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Observations of Moving Self-Foci in Sapphire*

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Observations of the time development of damage tracks in sapphire are explained in terms of a backward-moving self-focused spot. The detailed characteristics of the dynamics of the track evolution depend on the temporal shape of the incident laser pulse. Both qualitative and semiquantitative features of the observed phenomena are explained using a theory which shows that the results are consistent with a combination of both electrostrictive and electronic self-focusing mechanisms.

The importance of self-focusing in enhancing optical intensities in solids to material-rending levels has long been recognized.[1,2] Unfortunately, it has been difficult to study the self-focusing process quantitatively in solid materials because, at the very high incident power levels required, the source-beam quality is ofter poor and not reproducible. In this note, we present experimental data on the space and time evolution of damage tracks in sapphire, induced with a high-quality, reproducible, ruby laser-amplifier source.[3] We show that these tracks may be understood qualitatively, and to some extent quantitatively, as forming at the moving focus of the self-focusing source beam.[4] Experiments supporting the existence of a moving self-focus in nonlinear liquids have been reported previously.[5] It is hoped that this unambiguous evidence of a moving self-focus in sapphire will help clarify the interpretation of other "filamentary" phenomena observed concomitantly with self-focusing in other media.

The experimental arrangement has been described in detail elsewhere.[3] All experiments were performed using as a source a mode-controlled, Q-switched ruby oscillator and amplifier. The far-field beam profile was measured to be Gaussian down to 8% of the peak using a modified multiple-lens camera technique.[6] The light was focused into the sample using a lens ($f = 19$ cm) designed for minimum spherical aberration. The sapphire samples were typically 3-in.-long by 0.25-in.-square bars. Fast streak photographs were taken using an STL image-converter camera operating in the streaking mode. In most experiments a Corning 4-94 filter was placed between the camera and the sample which blocked light at 6943 Å, passing only the blue-green portion of the broad-band light from the self-luminous damage track. In addition a portion of the main beam was allowed to enter the camera directly, giving a marker streak relating the time of formation of a particular point on the damage track to the peak of the incident pulse.

Figure 1(a) shows a typical damage track. It has a "head" of relatively massive damage followed by a tapering tail which always ends at or

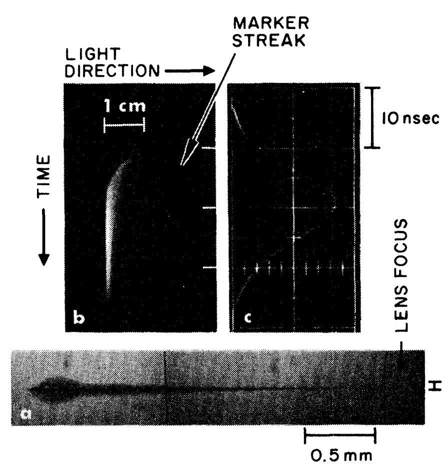

FIG. 1. Typical examples of (a) damage filament, (b) streak photograph, and (c) oscilloscope trace for a temporally smooth incident pulse.

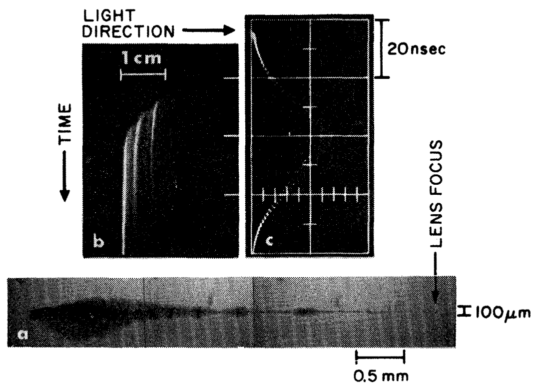

FIG. 2. Typical example of (a) damage filament, (b) streak photograph, and (c) oscilloscope trace for a modulated (~ 750-MHz) incident pulse.

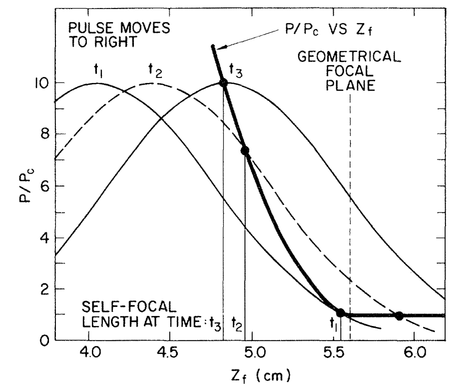

FIG. 3. Construction yielding trajectory of self-focus for instantaneous nonlinear response. For this figure $ka^2 = 220$ cm and $R = -5.6$ cm. The e^{-1} total pulse width of 0.12 nsec was chosen for illustrative purpose only.

prior to the low-intensity focal plane. Only one track is formed per pulse, and its appearance and position are highly reproducible. If the laser pulse is not temporally smooth, a more complicated track is induced as shown in Fig. 2(a). Corresponding traces of the temporal pulse shape are also shown in Figs. 1 and 2.

