Experimental Test of the Theory of Muonic Atoms*†

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We have measured muonic x rays in the energy region 150 to 440 keV in nine elements with an absolute precision of 15 to 21 eV for transitions with small nuclear effects. Calculated transition energies were found to be consistently larger than those measured by an amount that varied from 15 ± 16 eV at 157 keV to 137 ± 22 eV at 438 keV. For these transitions, the principal correction to the Dirac energy is the vacuum polarization. The discrepancy, however, lies outside the expected validity of quantum-electrodynamic calculations and we are unable, at present, to offer an explanation for this effect.

In the past, muonic x rays have been used principally in the study of the electromagnetic structure of nuclei. Measurements have been made primarily on the $2p$-$1s$ and other low-lying transitions because these are most sensitive to nuclear effects. On the other hand, the higher transitions which are little affected by the nucleus have not been studied as extensively so far. In our recent work at the Chicago cyclotron a special effort was made to measure such transitions with high accuracy and to test the extent to which such measurements could be accounted for by the existing theory of such hydrogen-like atoms, namely, the Dirac equation together with the applicable atomic, vacuum-polarization, and self-energy corrections.[1]

In this energy range the principal correction to the Dirac energy is the e^+-e^- vacuum polarization because this effect extends over a distance of the order of the electron Compton wavelength, and many muonic orbits lie well within this region. Self-energy corrections are small due to the large mass of the muon. The early precise measurements of the $3d$-$2p$ transition energy in muonic phosphorus[2,3] were interpreted[4] to show that the vacuum polarization effect in muonic phosphorus was given correctly by the theory to within 4%. Recent measurements[5] in muonic Pb verified the theory to a similar accuracy. Still more recently, Backenstoss *et al.*[6] claim to have verified the vacuum polarization effect in muonic Pb and Bi to within 1%.

The following muonic transitions in the energy region between 150 and 440 keV were measured: Ca, Ti, and Fe (the 3 to 2 transitions); Sr, Ag, Cd, Sn, and Ba (the 4 to 3 transitions); and Ba and Pb (the 5 to 4 transitions). We used a $\frac{1}{2}$-cm^3 planar Ge(Li) detector with a full width at half-maximum resolution of 600 eV at 136 keV and 1000 eV at 468 keV. The following γ-ray sources were used as standards[7]: Co^{57} (136.471 ± 0.010), Ce^{139} (165.853 ± 0.007), Th^{228} (238.623 ± 0.007), Ir^{192} (295.949 ± 0.007, 308.445 ± 0.007, 316.497 ± 0.007), Au^{198} (411.794 ± 0.008), and Ir^{192} (468.056 ± 0.013), all energies given in keV. We measured x rays from two targets simultaneously. The experiment utilized a PDP-9 computer on line. Prompt x rays, delayed γ rays, calibration sources, and the linearity spectrum from a sliding pulser using a digital-to-analog converter (DAC) were measured concurrently and stored separately. The spectrometer was digitally gain and zero stabilized on source peaks outside the range of the DAC. Peaks were fitted and linearity corrections made as described in McKee *et al.*[8] With these corrections and a correction for an angular effect discussed below we obtained a straight-line fit to the eight calibration lines with an rms deviation of 4 eV.

Considerable care was taken to avoid rate effects. To simulate the time structure of the muonic x rays we accepted only those source events which occurred within a wide time gate in accidental coincidence with the muon telescope. A rate inspector was used to insure that data was accepted only when the beam had more than a certain intensity. Moreover, the singles rate in the Ge(Li) detector was kept low, about 2000 counts/sec. A Ge pulse was vetoed if preceded by another pulse within 50 μsec or if followed by another within 4 μsec.

Systematic shifts between x-ray and source spectra were checked in various ways. We looked for the positions of some of the more intense source peaks which appeared in the x-ray spectrum by accidental coincidence. The relative shift between these two spectra for the Co^{57} (122 keV) amounted to 3 ± 8 eV. In a similar experiment this shift for the Au^{198} (412 keV) line amounted to $+15 \pm 30$ eV.

A possible energy shift due to small-angle Compton scattering was found to be negligibly small. On the other hand, a correction varying between 3.2 ± 1.6 eV at 200 keV and 7.0 ± 3.5 eV at 440 keV, as determined by a separate measurement, was applied to take into account the energy shift[9] due to the direction of motion of the photoelectrons relative to the field in the planar diode we used.

