma will be given by the space-charge limit,

$$N=5.17\times10^9 k T_1^{3/2}\sqrt{m}/B\,(\text{kG}). \qquad (1)$$

N is about 5×10^{10} and 1.5×10^{12} for 100-keV and 1-MeV deuterons, respectively, in a field of 60 kG with $k=0.4$. At these magnetic fields the effective volume of the migma, V_{eff} (the central high-density region) is ~0.1 cm^3 and numerically $\rho_i \sim 3N_i$. For example, at 100 keV the rate is $I \simeq \langle\sigma v\rangle 0.9N^2$ which gives 6×10^6 fusions per second. (We assume $N_1=N_2=\frac{1}{2}N$.)

Even at these low densities criticality can be achieved in the migmatron; admittedly the net energy output under these conditions would only be 10 to 100 μW but this encourages us to attempt to increase the space-charge limit by order-of-magnitude steps through various techniques.

(6) To raise the space-charge limit we propose

to inject a pencil beam of monoenergetic (~keV) electrons along the magnetic field lines into the central high-density zone. Neutralization occurs only over some small fraction ($\sim\frac{1}{60}$) of each ion orbit. This coupled to the facts that, unlike in a plasma, the electrons are faster than ions and move perpendicular to them should increase the relaxation time for energy transfer from the ions to the electrons by a large factor over the plasma value, the latter being $\sim 10^5$ sec for 5-keV electrons and $\rho_e \sim 10^{10}$ cm^{-3}. The corresponding d-t reaction time when $\rho_i \sim 10^{10}$ at 100 keV (or d-d at 1.5 MeV) is $\tau_R \sim 3\times 10^4$ sec.

(7) The gyroradius of migma orbits, a, is large by comparison to that in most of the plasma devices, and the particles experience significant focusing. This, in turn, reduces the losses due to the multiple scattering (to be referred to as MS) by a large factor. For example, the lateral "blow-up" of the beam, Δ^2, normally proportional to the third power of the distance traversed, X^3, becomes proportional to $X\lambda\!\!\!^{-2}$ in the focusing medium.[3] $\lambda\!\!\!^{-}$ is the wavelength of the horizontal ($\lambda\!\!\!^{-}_x$) or vertical ($\lambda\!\!\!^{-}_z$) oscillations; $\lambda\!\!\!^{-}_x = a$ and $\lambda\!\!\!^{-}_z = (k^{-1}-1)^{1/2}\lambda\!\!\!^{-}_x$, with k the field index. The interaction parameter for the horizontal beam spread Δ^2 (cm^2) due to MS is given by

$$\langle\sigma(\geq\Delta^2)v_{12}\rangle_{\rm MS} \simeq 3.75\times 10^{-11}\lambda\!\!\!^{-}_x{}^2/T_{\rm eff}{}^{3/2}\Delta^2 \text{ cm}^3\text{ sec}^{-1}. \tag{2}$$

$T_{\rm eff}$ is the T_{12} relative kinetic energy at a given radius which also depends on the energy spread of the beam. Typically, in the peripheral zone, $T_{\rm eff} \simeq \frac{1}{3}T_1$. We consider particles which deviated by $\Delta^2 > a^2$ as lost and use Eq. (2) to evaluate the peripheral losses.

(8) The multiple scattering between migma ions in the central zone does not change the orbit density there because Liouville's theorem requires that all particles scattered out at that point return to it, provided their vertical velocity component is below the vertical confinement angle $\theta_z = \cos^{-1}(B/B_{\rm max})^{1/2}$. The loss parameter for the vertical escape of ions due to MS beyond the vertical focusing angle θ_z, $\langle\sigma(\geq\theta_z)v_{12}\rangle_{\rm MS}$, has been calculated for our geometry using Monte Carlo techniques.

(9) A migma need not be composed of gas ions. It could also contain metallic ions such as lithium which, together with stored d's, results in all-charged-particle reaction products: $\text{Li}^6 + d \to 2\text{He}^4 + 22$ MeV, thus implying direct energy conversion.

The interaction rate I in the migma for a process whose cross section is a function of the relative velocity of approach of the particles v_{12} is given by

$$I = \int_{\rm Vol}\sigma(v_{12})v_{12}\rho_1\rho_2 d^3r \simeq \langle\sigma(v_{12})v_{12}\rangle N_1N_2/V_{\rm eff}, \tag{3}$$

where $v_{12}{}^2 = v_1{}^2 + v_2{}^2 - 2v_1v_2\cos\alpha$ and $d^3r = d(r^2)\,d\theta\,dz$. An estimate of I may be obtained from the approximation in Eq. (3), where N_i is the total number (*not* density) of ions in the migma and $V_{\rm eff}$ is defined in section (5).

