Comparison of Analogous $\Delta T = 0$ β- and γ-Ray Decays in the Nuclear 2*s*-1*d* Shell*

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$M1$, $\Delta T=0$ radiative widths Γ_γ in the $2d$-$1s$ shell are compared with the widths computed with the Gamow-Teller β-decay ft values, Γ_β. For twelve of nineteen cases, $\Gamma_\gamma = (2\pm1)\Gamma_\beta$.

Kurath[1] has taken advantage of the formal similarity between the $M1$ γ-ray transition operator and the operator for Gamow-Teller β decay and has obtained an expression which may be used to relate the strengths of analogous $\Delta T=1$ β- and γ-ray transitions among members of an isospin multiplet. This expression may be used to deduce the importance of the orbital contribution to the $M1$ matrix element; alternatively, if this contribution is assumed to be small, the $M1$ transition strength may be estimated given the β-decay ft value, and vice versa. This expression has been very useful: Kurath originally used it to relate the C^{10} β-decay strength to the $\Delta T=1$ γ decay of the B^{10} 1.74-MeV level, while Youngblood *et al.*[2] subsequently made similar comparisons for $A=30$. Hanna[3] extended this comparison to many other nuclei with mass <30 in discussing the γ-ray decay of iosbaric analog states. An expecially striking example may be found in the mass-12 nuclei.[4]

The expression given by Kurath[1] makes use of the vanishing of the isoscalar contribution to the $M1$ matrix element for $\Delta T=1$ transitions and has been applied only to transitions of that type; here we point out that this relationship is also useful for transitions with $\Delta T=0$ where the isoscalar term contributes to the transition strength. This is best illustrated by a detailed consideration of the formulas for the transition rates, followed by the specific examples for conjugate nuclei in the nuclear $2s$-$1d$ shell.

The ft value for allowed Gamow-Teller (GT) β transitions from a state in nucleus A $(J_a;\, T_a,\, T_{3a})$ to a state in nucleus B $(J_b;\, T_b,\, T_{3b})$ can be written as

$$ft_{\rm GT} = D\left(\frac{g_v}{g_a}\right)^2 (2J_a+1)\left[(-1)^{T_b-T_{3b}}\begin{pmatrix} T_b & 1 & T_a \\ -T_{3b} & \pm1 & T_{3a}\end{pmatrix}\langle J_b, T_b \| \sum_{i=1}^{A} \vec{\tau}(i)\vec{\sigma}(i) \| J_a, T_a\rangle\right]^{-2}, \tag{1}$$

where $D = 2\pi^3\hbar^7 \ln 2/g_v^{\,2} m_e^{\,5} c^4 \approx 6280$ sec, and g_a/g_v is the ratio of the axial-vector coupling constant to the vector coupling constant, $(g_a/g_v)^2 = 1.51$. The reduced matrix elements of the single-particle spherical tensor operators $\vec{\sigma}\vec{\tau}$ are reduced in both the ordinary and the isospin space; $\vec{\sigma}$ is the Pauli spin spherical tensor, while the reduced matrix elements of $\vec{\tau}$ may be obtained from the relationship $\langle n | t_+ | p\rangle = \langle p | t_- | n\rangle = 1$. The expression for the width Γ_γ of an $M1$ γ-ray transition of energy E_γ from the state $A(J_a;\, T_a,\, T_{3a})$ to the state $B(J_b;\, T_b,\, T_{3b})$ may be written in terms of the transition strength Λ as

$$\Gamma_\gamma(M1{:}J_a;\, T_a,\, T_{3a} \to J_b;\, T_b,\, T_{3b}) = \frac{1}{3}\frac{e^2}{\hbar c} M_p c^2 \left(\frac{E_\gamma}{M_p c^2}\right)^3 \Lambda(M1). \tag{2}$$

