but more general tests at larger n and Z listed in Ref. 8 all agree with theory. We also find that the recent measurements[15] in hydrogen for $n=3$, 4, and 5 all agree with theory and the expected 32% ($\frac{5}{15}$) differ by more than 1 standard deviation. In summary, we find quite satisfactory confirmation of these predictions of QED calculated with an uncertainty often better than 100 ppm, and sometimes approaching 10 ppm.

These results may thus be used to calculate atomic energy levels to a higher degree of precision and for larger values of Z than previously attainable.[16] The entire calculation (including the usual expressions for relativistic binding energies and recoil corrections) has been computerized, and so results (with details of various terms) for any energy levels or splittings (including the associated uncertainties) may be provided on request. However, the previous tabulation[16] is still quite valid for all except very high-precision purposes.

A more complete discussion of this calculation and its results will be published in the near future.

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[1]See G. W. Erickson and D. R. Yennie, Ann. Phys. (New York) 35, 271 (1965), for references to the earlier calculations, for the covariant formulation used here, and for a summary of the higher-order terms. We will refer to this paper as I.

[2]G. W. Erickson and D. R. Yennie, Ann. Phys. (New York) 35, 447 (1965).

[3]See W. M. Frank, D. J. Land, and R. M. Spector, Rev. Mod. Phys. 43, 36 (1971); P. G. Reinhard, Lett. Nuovo Cimento 3, 313 (1970); V. S. Popov, Yad. Fiz. 12, 429 (1970) [Sov. J. Nucl. Phys. 12, 235 (1971)], for discussions and references related to the use of Dirac wave functions for $Z\alpha > 1$.

[4]The results, mostly integrals like Eq. (2), can all be expressed in closed form except for some trilogarithmic Spence functions which can be handled numerically.

[5]A. M. Desiderio and W. R. Johnson, Phys. Rev. A 3, 1267 (1971).

[6]G. E. Brown, J. S. Langer, and G. W. Schaefer, Proc. Roy. Soc., Ser. A 251, 92 (1959).

[7]G. E. Brown and D. F. Mayers, Proc. Roy. Soc., Ser. A 251, 105 (1959).

[8]T. Appelquist and S. J. Brodsky, Phys. Rev. A 2, 2293 (1970).

[9]A. Peterman, Phys. Lett. 35B, 325 (1971).

[10]R. Barbieri, J. A. Mignaco, and E. Remiddi, Lett. Nuovo Cimento 3, 588 (1970).

[11]M. F. Soto, Jr., Phys. Rev. Lett. 17, 1153 (1966), and Phys. Rev. A 2, 734 (1970).

[12]R. Hofstadter and H. R. Collard, in *Landolt-Börnstein Numerical Data and Functional Relationships in Science and Technology*, edited by K.-H. Hellwege and H. Schopper (Springer, Berlin, 1967), New Series, Group I, Vol. 2, p. 21.

[13]F. A. Bumiller, F. R. Buskirk, J. W. Stewart, and E. B. Dally, Phys. Rev. Lett. 25, 1774 (1970), find $r_d = 1.95 \pm 0.02$ F for the effective radius in the deuteron wave function. We require instead the deuteron form-factor slope $-6dG_E(d)/dq^2 = \langle r_d^2\rangle - \langle r_n^2\rangle + \langle r_p^2\rangle = (2.082 \pm 0.019\ \mathrm{F})^2$, where $\langle r_n^2\rangle = 0.116\ \mathrm{F}^2$ is obtained from the neutron-electron slope and $\langle r_p^2\rangle = 0.648 \pm 0.018$ F is given by L. N. Hand, D. G. Miller, and R. Wilson, Rev. Mod. Phys. 35, 335 (1963).

[14]B. N. Taylor, W. H. Parker, and D. N. Langenberg, Rev. Mod. Phys. 41, 375 (1969).

[15]C. W. Fabjan, F. M. Pipkin, and M. Silverman, Phys. Rev. Lett. 26, 347 (1971).

[16]J. D. Garcia and J. E. Mack, J. Opt. Soc. Amer. 55, 654 (1965).

Spatial Determination of the Direction of the Magnetic Field in a Tokamak Configuration

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A method is suggested by which the local magnetic field direction can be determined in Tokamaks. A numerical application is made to the Princeton ST Tokamak.

