Mixed-State Ultrasonic Attenuation in Clean Niobium Near H_{c2}: Field Dependence for $\vec{q} \parallel \vec{H}$†

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Ultrasonic attenuation measurements are reported for clean niobium in the region from $H_{c2}-H=100$ G to $H_{c2}-H=0$. The results agree qualitatively with the theory of Maki and Houghton which develops regimes of attenuation which are "gapless" near the transition and BCS-like farther away.

Recently there has been much theoretical[1-4] and experimental[5,6] interest in the coefficient of ultrasonic attenuation for pure type-II superconductors in the mixed state. In this paper we present some results for attenuation in the experimental situation $\vec{q} \parallel \vec{H}$, where $q=2\pi/\lambda$ is the phonon wave vector, H is the external applied field, λ is the phonon wavelength. We find qualitative agreement with the theory of Houghton and Maki[1] (hereafter HM) insofar as that theory claims pertinence.

The attenuation was continuously measured using the Matec Ultrasonic Attenuation system in the conventional pulse-echo mode. The sample temperature was controlled to ± 1 mK using a bridge-off-balance-amplifier–type controller. The magnetic field was found by measuring the voltage in a precision resistor wired in series with a superconducting magnet. Field strength was known to ± 30 G but field changes of $\frac{1}{3}$ G were easily seen. There were two important sources of error. Temperature drift and magnetic field noise, however small, influence data taken for $H_{c2}(T)$. In fact temperature excursions vary $H_{c2}(T)$, and magnetic noise changes the linearly increasing (or decreasing) field H, so that both affect $H_{c2}-H$. This effect will be comparable to $H_{c2}-H$ at some point since $H=H_{c2}(T)$ at the transition. Also the magnet current possessed a ripple, visible in the data (Figs. 1 and 2), of 2% of its magnitude. Slopes taken from the data are good to $\pm 10\%$.

In the region below the transition, HM and Cerdeira and Houghton[2] (hereafter CH) use the Green's function of Brandt[7] to show that the ultrasonic attenuation coefficient for clean niobium can be described in terms of a parameter $\mu=2\sqrt{\pi}(\Delta/\hbar k_c v_0)^2 k_c l$ where Δ is the space-averaged order parameter, $k_c^2=2eB/c=2\pi H_{c2}/\varphi_0$ is the reciprocal lattice vector of the flux line lattice, v_0 is the Fermi velocity, and $\varphi_0=2.07\times10^{-7}$ G cm^2 is the magnetic flux quantum. The dependence of μ on l represents a departure from earlier theoretical work.[4,8] In particular two identifiable regions are theoretically predicted. In the region very close to the transition, "gapless" behavior dominates such that $\Delta\alpha/\alpha \propto \Delta^2 \propto H_{c2}-H$ or

$$
\begin{aligned}
(\Delta\alpha/\alpha)_t &\simeq 2.68\mu, \\
(\Delta\alpha/\alpha)_l &\simeq 4.61\mu
\end{aligned}
\tag{1}
$$

for transverse (shear) and longitudinal waves, respectively, while farther away from the transition more traditional BCS behavior is anticipated where

$$
\begin{aligned}
(\Delta\alpha/\alpha)_t^2 &\simeq 1.63(\mu-0.082), \\
(\Delta\alpha/\alpha)_l^2 &\simeq 2.22(\mu-0.043).
\end{aligned}
\tag{2}
$$

HM contend that these results will hold in moderately pure niobium where $l/\xi \gg 1$ and $\mu < 1$ and for phonons of low frequency ($ql<1$). The somewhat more generalized result of CH finds the same form for $\Delta\alpha/\alpha$ as long as $\mu \ll 1$ and for phonons having $ql>1$ but frequency such that $\omega_{\mathrm{ph}} \ll v_0/l + \Delta^2/k_c v_0$. Hence we consider the theory of HM and CH appropriate to a discussion of our data taken for ql values from 0.7 to 3.

The conclusions of HM and CH are found by evaluating the imaginary part of the temperature-dependent response function at $T=0$°K. At higher temperatures, electrons in a superconducting portion of the sample at the Fermi surface will spend more time in the excited state. This is not accounted for in the $T=0$ limit. However, the theory is expected to be valid up to about 5°K.