Evaluating the constants leads to the usual expression $\Gamma_\gamma({\rm eV}) = 2.76\times10^{-3} E_\gamma^{\,3}({\rm MeV})\Lambda(M1)$. The $M1$ γ-ray transition strength $\Lambda(M1)$ consists of two parts, corresponding to the isoscalar λ_0 and the isovector λ_1 matrix elements:

$$\Lambda(M1) = (\lambda_0+\lambda_1)^2,$$

where

$$\lambda_0 = \frac{1}{(2J_a+1)^{1/2}}(-1)^{T_b-T_{3b}}\begin{pmatrix} T_b & 0 & T_a \\ -T_{3b} & 0 & T_{3a}\end{pmatrix}\left\langle J_b, T_b \left\| \sum_{i=1}^{A} \frac{\mu_p+\mu_n}{2}\vec{\sigma}(i) + \tfrac{1}{2}\vec{1}(i) \right\| J_a, T_a\right\rangle; \tag{3}$$

$$\lambda_1 = \frac{1}{(2J_a+1)^{1/2}}(-1)^{T_b-T_{3b}}\begin{pmatrix} T_b & 1 & T_a \\ -T_{3b} & 0 & T_{3a}\end{pmatrix}\left\langle J_b, T_b \left\| \sum_{i=1}^{A} \frac{-\vec{\tau}(i)}{\sqrt{2}}\left[(\mu_p-\mu_n)\vec{\sigma}(i) + \vec{1}(i)\right] \right\| J_a, T_a\right\rangle. \tag{4}$$

For $\Delta T=1$ transitions, only the isovector term λ_1 contributes to the $M1$ transition strength. One can then relate the GT β-transition rate to the analogous $\Delta T=1$ $M1$ γ-transition rate by using Eqs. (1) and (2):

$$ft_{\mathrm{GT}}(\mathrm{sec})\Gamma_\gamma(\mathrm{eV}) = 126E_\gamma^{3}(\mathrm{MeV})C\Big[1+0.21\langle b\| \sum_{i=1}^{A} \vec{\tau}(i)\vec{1}(i)\| a\rangle\langle b\| \sum_{i=1}^{A} \vec{\tau}(i)\vec{\sigma}(i)\| a\rangle^{-1}\Big]^2; \tag{5}$$

$$C=\left[\begin{pmatrix} T_b & 1 & T_a \\ -T_{3b}{}^{\gamma} & 0 & T_{3a}{}^{\gamma}\end{pmatrix}\Big/\begin{pmatrix} T_b & 1 & T_a \\ -T_{3b}{}^{\beta} & \pm 1 & T_{3a}{}^{\beta}\end{pmatrix}\right]^2. \tag{6}$$

The superscripts β and γ refer T_3 specifically to the β- or γ-ray transition. Eq. (5) is the expression due to Kurath; a generalized form of this expression relating the matrix elements of $\Delta T=1$ ML transitions and those of unique n-forbidden β decay is given by Warburton and Weneser.[5]