Much interest is presently being given to the spatial determination of the direction of the total magnetic field (or local current density distribution) in a Tokamak configuration. Methods have been proposed using laser scattering[1-5] or particle deflection.[6] The possibility of using coupling of the characteristic electromagnetic modes has also been suggested.[7-8]

In the new method discussed in this Letter, a locally extraordinary electromagnetic wave is generated at a point x_0 in the plasma. If plasma conditions are such that the ordinary and extraordinary indices are near unity for $x > x_0$, the wave propagates as in vacuum, and a measurement of its polarization yields directly the direction of the total magnetic field at x_0.

Our model is a weakly inhomogeneous slab plasma in which the density n_e varies along x in a direction perpendicular to the slab face. The total magnetic field B_T (poloidal plus toroidal) remains in the y-z plane, parallel to the interface, and is slowly varying in the x direction. An electromagnetic wave is propagated along x in the direction of the decreasing magnetic field. The scheme is shown in Fig. 1. For the validity of the slab approximation we require that the minor radius of the Tokamak configuration be large compared to the wavelength ($R/\lambda \gg 1$), a condition that is easily satisfied.

In a cold, shear-free plasma, there are two independent electromagnetic modes of propagation in a direction transverse to the magnetic field; they are the ordinary and extraordinary waves, characterized by the refractive indices N_0 and N_x, respectively. If the effect of a small shear is introduced the modes are coupled, and the equations governing the propagation are given by[8]

$$\frac{d^2E_\parallel}{dx^2}+\left[\frac{\omega^2}{c^2}N_0^2-\varphi^2\right]E_\parallel=2\varphi\frac{dE_\perp}{dx}+\frac{d\varphi}{dx}E_\perp, \qquad (1)$$

$$\frac{d^2E_\perp}{dx^2}+\left[\frac{\omega^2}{c^2}N_x^2-\varphi^2\right]E_\perp=-2\varphi\frac{dE_\parallel}{dx}-\frac{d\varphi}{dx}E_\parallel, \qquad (2)$$

where

$$N_0^2=1-X, \quad N_x^2=1-\frac{X(1-X)}{1-X-Y};$$

$$X=\nu_p^2/\nu^2, \quad Y=\nu_c^2/\nu^2;$$

ν_p, ν_c, and ν are the plasma, electron cyclotron, and wave frequencies, respectively ($\omega/c = 2\pi\nu/\lambda$); $\varphi = d\theta/dx$, where $\theta(x)=\tan^{-1}(B_y/B_z) \approx B_y/B_z$ [angle made by the total magnetic field with respect to the z axis (Fig. 1)].

$E_\perp$ and $E_\parallel$ are the wave components in the local magnetic field frame and are related to the field components in the fixed laboratory frame as follows:

$$E_\parallel = E_z\cos\theta+E_y\sin\theta\approx E_z+\theta E_y,$$
$$E_\perp=-E_z\sin\theta+E_y\cos\theta\approx-\theta E_z+E_y; \qquad (3)$$

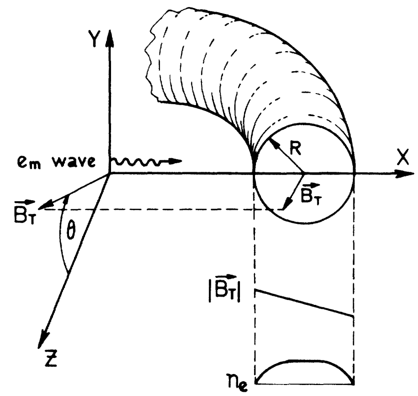

FIG. 1. Model configuration. θ is the angle made by the total magnetic field with the z axis.

or, reciprocally,

$$E_y\approx\theta E_\parallel+E_\perp,$$
$$E_z\approx E_\parallel-\theta E_\perp. \qquad (4)$$

Defining $u=E_\parallel/E_\perp$, we have from Eqs. (1) and (2)

$$\frac{d}{dx}E_\perp^2\frac{du}{dx}+\frac{\omega^2}{c^2}(N_0^2-N_x^2)E_\perp^2u=\frac{d}{dx}\varphi(1+u^2)E_\perp^2. \qquad (5)$$