Our sample was taken from Oak Ridge niobium No. 68 given to us by Richard E. Reed. Its resistivity ratio was about 7000 and normal-state data over a wide range of frequencies indicated that $l(T)=[l(0)^{-1}+1.27T^3]^{-1}$, where $l(0)=(40\pm12)\times10^{-4}$ cm. For our purposes we use the equation for the gap parameter due to Maki and Tsuzuki,[9]

$$\Delta^2=\frac{1}{2\pi N}\frac{H_{c2}-H}{1.16(2\kappa_2{}^2-1)+0.187}\left(H_{c2}-\frac{t}{2}\frac{dH_{c2}}{dt}\right),$$

and the convenient expression (modified) of Gorter and Casimir,

$$H_{c2}(t)=H_{c2}(0)(1-t^2)/(1+0.8t^2).$$

This empirical expression fits the experimental $H_{c2}(t)$ of this sample to <1% below $t=0.5$. We can now write μ in more familiar variables:

$$\mu=\left[\frac{l(T)}{\hbar^2 v_0{}^2N}\right]\left[\frac{\varphi_0H_{c2}(t)}{2\pi^2}\right]^{1/2}\times\frac{1+1.9t^2/(1+0.8t^2)(1-t^2)}{1.16[2\kappa_2{}^2(t)-1]+0.187}(H_{c2}-H).$$

We have used $v_0=2.67\times10^7$ cm/sec, $N=5.63\times10^{34}$ cm^{-3} erg^{-1}, $H_{c2}(0)=3992$ G, $\varphi_0=2.07\times10^{-7}$ G cm^2, and values of κ_2 from Finnemore, Stromberg, and Swenson.[10]

Figure 1 shows longitudinal data at 15 and 42 MHz. Figure 2 shows corresponding data for shear waves at 16 and 45 MHz. In Figs. 1(b), 1(d), 2(b), and 2(d), data very near H_{c2} are shown by plotting the reduced attenuation at constant temperature, $\Delta\alpha/\alpha=(\alpha_n-\alpha_s)/\alpha_n$, versus $H_{c2}-H$ in gauss. Where they are available, data are shown for both increasing ("up") and decreasing ("down") field. In each case the straight lines are linear fits to the data. In all cases investigated there is a region called the gapless region near T_c where $\Delta\alpha/\alpha\propto H_{c2}-H$ in agreement with Eq. (1). Figures 1(a) and 1(c) show data for $(\Delta\alpha/\alpha)^2=[(\alpha_n-\alpha_s)/\alpha_n]^2$ vs $H_{c2}-H$. This region has been observed often on less pure samples in the recent past.[5,11,12] Each run through the transition, whether in increasing or decreasing field, resulted in a region of data called the BCS region where $(\Delta\alpha/\alpha)^2\propto H_{c2}-H$. Figure 1(a) also shows the attenuation as predicted by Tang,[4] the approximate fit to the data of Forgan and Gough,[5] and the predicted fit to the data from Houghton and Maki,[1] where the quantity k_cl is chosen to agree with the straight line of Fig. 1(b). The linear fit of Fig. 1(b) is made to data in decreasing field because the slopes of those data in the BCS and gapless regimes are in good agreement with the slopes of HM. The authors consider the preference of the theory for the "down" data to be an interesting result which may indicate a direction for further theoretical work. The raw data show a 30-G "hysteresis" very common to niobium work.[11] Figures 1(a) and 1(b) show data for $t=T_c=0.211$; Figs. 1(c) and 1(d) show the case $t=0.77$. Clearly the data still show gapless and BCS regions even though the temperature is well in excess of the range of prediction of HM. In Fig. 1(d) linear fits are made to both "up" and "down" data. In Fig. 1(c) the attenuation curves expected from those fits are shown. Here not even the downgoing data behave exactly as expected.

Figures 2(a) and 2(b) show shear data at 45 MHz for increasing fields at a low and a high temperature. For both temperatures, gapless and BCS regions are visible. For both regimes the