For $\Delta T=0$ transitions, the isovector term is still expected to dominate the $M1$ matrix element. This may be seen by using Eqs. (3) and (4) to estimate the magnitude of λ_0/λ_1 to be $|(\mu_n+\mu_p)/(\mu_n-\mu_p)|=0.19$; thus it is reasonable as a first step to neglect the isoscalar contribution to the $M1$ matrix element and use Eq. (5) to compare the transition rates of analogous $\Delta T=0$ transitions in conjugate nuclei. The relevant transitions are schematically illustrated in Fig. 1. The states of the conjugate nuclei are related by the isospin-raising (or -lowering) operator, and Eq. (5) may be used to compare the strength of the β transition from the ground state in nucleus ${}_{Z+1}A_N$ with the excited state of energy E_x of the nucleus ${}_ZA_{N+1}$, i.e., $b\to a$, with the "upward" $M1$ decay strength of the γ-ray transitions $a\to b$ in both the $T_3=+\frac{1}{2}$ and $-\frac{1}{2}$ nuclei. This comparison is made in Table I for conjugate nuclei with $10<Z<20$, using experimental data taken mainly from the compilation of Endt and van der Leun,[6] supplemented by results from the recent literature. To compare the electromagnetic transition rate with the β-decay transition rate, we used Eq. (5) to define a hypothetical radiative width $\Gamma_\beta(\mathrm{GT})$ and form the ratio $\Gamma_\gamma(M1)/\Gamma_\beta(\mathrm{GT})$, where Γ_β is obtained from Eq. (5) ignoring the orbital contribution to the matrix element. Examining the ratio $\Gamma_\gamma/\Gamma_\beta$ given in Table I, we see a clustering of values and a tendency for $\Gamma_\gamma/\Gamma_\beta>1$; for twelve of the nineteen examples, $\Gamma_\gamma=(2\pm1)\Gamma_\beta$. The range of ft values in the table is somewhat more than an order of magnitude: $4.25\leq\log ft\leq5.9$. We conclude that Eq. (5) can reasonably be applied to $\Delta T=0$ transitions. The departure of $\Gamma_\gamma/\Gamma_\beta$ from unity may be due to (1) the isoscalar contribution to Γ_γ or (2) a significant contribution to Γ_γ from the orbital part of the operator. We take up point (1) first. Most of the entries in Table I are due to γ decays in $T_3=+\frac{1}{2}$ nuclei, and it is possible that the isoscalar and isovector terms of the $M1$ operator are in phase for the majority of these cases, resulting in values $\Gamma_\gamma/\Gamma_\beta>1$. Because of the T dependence of λ_0 and λ_1 [Eqs. (3) and (4)], the magnitude and relative phase of λ_0/λ_1 may be deduced for those nuclei where $\Lambda(M1)$ is known for both mirror transitions. These are given in the last column of Table I. We see that not only is the phase such that it is not reasonable to expect addition in the $T_3=+\frac{1}{2}$ nuclei, but that in any case the absolute magnitude is insufficient to account for the ratio $\Gamma_\gamma/\Gamma_\beta$. Note that the ratio λ_0/λ_1 is in accord with the estimate given earlier, $|\lambda_0/\lambda_1|=0.19$. Thus we need to look to point (2) to explain the magnitude of $\Gamma_\gamma/\Gamma_\beta$. This means we need to estimate the orbital contribution to the $M1$ matrix element $0.21\langle b\|\sum\vec{\tau}\vec{1}\|a\rangle/\langle b\|\sum\vec{\tau}\vec{\sigma}\|a\rangle$. Since the wave functions for the states involved are quite complex, it is not clear what to expect for this term. We can, however, get an idea of its magnitude by evaluating the orbital contribution for the single-particle transitions involved, i.e., $d_{5/2}\to d_{5/2}$, $d_{5/2}\to d_{3/2}$, and $d_{3/2}\to d_{3/2}$. We find the above term equal to $+0.43$, -0.11, and -0.64, respectively. The magnitude of these quantities suggests that this term is responsible for the departure of Γ_γ from the value Γ_β. For example, including the orbital term (but still neglecting the isoscalar term) we find that for a $d_{5/2}\to d_{5/2}$

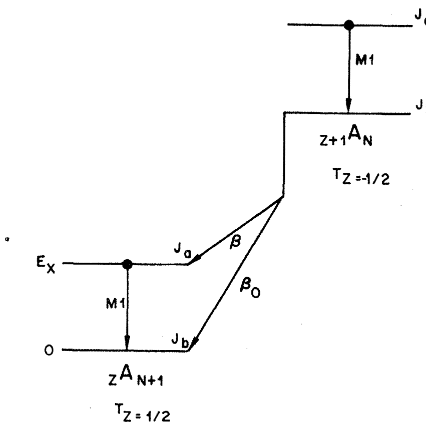

FIG. 1. Schematic representation of the β decays and $M1$ transitions under consideration.

