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Normal-State Resistance as the Determining Parameter in the Behavior of Dayem Bridges with Sinusoidal Current-Phase Relations

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We have shown experimentally that both the ac and dc properties of small Dayem bridges are fully determined at all temperatures by their normal-state Ohmic resistances. The behavior can be described by a two-fluid model where the normal component is governed by the normal-state resistance and the superfluid component determined by the two Josephson equations. The normal-state resistance also determines the maximum supercurrent.

Considerable effort by many workers has been put into devising equivalent circuits to describe the dc and ac behavior of the various types of weak links. A description of the Josephson junction and the superconductor-normal-superconductor junction has been attempted,[1,2] but this is difficult as the spatial distribution of the current along the barrier must be taken into account. A general description of point contacts is also difficult because of insufficient knowledge of their geometrical structure. We report here experimental results on superconducting thin-film microbridges (Dayem bridges), whose behavior is shown to be very well described by a simple equivalent circuit consisting of a resistor in parallel with a component determined by the two Josephson equations and, moreover, is fully determined at all temperatures by the resistance of the bridge in the normal state.

We have investigated the I-V characteristics of the bridges with an applied X-band microwave field and found current steps at voltages $V = n(\hbar/2e)\omega$, where n is an integer and ω is the frequency. This is similar to the findings of Anderson and Dayem.[3] However, we found[4] that for bridges which were much smaller than any previously reported,[3,5] a periodic variation of the step heights with microwave power was observed. The geometry of these smaller bridges was investigated using a scanning electron microscope. The lengths and widths were typically found to be 0.2 and 0.5 μm, respectively, for film thicknesses of about 0.1 μm.[6] In Fig. 1 we have plotted the heights of several current steps as functions of the microwave field for various values of n. On comparison with the Bessel-function behavior found in Josephson junctions, two differences become apparent as the temperature is reduced from T_c. First, the supercurrent increases relatively faster than the magnitudes of the steps; and, second, the first period of the Bessel-function–like behavior becomes more extended. Also an initial increase in the zero-voltage current step at small microwave power was observed in accord with the earlier investigations on larger Dayem bridges.[5]

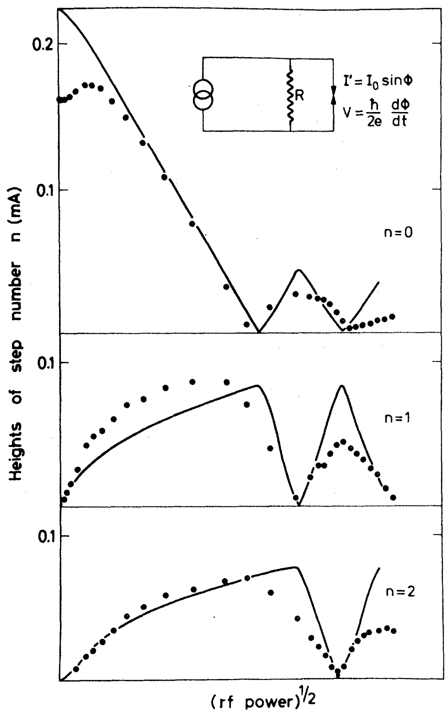

FIG. 1. The magnitudes of the zeroth, first, and second current steps ($n = 0, 1, 2$) as functions of the microwave field amplitude for a small Dayem bridge. $\omega/2\pi = 9.054$ GHz; $T = 3.677$ K. The solid lines are taken from Ref. 7 and calculated on the basis of the equivalent circuit shown in the insert, using the parameter $(\hbar\omega/2e)R^{-1}I_0^{-1} = 0.22$. Inserting I_0, this parameter yields $R = 0.38\ \Omega$. Equation (2) yields $R = 0.37\ \Omega$. The plateau bridge resistance is $0.4\ \Omega$.

Comparison of these measurements with analog computations reported recently[7,8] on the equivalent circuit shown as an insert in Fig. 1 led to some very interesting conclusions. When a combined dc and ac current source is applied to the equivalent circuit, then the following differential equations govern the behavior[2]:

$$\frac{I}{I_0} + \frac{I_1}{I_0}\sin\omega t = \sin\varphi + R^{-1}I_0^{-1}\frac{\hbar}{2e}\frac{d\varphi}{dt},$$
$$V = \frac{\hbar}{2e}\frac{d\varphi}{dt}, \tag{1}$$