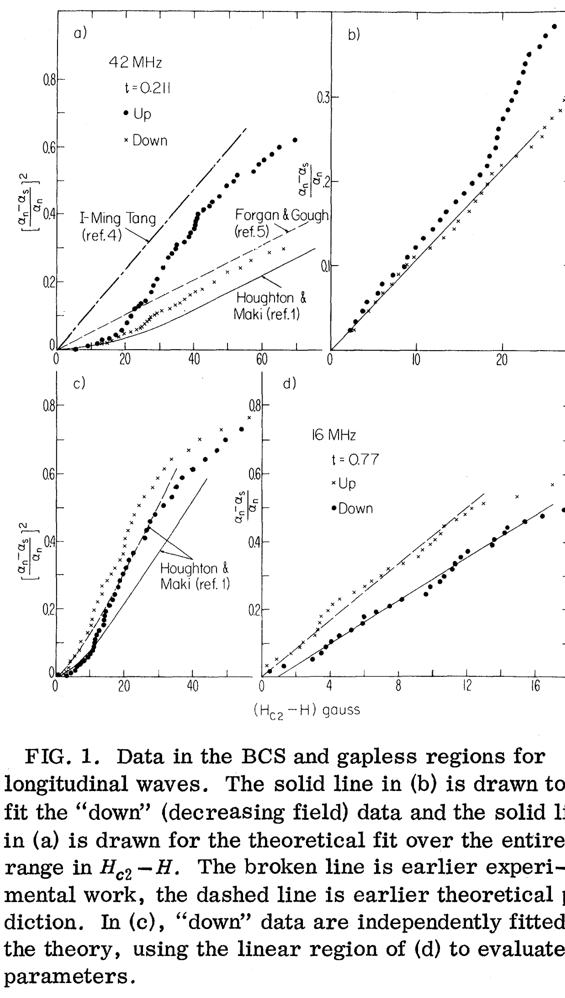

FIG. 1. Data in the BCS and gapless regions for longitudinal waves. The solid line in (b) is drawn to fit the "down" (decreasing field) data and the solid line in (a) is drawn for the theoretical fit over the entire range in $H_{c2}-H$. The broken line is earlier experimental work, the dashed line is earlier theoretical prediction. In (c), "down" data are independently fitted by the theory, using the linear region of (d) to evaluate parameters.

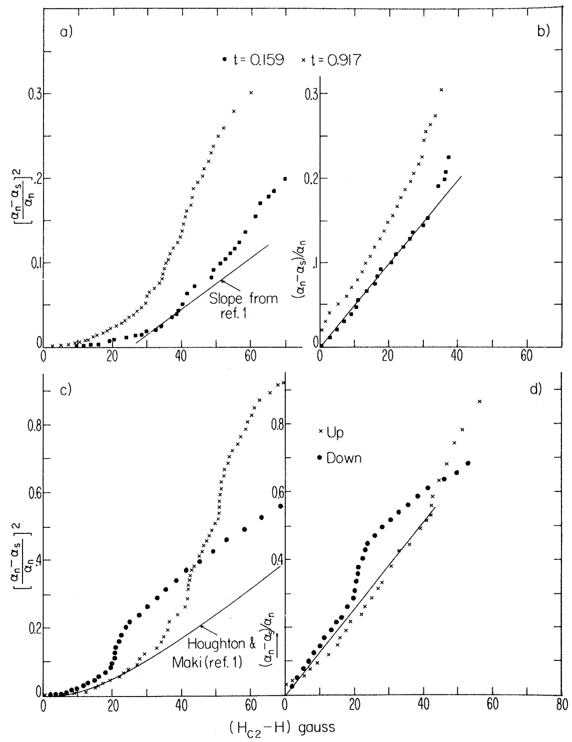

FIG. 2. Data in the BCS and gapless regions for shear waves. In (a) and (b) high- and low-temperature 45-MHz "up" data are plotted. The solid line in (b) is a linear fit to data for $t = 0.159$. In (a) the slope theoretically implied by that linear fit is shown. In (c) and (d) increasing field and decreasing field data for 16-MHz phonons at $t = 0.56$ are plotted. The best-fit line in (d) is used to evaluate the solid line in (c).

high-temperature data are characterized by a steeper slope. In Fig. 2(b) the data for $t = 0.159$ are compared with the linear least-squares best fit, and in Fig. 2(a) the slope of the BCS region expected from HM is shown. Figures 2(c) and 2(d) show shear data at 16 MHz and at $t = 0.56$. In Fig. 2(d) a least-squares fit to the "up" data is shown and in 2(c) the corresponding prediction for $(\Delta\alpha/\alpha)^2$ is shown. In 2(d) the "down" data and "up" data have a common slope to within the experimental precision. In both 2(c) and 2(d) "up" and "down" data are shifted apart by 3 G to avoid confusion.