TABLE I. Comparison of the upward γ radiative width, $\Gamma_{M1}(\uparrow)$, with the radiative width predicted from the β-decay ft value, Γ_β. The ratio of isoscalar to isovector matrix elements λ_0/λ_1 is also presented.[a]

		E_a (MeV)		$\Lambda(M1)$			$\Gamma_{M1}(\uparrow)/\Gamma_\beta$ [b,c]		
A	$J_a \to J_b$	$T_3 = +1/2$	$T_3 = -1/2$	$T_3 = +1/2$	$T = -1/2$	log ft ($0 \to E_a$)	$T_3 = +1/2$	$T_3 = -1/2$	λ_0/λ_1 [d]
21	$5/2 \to 3/2$	0.347	0.332	0.26 ± 0.04 [e,f]	0.46 ± 0.10 [f]	4.93 ± 0.04	1.46 ± 0.15	2.58 ± 0.43	-0.14 ± 0.05
23	$5/2 \to 3/2$	0.439		1.76 ± 0.08		4.45 ± 0.02	3.22 ± 0.16		
25	$3/2 \to 5/2$	0.975		0.012 ± 0.004 [g,h]		6.25 ± 0.18 [i]	0.63 ± 0.25		
25	$7/2 \to 5/2$	1.614	1.61	2.49 ± 0.60 [j]	1.97 ± 0.56 [k]	4.24 ± 0.13 [i,l,m]	2.51 ± 0.68	2.00 ± 0.58	0.05 ± 0.065
27	$7/2 \to 5/2$	2.209		0.46 ± 0.02		5.01 ± 0.08	2.74 ± 0.39		
29	$3/2 \to 1/2$	1.273	1.38	0.31 ± 0.05 [n]	0.47 ± 0.07 [o]	4.84 ± 0.07	1.88 ± 0.31	2.84 ± 0.45	-0.10 ± 0.04
29	$3/2 \to 1/2$	2.427		0.69 ± 0.35 [n]		4.24 ± 0.11	1.05 ± 0.42		
31	$3/2 \to 1/2$	1.267		0.15 ± 0.02		5.00 ± 0.04	1.32 ± 0.15		
33	$1/2 \to 3/2$	0.840	0.809	0.24 ± 0.05 [g,p]	0.26 ± 0.04 [q]	5.60 ± 0.08 [m]	2.06 ± 0.43	2.26 ± 0.60	-0.01 ± 0.04
33	$5/2 \to 3/2$	1.967	1.99	0.11 ± 0.03 [p]	$0.20^{-0.13}_{+0.07}$ [r]	4.87 ± 0.06 [m]	0.60 ± 0.13	$1.10^{-0.52}_{+0.29}$	$-0.15^{-0.14}_{+0.08}$
33	$5/2 \to 3/2$	2.869	2.85	0.77 ± 0.30	0.10 ± 03 [r]	4.06 ± 0.19	0.65 ± 0.19	0.084 ± 0.021	-0.47 ± 0.11
35	$1/2 \to 3/2$	1.220		1.02 ± 0.24		5.19 ± 0.09 [u,v]	3.44 ± 0.76		
35	$5/2 \to 3/2$	1.762		$(7.3 \pm 0.3) \times 10^{-3}$		5.48 ± 0.13 [u]	0.15 ± 0.06		

[a] The bulk of experimental information is taken from Ref. 6.

[b] "Upward" and "downward" radiative widths are related by $\Gamma(\uparrow;\, a \to b) = \Gamma(\downarrow;\, b \to a)(2J_a+1)/(2J_b+1)$.

[c] For these conjugate nuclei, $\Gamma_\beta(\text{eV}) = 63E_\gamma^3(\text{MeV})/ft(\text{sec})$.

[d] The reciprocal of these entries is also a solution for λ_0/λ_1; only the solution for $\lambda_1 > \lambda_0$ is given here.