Peak fitting was done mainly using the polynomial line shape as described in Ref. 8. Gaussian and Lorentzian shapes reproduced the energy values to within 10 eV. The x-ray lines were found to be somewhat broader than the source lines. Assuming that the instrumental response is Gaussian and determined by the source lines (which have negligible natural widths) and that the intrinsic x-ray line shape is Lorentzian, we obtained values for the natural widths in reasonable agreement with theory.

Account was taken of interfering lines of different l values in determining the position of a line by computing their energies and relative intensities[10], and including them in the fit. At worst, the error in the energy of a line, such as $5g_{7/2} \to 4f_{5/2}$ in lead, amounted to ±7 eV due to the uncertainty in the cascade intensity of the weak interfering $5f_{7/2} \to 4d_{5/2}$ transition.

The measured transition energies are tabulated in Table I. The errors shown were compounded from a statistical error, typically ±10 eV, a calibration error of ±10 eV, corresponding to the precision with which the calibration sources are known, an error of ±2.2 eV due to DAC statistics, and an error of ±10 eV to allow for any bias introduced by the peak fitting procedures. Additionally, errors due to the effect of the diode electric field and to interfering lines were included as mentioned above.

The theoretical values of the energies given in Table I were calculated as described in Ref. 5 but with certain improvements. The lowest-order vacuum polarization potential was evaluated using the Glauber expansion[11] and added to the Coulomb potential before solving the Dirac equation. This calculation has been verified by McKinley[12] using an exact evaluation of the vacuum polarization Green's function for the levels concerned. The electron screening was calculated for the $1s^2$ electrons only. These contribute approximately 90% of the screening correction and it may be that we are underestimating the screening effect by 10% or so. On the other hand, it is not clear which other electrons are present inasmuch as many are ejected by the Auger effect in the initial stages of the atomic cascade. The screening effect is small, between 0 and -27 eV in all cases except for the $5g \to 4f$ transitions in Pb, where it amounts to -72 eV. Even here, the contribution of all the other electrons[1] would add only -6 eV, not enough to affect our results significantly.

In addition, the nuclear polarization corrections were obtained from the work of Cole,[13] the values for electron vacuum polarization of order α^2 and $(Z\alpha)^3$ from Fricke,[14] and the effect of the relativistic correction to the reduced mass from Barrett *et al.*[15]

Only the values of the largest corrections, vacuum polarization and finite size effect, are shown in Table I. Apart from the electron screening effect just mentioned, the additional corrections taken into account (Lamb shift, nuclear polarization, reduced mass correction) contribute only a few eV to the theoretical value.

Uncertainties in the theory included contributions from the finite-size correction calculated by assuming a Fermi-type charge distribution, 10% of the electron screening correction, 30% of the Lamb-shift correction,[15] and uncertainties in the fundamental constants and the muon mass[16] of 14 ppm in the energy.

The comparison of the theoretical with the measured values for the 20 transitions studied in this work is given in Table I and shown plotted in Fig. 1. The discrepancy is small for the low-Z elements with transition energies below 200 keV. However, a consistent trend is evident in which the calculated energy exceeds the measured value by an increasing amount for transitions of higher energy in elements of higher Z. In the case of the $5g \to 4f$ transition in Pb at 435 keV the discrepancy amounts to 296 ± 33 ppm. If this were the only sizable discrepancy we

TABLE I. Comparison between the theoretical and experimental energies (keV) for the muonic x rays. Important corrections to the point-nucleus Dirac energies are also shown.