Figure 3 summarizes the data for our experiments measuring attenuation versus field at constant temperature. Each quadrant shows a measured slope: 3(a) the slope in the BCS regime for longitudinal waves, 3(b) the slope in the gapless regime for longitudinal waves, 3(c) the slope in the BCS regime for shear waves, and 3(d) the slope in the gapless regime for shear waves. In 3(b) the theoretical line is that from the temperature dependence of μ, with the constants contained in μ including $k_c l(0)$ adjusted to fit the data for low temperature. In 3(a) the theoretical line is the line implied from HM for the value of μ found for 3(b). Similarly the solid line of Fig. 3(d) is the temperature dependence of μ previously obtained for 3(b), and the line of 3(c) is the result implied by that value for μ. We are considering the gapless region to be fundamental

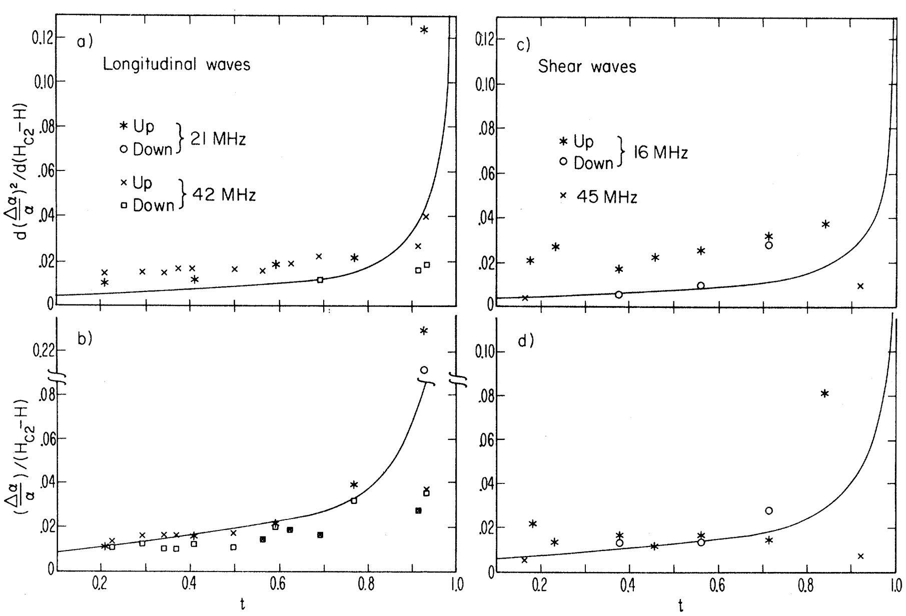

FIG. 3. A summary of slopes for the attenuation-versus-field data at constant temperature for $\vec{q}\parallel\vec{H}$. The lines are drawn from the theory of Houghton and Maki (Ref. 1) for the case $\mu(T=0) = 0.021(H_{c2}-H)$.

and are using data taken on it to evaluate the constant terms contained in μ and subsequently using that resulting μ to arrive at a prediction for the BCS region. We have used the value $\mu/(H_{c2} - H) = 0.021$ at $T = 0°K$ for the theoretical lines in 3(a), 3(b), 3(c), and 3(d). Although this is only an extrapolation of low-temperature results, there is surprising agreement between the theoretical and experimental slopes for the gapless region. The experimental slopes in the BCS region lie above the theoretical curve. However, the temperature dependence appears to be similar.

In conclusion we find the predicted BCS and gapless regions in the field dependence of the attenuation of both longitudinal and shear waves. At lower temperatures we find relative agreement between experiment and theory in the shapes of the curves. There is qualitative agreement with the temperature dependence of the slopes in the gapless region. In the BCS region these slopes follow the shape of the temperature-dependent theoretical curves. Clearly more theoretical work is necessary to improve the exact understanding of the regimes as well as the transition from one to the other.

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New Perturbation Theory for Low-Energy Electron-Diffraction Intensities

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Standard perturbation theory applied to the low-energy electron-diffraction problem does not appear to give accurate results even when carried to second order. A new scheme, renormalized forward-scattering perturbation theory, retains the advantage of being fast to execute, but at the same time is very accurate. The new theory is subjected to the stringent test of comparison with exact results for a realistic model of the copper (100) surface.

There are two decisions to be made before embarking on calculations of the diffraction of a monoenergetic, well-collimated beam of electrons incident on a crystal surface. Firstly a model must be postulated for the surface, including the quantities strongly affecting the diffraction process, and omitting quantities that do not strongly affect the process but are possibly difficult to calculate. Secondly, having decided the nature of the environment in which the electron finds itself, a method of solution of the diffraction process must be fixed upon.

In recent years it has become apparent that a model of the surface taking account of the strong elastic scattering by ion cores and of inelastic scattering processes (the ion-core scattering model),[1-3] when combined with an accurate nonperturbative method of calculation, gives good agreement with experiment.[1,3-5] The main concern of this Letter is to suggest a fast, accurate