For the case of weak shear, $\theta(x)$, $\varphi(x)\ll 1$, if $u(x_0)=0$ (x_0 denotes some point inside the plasma) then we expect that $u(x)$ for $x>x_0$ will remain small, i.e., $u(x\geqslant x_0)\ll 1$. If no resonance or cutoff is encountered by the wave for $x>x_0$, and if the plasma is weakly inhomogeneous, the solution of Eq. (2) is just

$$E_\perp\sim\exp\left(\frac{i\omega}{c}\int_{x_0}^x N_x\,dx\right)$$

(propagation in the direction of increasing x is assumed). Substituting this expression for $E_\perp$ into Eq. (5) results in an equation having the following solution for $u(x)$:

$$u(x)=\int_{x_0}^x dx'\varphi e^{i\psi(x')}, \quad N_x/N_0\approx 1, \qquad (6)$$

where

$$\psi(x')=(\omega/c)\int_{x'}^x(N_0-N_x)dx''.$$

Letting b refer to some point in the vacuum outside the plasma, we have from Eq. (4)

$$E_z(b)/E_y(b)\approx-\theta(b)+u(b). \qquad (7)$$

From Eq. (6) we see, for $|\psi|\ll 1$ (corresponding to essentially free-space propagation),

$$u(b)\approx\theta(b)-\theta(x_0). \qquad (8)$$

Thus,

$$E_z(b)/E_y(b)\approx-\theta(x_0)\approx-B_y(x_0)/B_z(x_0);$$

this is just the wave polarization with respect to a fixed system at the point x_0. If plasma conditions for $x > x_0$ are such that $|\psi| \gtrsim 1$ and changing, $u(b)$ no longer has the simple form given by Eq. (8). In this case the final polarization of the wave is determined by an integrated rather than by a local effect. However, even in this case it may still be possible to extract information regarding the shape of the B_y profile, as will be seen in the numerical application.

The system we adopt for illustration of the technique is the Princeton ST Tokamak.[9] The configuration profiles used in the calculations are shown in Fig. 2. The B_y profile was obtained by assuming the current density $j(x) \propto T_e^{3/2}(x)$,[9] with a total current of 50 000 A. The lateral displacement of the temperature maximum from the geometrical center[9] has been neglected in these calculations but should have no bearing on the applicability of the method.

An incident electromagnetic wave is propagated along the decreasing magnetic field towards the point at which the upper hybrid resonance matches it in frequency. At the resonance it has been shown theoretically[10] and verified experimentally[11] that a locally extraordinary wave can be created at the second harmonic, 2ν, of the incident wave frequency by nonlinear generation. We assume that a detectable harmonic can be similarly observed in Tokamaks. In this case, then, x_0 is the point in the plasma where $\nu^2 = \nu_p^2 + \nu_c^2 = \nu_{uh}^2$. We emphasize that locally extraordinary means that the electric field of the harmonic is perpendicular to the local magnetic field.

The coupled system of Eqs. (1) and (2) is solved numerically as a mixed boundary-value problem. A transmitted plane wave is assumed for $E_\perp$ in the vacuum outside the plasma at a point where $B_y = 0$ (at this point $E_\perp = E_y$). Since the wave is locally extraordinary at the resonance, we can write

$$E_\parallel(x_0) = dE_\parallel(x_0)/dx = 0.$$

The computed solutions for $|E_z/E_y|$ for several values of density are shown in Fig. 3 along with the assumed $\theta(x)$ profile. As the density is decreased, the propagation takes on an increasingly free-space character; if the density is sufficiently small, $n_{e\,\max} \approx 10^{12}$, the profile is produced with good precision. As the location of the resonance approaches the plasma boundary, for all densities studied, we obtain good agreement of the profile deduced from $|E_z/E_y|$ with the assumed profile. We see that even for the case of high density, a reasonable indication of the profile shape is obtained far from the bound-

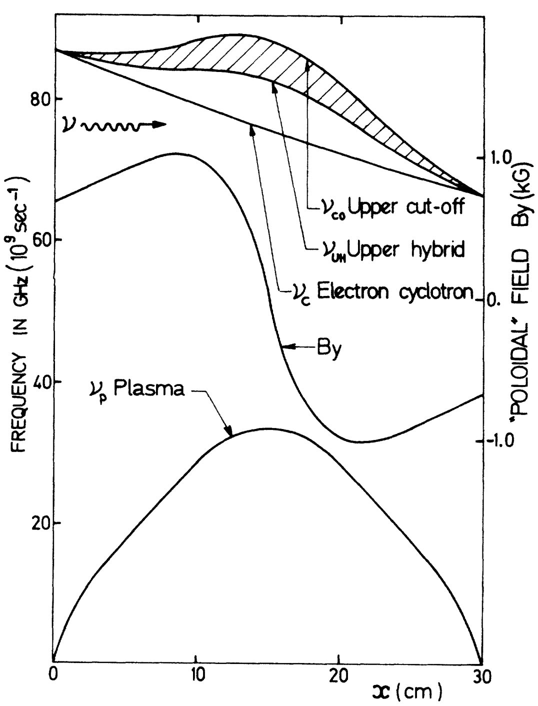

FIG. 2. Princeton ST Tokamak configuration. Spatial variation of characteristic frequencies ν_p, ν_c, ν_{uh}, and ν_{co} and "poloidal" field B_y (assuming $j \propto T_e^{3/2}$, $I = 50\,000$ A).