Z	Element	Transition	Finite Size Effect	Vacuum Polarization of Order α	Vacuum Polarization Higher Order $(\alpha^2(Z\alpha)^3)$	$E_{\text{total(theory)}}$	$E_{\text{experiment}}$	$\Delta E_{\text{theory-exp.}}$ (eV) (discrepancy ppm)
20	Ca	$3d_{3/2} \to 2p_{1/2}$	-.078	0.734	.006	158.181±.003	158.173±.018	8±18 (51±114)
		$3d_{5/2} \to 2p_{3/2}$	-.028	0.716	.006	156.845±.002	156.830±.016	15±16 (96±102)
22	Ti	"	-.154	0.947	.009	191.921±.006	191.921±.018	0±19 (0±99)
			-.058	0.920	.009	189.977±.004	189.967±.017	10±18 (53±95)
26	Fe	"	-.439	1.473	.016	269.462±.010	269.427±.017	35±20 (130±74)
			-.163	1.419	.016	265.727±.006	265.705±.016	22±17 (83±64)
38	Sr	$4f_{5/2} \to 3d_{3/2}$	-.004	0.852	.008	200.275±.003	200.254±.020	21±20 (105±100)
		$4f_{7/2} \to 3d_{5/2}$	-.002	0.833	.008	198.712±.003	198.708±.018	4±18 (20±91)
47	Ag	"	-.029	1.519	.017	308.472±.005	308.428±.019	44±20 (143±65)
			-.011	1.470	.017	304.794±.005	304.759±.017	35±18 (115±59)
48	Cd	"	-.036	1.608	.019	322.012±.005	321.973±.018	39±19 (121±59)
			-.014	1.555	.019	318.006±.005	317.977±.017	29±18 (91±57)
50	Sn	"	-.050	1.795	.022	350.000±.006	349.953±.020	47±21 (134±60)
			-.019	1.731	.022	345.276±.005	345.226±.018	50±19 (145±55)
56	Ba	"	-.140	2.435	.033	441.398±.007	441.299±.021	99±22 (224±50)
			-.053	2.328	.033	433.943±.007	433.829±.019	114±20 (263±46)
56	Ba	$5g_{7/2} \to 4f_{5/2}$	.000	0.762	.009	201.291±.004	201.260±.016	31±17 (154±84)
		$5g_{9/2} \to 4f_{7/2}$	.000	0.748	.009	199.924±.004	199.902±.015	22±16 (110±80)
82	Pb	"	-.010	2.190	.037	437.824±.010	437.687±.020	137±22 (313±50)
			-.004	2.106	.035	431.407±.009	431.285±.017	122±19 (283±44)

might suspect a shift due to a resonance with a nuclear level. However, all our measurements show this trend, as is evident from Fig. 1.

Such a discrepancy contradicts the work of Backenstoss *et al.*[6] Indeed, our measurement of the $5g \to 4f$ transitions in Pb show that they lie lower by 120 eV (2.8 times the combined standard deviation). We have no ready explanation for this discrepancy. However, we had better resolution, higher precision, and a larger measurement set. On the other hand, if our results are accepted at face value it is no simple task to resolve the discrepancy between theory and experiment.

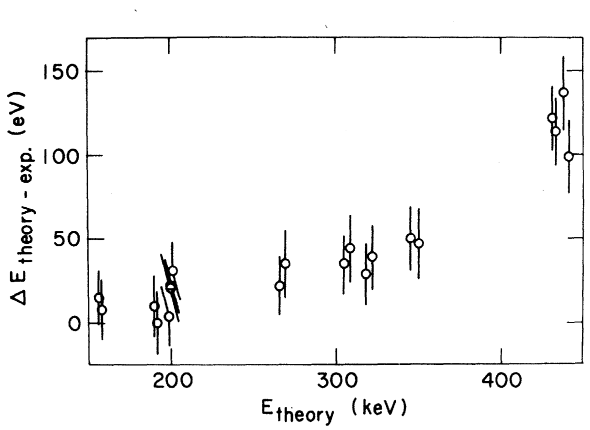

FIG. 1. The discrepancy $\Delta E_{\text{theo-expt}}$ (eV) plotted against the theoretical transition energies for 20 muonic x-ray transitions.

If we suppose that the discrepancy is due to an overestimate of the vacuum polarization correction we have to believe that this can be as large as (3.4 ± 0.4)%. The validity of quantum-electrodynamic calculations in general has been shown to be much better than this.[17] We cannot claim that the present evidence is convincing as long as other ways to explain the discrepancy might exist. Among other possibilities, it may be that the finite size effect has not been taken properly into account. Nothing guarantees that the Fermi distribution gives an adequate description of the nuclear charge distribution to the accuracy indicated here. A nuclear halo as postulated by Barrett *et al.*[15] with a small part of the nuclear charge contained in a long tail could possibly improve the agreement. However, we have not been able to find a prescription that would remove the discrepancy and still give a good account of the other x-ray data as well.

Additional work, both theoretical and experimental, will be needed to clarify the issues raised by this experiment.

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New Results on $n + p \to d + \gamma$, and Time-Reversal Invariance*

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The angular distribution for the reaction $n + p \to d + \gamma$ has been measured at neutron energies of 475, 560, 625, and 750 MeV. Results based on 31 000 events at nine scattering angles are reported and compared with existing data for the inverse reaction, $\gamma + d \to n + p$. The angular distributions are found to agree, as predicted by time-reversal invariance.

We are reporting a new measurement of the angular distribution for the reaction

$$n + p \to d + \gamma \tag{1a}$$

at neutron laboratory energies of 475, 560, 625, and 750 MeV. A comparison of the angular distribution for this reaction with that of its inverse

$$\gamma + d \to n + p \tag{1b}$$

offers a direct test of time-reversal invariance