The evolution of these tracks can be inferred from streak photography if it is assumed that the processes causing intense self-luminescence also lead to local damage. The streak photographs in Figs. 1 and 2 show clearly that the track begins at the tail which is located at or near the low-intensity beam waist and propagates upstream terminating at the head. We also see that the damage track reaches its full length at the peak of the laser pulse. The time between initiation of the bright streaks in Fig. 2(c), for a modulated incident pulse, coincides with the modulation period of the pulse. The positions of the streaks coincide with those of the more heavily damaged regions of the track.

To correlate these facts with the self-focusing theory, let us first assume that the induced refractive-index change δn responds instantly to local changes in optical intensity. Then, above the critical power for self-focusing, a self-focus always forms at a unique point z_f. The position of the self-focus at any time t may be obtained from the intersection of the pulse-shape curve $P(z_f - ct)$ and the curve of P vs z_f.[7] The latter curve has been obtained to a very high degree of accuracy from numerical solutions of the nonlinear wave equation for a variety of incident beam shapes and phase profiles.[8]

Figure 3, which shows this construction, employs a curve of P vs z_f which was plotted from the numerically determined formula[8] (for $P > P_c$)

$$[(P/P_c)^{1/2} - 0.858]^2 = 0.0202 + 0.136\,[ka_0^2/z_f(\infty)]^2 \quad (1)$$

and[9]

$$z_f^{-1}(R) = z_f^{-1}(\infty) - R^{-1}. \quad (2)$$

Here, for an incident Gaussian equiphase beam of wavelength 0.69 μm,

$$P_c = 1.7\times10^{-13}/n_2 \text{ MW}; \quad (3)$$

a_0 is the e^{-1} radius of the intensity profile at $z = 0$; k is the wave vector in the crystal; and $z_f(R)$ is the distance to the self-focus when the incident beam has phase curvature R ($R<0$ for a converging beam). The quantity n_2, the nonlinear index in Gaussian units, is defined through

$$\delta n = n_2\langle E^2\rangle, \quad (4)$$

the brackets denoting time average. Equations (1) and (2) are essentially exact for Gaussian incident beams and spherical phase surfaces as long as δn has the form (4).[8,9]

A glance at Fig. 3 shows that the self-focus occurs at or just prior to the lens focus, and splits into two foci, one of which travels downstream and one upstream. The upstream focus reaches its minimum extent as the pulse maximum passes, and therefore dwells at this position for a while. This presumably allows massive damage to occur at the head of the track, which scatters subsequent light in the trailing portion of the pulse. The narrowness of the damage tail near its downstream end is presumably due to the rapid motion of the focal spot in this region.

According to this picture, the "discrete" damaged regions associated with incident-beam modulation are created during the passage of local maxima on the leading edge of the pulse. Each consecutive peak has slightly more power than the preceding one and therefore causes a self-focus which dwells at slightly smaller z. Thus what appears to be evidence of multiple or repeated focusing could be simply an artifact of temporal spiking on the input pulse which causes *one* focus to pause occasionally as it sweeps upstream. No foci are created during the passage of the trailing edge because the near-axis rays there, which are primarily responsible for self-focusing, are scattered in all directions by the damage caused at the head by the pulse peak.

These assertions are strongly supported by a plot of $P^{1/2}$ vs z_f^{-1} obtained directly from the streak and beam diagnostic photographs. For powers greater than about $2P_{\mathrm{cr}}$ this curve should be a straight line [the asymptote of (1)]. Figure 4 shows that this is nearly so.

The evident curvature of the lines in Fig. 4, which exceeds that predicted by (1), and their dependence upon pulse duration and peak power, can be understood by invoking the slow response of the index change δn which was ignored in the analysis above. Of the various possible mechanisms for δn, those arising from nonlinearity of electronic response[10] and "libration"[11] are essentially instantaneous.

The largest slow mechanism for δn is electrostriction, which leads to an effective n_2 which decreases as the dimensionless quantity $x = a/u\tau$ increases.[12] Here u is the velocity of radial compression waves, and τ and a are usually taken to be the incident-pulse length and beam radius. But the index change responsible for a self-focus at z_f can only be induced by the portion of the pulse which has passed z_f prior to the focus formation. For focal points formed on the leading edge of the pulse, this suggests that we take τ to be a decreasing function of z_f. Similar reasoning suggests, for a converging incident beam, that a also be replaced by a decreasing function of z_f: A self-focus formed prior to the "lens" focus cannot take advantage of the small beam radius at the low-intensity focal plane.

