i.e., it corresponds to the Coulomb interaction of three particles of charge e_1, e_2, and e_3. It is then easily seen that the boundedness conditions (10) yield

$$e_1 e_3 < 0 \tag{22a}$$

and

$$|e_1| > 2|e_3| \text{ or } |e_3| > 2|e_1|. \tag{22b}$$

Thus, if these conditions are satisfied, the ground-state of the system is the three-body bound state described by the function ψ of Eq. (3), and the corresponding energy is

$$E = -\tfrac{1}{3} m \hbar^{-2} \alpha^{-2} e_1^2 e_3^2 \frac{(e_1^2 + e_1 e_3 + e_3^2)}{(e_1 + e_3)^2}. \tag{23}$$

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Two Experimental Inputs to the λ-Point Helium Paradox*

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Interpretations of the "λ-point paradox" are narrowed significantly by recent experimental findings. Penney's proposal that the paradox originates in effects of room-temperature radiation falling on the disk is disproved. Further, by rotating the disk through the helium rather than rotating the liquid past the disk, the paradox is found to be dependent only on relative rotation. This invariance to absolute rotation of helium evidently eliminates any role of the rotational properties *per se* of the liquid.

Although by no means offering a complete resolution to the λ-point rotation paradox, the two experimental results reported here do greatly narrow the scope of the problem. One should be reminded that the rotation paradox[1,2] concerns the observed temperature dependence of torque exerted by rotating liquid helium on a Rayleigh disk, in particular the decrease of this torque toward zero as the temperature is increased toward the λ point.

One of the present findings eliminates the effects of radiation as recently proposed to explain the paradox. The other reveals that λ-point phenomenon is in fact rooted in other than the rotation properties *per se* of liquid helium.

(1) *Penney's radiation hypothesis.*—Recently Penney[3] suggested that the rotation paradox might be attributed to the action of room-temperature radiation falling upon the disk. The heating generated over the surface of the disk by such radiation would set up internal convection currents between normal fluid and superfluid, and these in turn would modify the individual fluid flow patterns about the disk. According to Penney's calculations, the process would result in decreased net torque exerted upon the disk similar in behavior to the observed effect. This includes the total disappearance of torque at the λ point.

Perhaps the least convincing aspect of the argument is the apparent strictness placed upon the amount of radiation incident upon the disk. Rather than comprising a "saturation" effect (and thus some minimum threshhold), the process requires a *specific level* of radiation to suppress the torque at the λ point. Immediately the explanation hangs upon the fortuitous values of room-temperature radiation balanced against other factors.

Clearly a test by direct experiment was required. A helium-temperature "radiation envelope" was provided for the test region by painting the outer surface of the original precision glass rotor and the supporting glass tube with Aquadag. Thus, except for two small "windows" left in the Aquadag painting as observation apertures, the rotating liquid-helium sample and suspended disk probe were completely shielded from all but liquid-helium-temperature radiation. Such a geometry permitted the original experimental method of observing disk behavior by measuring deflections of a beam of light reflected from the disk surface. (See Ref. 2, Fig. 1 for the basic system to which Aquadag shielding was applied.)

One small circular hole was provided in the rotating container for admitting the light beam and one horizontally elongated slot for observing disk deflections, so that a single measurement could be taken per rotation. To protect against leakage of external radiation into the test region through these moving apertures, a stationary shield of cylindrical shape was also placed about the blackened rotor. It was also black (matte) and was provided with a matching pair of apertures (one circular and one elongated); external radiation could enter the apertures only during each brief period of coincidence per rotation. Measurements were taken in nearly complete darkness and the only visible light present was the brief (about 1 sec duration) and dimly visible beam activated during periods of aperture coincidence.

The results are presented in Fig. 1 where observed deflections are plotted (circles) versus temperature. Deflections are normalized to unity at the low-temperature limit. The familiar decrease in observed disk deflection with increase in temperature toward the λ point is clearly evident. We must conclude on the basis of these measurements that Penney's radiation hypothesis does not explain the λ-point paradox.

(2) *Principle of relative rotation.*—Probably more significant are the results of measuring disk deflections when the role of rotating and stationary components of the experiment are interchanged. If the paradox is attributable to some property of rotating liquid helium—such as quantized vorticity, for example—then a different response might be expected when the disk rather than the helium performs the rotation. This was accomplished experimentally by rotating the entire Dewar cap of the earlier system in the opposite direction at the same (2 rpm) rate. Such a rotational retrogression imposed upon the system left the sample chamber—and thus the test helium sample—stationary, but carried the disk probe about on a circular path through the sample. The same radiation shielding was retained (the apertures in the Aquadag coating were now stationary), and a high-speed light flash (10 μsec) system permitted one observation per rotation of the disk assembly. As before, deflections of the reflected light beam indicated the disk response.

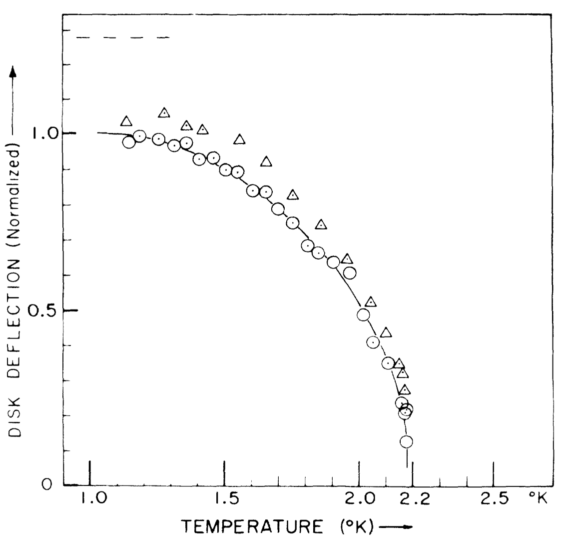

FIG. 1. Deflection produced on a Rayleigh disk probe in absence of room-temperature radiation plotted versus temperature. Circles and solid curve indicate results for helium rotating past a stationary disk (data normalized to low-temperature limit). Triangles represent torque for the disk rotating through stationary helium. Dashed line indicates theoretical value obtained by application of König formula (Ref. 4).

The results of these observations are plotted (triangles) in Fig. 1. Note that the λ-point paradox persists even though the liquid is not rotating and the only motion involves that of the disk through an otherwise stationary fluid background. Slightly larger values of deflection are observed for the disk rotating through stationary liquid, but this difference is not considered significant to the present argument.

Evidently the property of liquid helium responsible for the λ-point paradox does not originate in the rotational characteristics *per se* of the liquid. In particular, a role of quantized vorticity appears to be ruled out.

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