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transition, Γ_β will be increased by $(1+0.43)^2 = 2.04$, while for a $d_{3/2} \to d_{3/2}$ transition, Γ_β will be decreased by $(1-0.64)^2 = 0.13$. The relative phase of the orbital contribution also is in keeping with the observed trend in the data, viz., $\Gamma_\gamma/\Gamma_\beta > 1$ in the beginning of the $2s$-$1d$ shell, and <1 towards the end of the shell.

In conclusion, Eq. (5) is useful for relating rates of analogous β- and γ-ray transitions with $\Delta T = 0$ in the nuclear $2s$-$1d$ shell. The $M1$ transition width Γ_β predicted from the β-decay ft value is generally too low, and there is a suggestion that the ratio $\Gamma_\gamma/\Gamma_\beta$ is A dependent. Study of six cases in which the analogous radiative widths $\Gamma_\gamma(M1)$ are known in both of the conjugate nuclei together with the ft values suggests that the major departures from Eq. (5) are due to neglect of the orbital contribution. Two incidental results were obtained in the course of the analysis for these six transition pairs: (1) Five of the six pairs have approximately equal strengths and thus follow Morpurgo's rule,[8] and (2) the $E2/M1$ multipole mixing ratios change sign with T_3, which can be understood from arguments of Poletti, Warburton, and Kurath.[9] Glaudemans and van der Leun[10] have also made comments on point (2).

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Angular Distributions for $T_{np} = 1$ and $T_{np} = 0$ Transfers in (^{3}He, p) Reactions*

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Angular distributions of (^{3}He,p) reactions are investigated for some odd-even target nuclei, which allow orbital-angular-momentum transfers $L_{np}=0,2$ for $T_{np}=0,1$ transitions. "Pure" $L_{np}=0$ distributions are observed for the n-p pair transfer with $T_{np}=1$ leading to the analog state and to the corresponding state with the same nucleon configuration but with $T=T_<$. "Mixed" $L_{np}=0+2$ distributions are observed for $T_{np}=0$ transfers.

The two-nucleon (^{3}He,p) transfer reaction has frequently been used for studies of nuclear structure. Some of these studies on $0^+\to0^+$ and $0^+\to1^+$ transitions on every even-even target nucleus from ^{16}O to ^{28}Si were published recently.[1] The authors made some general observations in an attempt to obtain a better understanding of the peculiarities of the (^{3}He,p) reaction.

In the (^{3}He,p) reaction, the n-p pair can be transferred either as a singlet with spin assignment $S=0$, $T_{np}=1$ or as a triplet with $S=1$, $T_{np}=0$. Thus the $0^+\to0^+$ transitions occur via a singlet n-p pair absorption with orbital angular momentum $L_{np}=0$, whereas in the $0^+\to1^+$ transition both $L_{np}=0$ and $L_{np}=2$ may contribute and the n-p pair is transferred as a triplet. A number of experiments[2-4] have shown that all observed angular distributions for $0^+\to0^+$ transitions in both sd- and fp-shell nuclei—both for $T_A=0$ and $T_A\neq0$ targets—show the pattern characteristic of a pure $L_{np}=0$ transfer, with a sharp minimum at forward angles. In contrast, the angular distributions of all known $0^+\to1^+$ transitions in these nuclei deviate from the pure $L_{np}=0$ distribution; the filling in of the sharp minimum indicates an $L_{np}=2$ contribution. According to Ref. 1, the $L_{np}=2$ contribution to a mainly $L_{np}=0$ angular distribution should become much smaller than that observed in the $0^+\to1^+$ transitions whenever the selection rules allow the states of the final nucleus to be populated via the singlet as well as via the triplet pair, i.e., whenever $T_A=T_B\neq0$ and $J_A=J_B\neq0$, where J_A, T_A and J_B, T_B are the spin and isospin assignments of the target nucleus and the final nucleus, respectively. Measurements relevant to this particular prediction have recently become available.[5-7]