Let T_1 and T_2 be the injection kinetic energies for each species of particle, W the energy released in the reaction, I_R, $I_{\rm MS}$, and $I_{\rm CT}$ the rates of the fusion reaction, multiple scattering, and charge transfer $[\text{D}^+(\text{fast}) + \text{D}^0 \to \text{D}^0(\text{fast}) + \text{D}^+]$ in the residual gas, respectively. The condition for a critical reaction (energy output $\geq$ energy input) is obtained from $(T_1+T_2+W)I_R \geq (T_1+T_2)I$, the input rate I being balanced by the losses $I = I_R + I_{\rm MS} + I_{\rm CT}$:

$$\frac{W}{T_1+T_2} \geq \frac{\langle\sigma(\geq\theta_z)v_{12}\rangle_{\rm MS}}{\langle\sigma v_{12}\rangle_R} + \frac{\langle\sigma(\geq\Delta^2)v_{12}\rangle_{\rm MS}(\rho^2V)_{\rm PER}}{\langle\sigma v_{12}\rangle_R\rho^2V} + \frac{\langle\sigma v_{12}\rangle_{\rm CT}}{\langle\sigma v_{12}\rangle_R}\frac{3\times 10^{16}P}{N}, \tag{4}$$

where subscript "PER" refers to the peripheral zone, P is the pressure of the residual gas (in Torr), and N is the total number of ions. All three terms of Eq. (4) are plotted in Fig. 3 as functions of the injection energy. The charge-transfer parameter is very large[4]: At $T_d = 100$ MeV, $\langle\sigma v\rangle_{\rm CT} = 5.5\times 10^{-8}$, dropping to 1.4×10^{-11} at $T_d = 1$ MeV. In the same energy region $\langle\sigma v\rangle_R$ drops from 2.4 to 0.8×10^{-15}.

Our conclusion is that there appears to be no possibility of achieving a critical reaction even in a perfect vacuum unless the injection energy exceeds 40 keV and the vertical confinement angle $\theta_z > 30°$. If a residual gas pressure of 10^{-11} Torr is maintained in the migmatron, the d-t fusion reaction in an electron-free migma will be critical at an injection energy $T_d \geq 80$ keV. Because of the fast decrease of the loss processes with T_1, it might be easier to achieve critical d-t reactions with injection energies in the 1–2-MeV range. For d-d reactions the migmatron

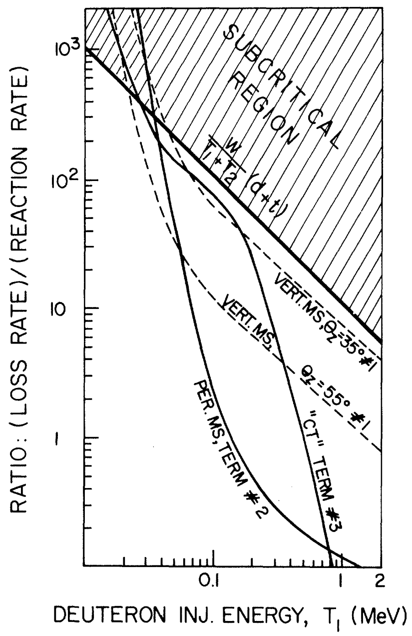

FIG. 3. Criticality condition for *d-t* migma as a function of the deuteron injection energy T_1 for the case of equal momenta, $p_d = p_t$, and equal numbers $N_d = N_t = \frac{1}{2}N$. The sum of the three terms in Eq. (4), labeled No. 1, No. 2, and No. 3, must lie below the $W/(T_1+T_2)$ line for a given T_1. Term No. 1 is plotted for two vertical confinement angles: $\theta_z = 35°$ and 55°. $P = 10^{-11}$ Torr and $N = 5\times 10^{10}$ are assumed.

would be critical for $T_1 \sim 1$ MeV. A Li^6+d migma will be critical at $T_1 \sim 600$ keV, if a vertical confinement angle $\geq 80°$ can be achieved.

The method of injection of the ions from an external accelerator into the migmatron is beyond the scope of this Letter.[5] It is intended to be studied experimentally at Rutgers using a *d-d* model.

While, in its present form, the migmatron cannot produce significant quantities of power, it is amusing that the concept of self-colliding orbits, introduced for high-energy physics studies, seems also to apply to observation of fusion reactions.

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Similarity Arguments and an Inverse-Frequency Noise Spectrum for Electrical Conductors

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A dimensional and similarity argument analogous to that used in the description of the flux of turbulent energy in fluid flow is applied to the problem of energy dissipation by Joule heating in electrical conductors. A spectral density in the space domain is obtained, and is used to derive a spectral density in the time domain by means of a dispersion relation. The spectral density in the time domain is proportional to the inverse frequency.

Low-frequency electrical noise usually has a component which is proportional to the square of the driving current and which has a spectral density varying approximately as the inverse of the frequency (viz., $1/f$ noise). It is fair to say that there is no completely satisfactory explanation of the specific source or sources of this noise.[1] However, recently there has been a significant