For our experiments in sapphire, the electrostrictive response time a/u varies from about 40 nsec at $z = 0$ to about 1 nsec at $z = R$. The streak photographs indicate that a focus may be formed near $z = R$ by the time the instantaneous power has reached about half its peak value, so

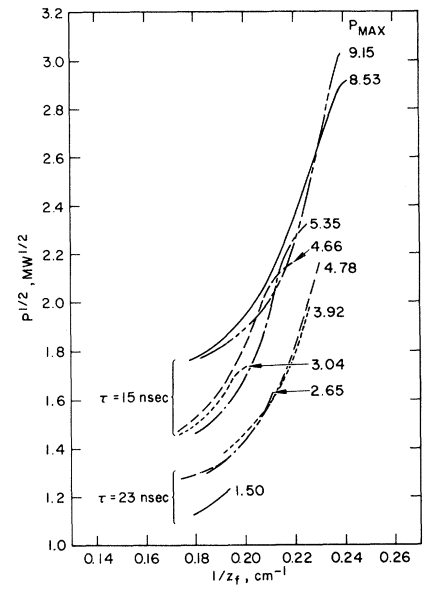

FIG. 4. Plot of $P^{1/2}$ vs $1/z_f$ from data taken from streak and oscilloscope photograph for different incident peak powers and pulse widths.

τ varies roughly between 5 and 15 nsec. Thus the entire transient regime of electrostrictive response is involved during the transit of the self-focus.

The dependence of the effective n_2 on z_f can be taken into account in a crude way by replacing P_c^{-1} in (1) by $P_{c_1}^{-1} + [P_{c_2} f(x)]^{-1}$, where f is an increasing function of x, P_{c_1} is the critical power due to "instantaneous" mechanisms, and fP_{c_2} is that due to electrostriction. As the fractional change in a during the development of a damage track is generally greater than that in τ, we expect f to be a decreasing function of z_f, causing positive curvature in a plot of $P^{1/2}$ vs z_f^{-1} according to Eq. (1). This is consistent with the data shown in Fig. 4.

Since the first self-focus always occurs at the low-intensity focal plane, a is the same for all curves at $z_f = [R]$. This leads us to expect that x, and thus f and the effective critical power at this point, will increase as the pulse duration is reduced, other parameters being equal. This accounts for the difference in vertical position of the two groups of curves in Fig. 4. A similar increase of the threshold power is anticipated when the peak power is increased at constant pulse duration. In this case the time τ to reach critical power is reduced and x again increases.

The curvature at the high-power ends of the curves in Fig. 4 is very sensitive to the time which is chosen on the pulse profile to coincide with the time at which the track reaches minimum z. Because of the slow response of the electrostrictive mechanism, the maximum upstream excursion of the track is expected to lag the pulse peak by an amount somewhat less than the response time. We did not attempt to take this into account in plotting the data, and this explains the negative curvature at the peak-power ends of several of the curves in Fig. 4. The adjustment required to remove this curvature is of the order of nanoseconds. Its sign is consistent with this explanation.

Using Eqs. (1)–(3) and the preceding qualitative theory of the influence of electrostriction, we can infer a range of values of critical powers and n_2's from Fig. 4. A slope on Fig. 4 is approximately equal to $0.369ka_0^2P_c^{1/2}$, where P_c is an effective critical power combining electronic and electrostrictive effects. The range of critical powers thus inferred is 180 kW to 5.4 MW, corresponding to values of n_2, given by (3), of 9.5×10^{-13} and 0.31×10^{-13} esu, respectively. These values are consistent at one extreme with the predictions of electrostrictive self-focusing theory for sapphire,[12] and at the other with values of n_2 arising from electronic nonlinearities in other materials (glass).[13]

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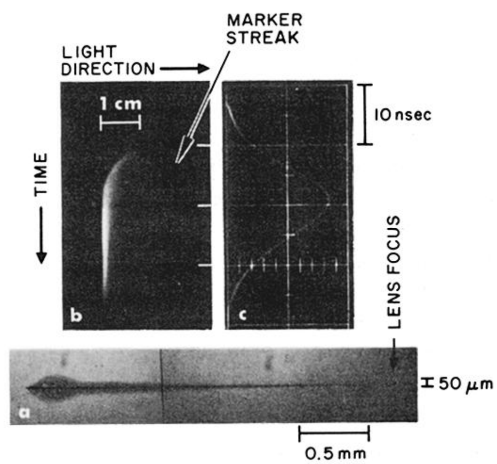

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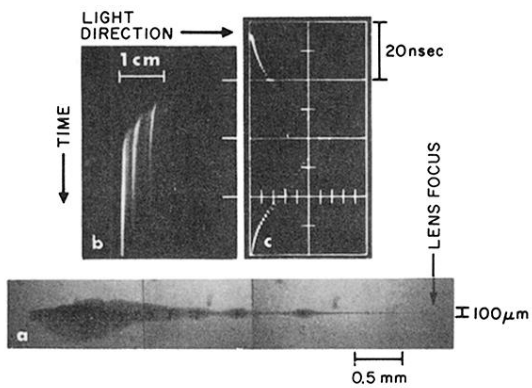

FIG. 2. Typical example of (a) damage filament, (b) streak photograph, and (c) oscilloscope trace for a modulated (~ 750-MHz) incident pulse.