where I and I_0 are the external applied dc current and the maximum supercurrent, respectively; I_1 and ω are the current amplitude and the frequency of the microwave signal, respectively; and R is the parallel resistance. The I-V characteristic is determined by the time average of the solution to these two equations. Such I-V characteristics have been calculated in Refs. 7 and 8 for various values of the normalized ac current I_1/I_0 and a few values of the parameter $(\hbar\omega/2e)R^{-1}I_0^{-1}$. A comparison of these calculations with our experimental results is shown in Fig. 1, where the heights of several current steps have been displayed as a function of I_1/I_0. In order to compare the theoretical curve with the experimental data, a choice was made for the parameter $(\hbar\omega/2e)R^{-1} \times I_0^{-1}$, which determines the general shape of the theoretical curve; the ac current amplitude, only known relatively in our experiments, was also scaled to fit the curve. The zero-voltage current step at zero microwave power (i.e., the supercurrent) is found to be smaller than the theoretical prediction but reaches its expected value on the application of a relatively small microwave signal. This so-called Dayem effect is thought to be due to some stabilizing influence of the microwave signal on the fluctuating phase difference φ. The reason for this stabilization is not known, but the close similarity of the equivalent circuit in Fig. 1 and a strongly damped pendulum perhaps indicate some effect analogous to the alternating-gradient stabilization of the inverted pendulum[9] as described by the Mathieu equation.

The good agreement between the experimental results and the theory clearly shows that a sinusoidal current-phase relation is relevant to small Dayem bridges as has recently been verified theoretically[10-12] using the one-dimensional Ginzburg-Landau equations. From the fact that our films are in the dirty limit, we can rewrite the expression for the maximum supercurrent I_0 by introducing the mean free path in the normal state and hence the normal-state resistance R of the bridge region. We find[13]

$$I_0 = (\pi/4e)R^{-1}\Delta^2(T)/k_BT_c, \tag{2}$$

where $\Delta(T)$ and T_c are the temperature-dependent energy gap and the transition temperature of the superconductor, respectively. The only unknown parameter in this equation is R. It was possible to separate the normal resistance of the bridge from that of the background film since the transition temperatures of these two regions were different, giving rise to a resistance plateau between the two transitions representing the normal-state resistance of the bridge. For temperatures not too far from T_c this value of R corresponds closely to that determined from Eq. (2) when the experimental value of I_0 is inserted. This normal-state resistance is also experimentally found to be the parallel resistance in the equivalent circuit from the fitted parameter $(\hbar\omega/2e)R^{-1}I_0^{-1}$. It is interesting to notice that Eq. (2)

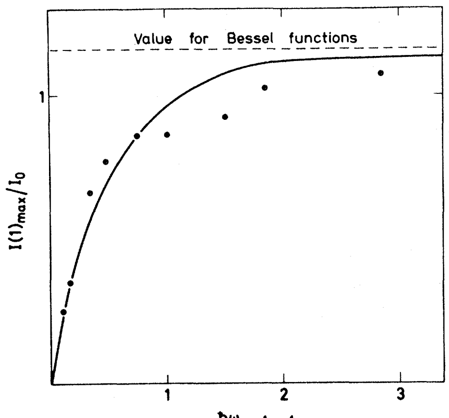

FIG. 2. The ratios between the maximum heights of the first current step $I(1)_{max}$ and the maximum supercurrent I_0 plotted as a function of the parameter $(\hbar\omega/2e)R^{-1}I_0^{-1}$ for varying I_0, i.e., varying temperature. These measurements are performed on another Dayem bridge, different from that used for the data of Fig. 1. The solid curve is taken from Ref. 8 and is based on the equivalent circuit shown in the insert of Fig. 1; $\omega/2\pi = 9.129$ GHz. The fit gives $R = 0.47\ \Omega$. The plateau bridge resistance is $0.5\ \Omega$.

has exactly the form found theoretically in the high-temperature limit for the Josephson tunnel junction[14] except for the interpretation of R which in our case is the resistance of the bridge as determined from the mean free path, whereas in the case of the Josephson tunnel junction it is the normal tunnel resistance.

The fitting of the experimental results by the analog computations of Ref. 8, based on the equivalent circuit of Fig. 1, involves the two parameters R and I_0. However, R and I_0 are found to be connected by Eq. (2); and, as R was found to be temperature independent, the only temperature dependence is that inherent in Eq. (2), i.e., the temperature dependence of the energy gap. As an illustration of the validity of the equivalent circuit at all temperatures we have plotted the ratio $I(1)_{max}/I_0$, where $I(1)_{max}$ is the height of the first current step ($n=1$) at maximum, as a function of the parameter $(\hbar\omega/2e)R^{-1}I_0^{-1}$ for varying I_0, i.e., for varying temperature. Such a curve has been calculated by Fack and Kose,[8] and the agreement with our results as shown in Fig. 2 is seen to be very good, confirming the validity of the equivalent circuit as well as the unique value of R.

The practical use of these bridges is particularly straightforward. They endure repeated thermal cycling and storage in air for periods of years with no degradation of their characteristics. We expect them to be great interest in device applications.

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