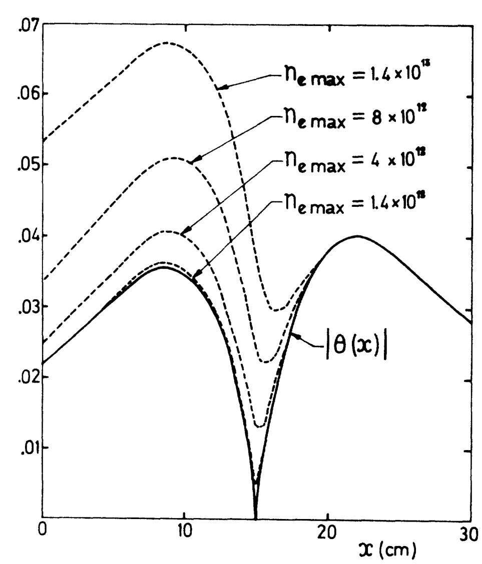

FIG. 3. Comparison of numerically obtained wave polarization $|E_z/E_y|$ (dashed curves) with assumed profile $|\theta(x)| = |B_y/B_z|$ for several values of maximum density $n_{e\,\max}$ (cm^{-3}).

ary, though there is a small displacement of the minimum.

Furthermore, as is evident from Fig. 3, it is possible to obtain an analytic representation for $B_y(x)$ valid very close to the point where $B_y=0$ in the plasma. If the $B_y(x)$ profile is symmetric, then by iteration the entire profile may be deduced from a numerical fit with the observed polarization results.

For experiments in which narrower spatial density profiles are obtained, we expect increasing applicability of the method (of course, the upper hybrid resonance must remain accessible). Finally, we suggest an experiment that can indicate whether a local or an integrated effect is being measured for the wave polarization. In this experiment the relative phase of the second-harmonic ordinary and extraordinary waves is measured. A small phase difference implies $|\psi|\ll 1$, and hence Eq. (8) is applicable (the difference must of course be less than 2π).

We mention briefly another method, using an incident ordinary wave, that may be appropriate to experiments having sufficiently narrow density profiles.

The frequency of the wave is chosen such that the ordinary wave must cross a point in the plasma, x_0, where $\nu=\nu_{co}$ (cutoff frequency for the extraordinary wave), i.e., $N_x(x_0)=0$. The $E_\perp$ component generated for $x<x_0$ is absorbed at the upper hybrid resonance (or converted into longitudinal waves). If wave tunneling is small, then very close to the cutoff, $E_\perp=0$ and $E_y=\theta(x_0)E_\parallel$. As before, if conditions for $x>x_0$ approach those of free space we again have a local determination of the magnetic field direction from a polarization measurement of the transmitted wave, $E_y(b)/E_z(b)=\theta(x_0)$. The utility of this method is more restricted than the harmonic technique as free-space propagation is more readily obtained in the latter case for $x>x_0$.

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[1]J. Sheffield, Center for Plasma Physics and Thermonuclear Research, University of Texas, Report No. CPPT-24, 1970 (unpublished).

[2]M. Murakami, J. F. Clarke, G. G. Kelley, and M. J. Lubin, ORNL Report No. ORNL-TM-3093, 1970 (unpublished).

[3]F. W. Perkins, Princeton University Report No. MATT-818, 1970 (unpublished).

[4]D. E. Evans and P. G. Carolan, Culham Laboratory Report No. CLM-P 252, 1970 (unpublished).

[5]L. Kellerer, in Proceedings of the Fourth European Conference on Controlled Fusion and Plasma Physics, Rome, Italy, August 1970 (to be published).

[6]F. C. Jobes and R. L. Hickok, Nucl. Fusion 10, 195 (1970).

[7]C. Etievant and I. Fidone, Plasma Phys. 12, 985 (1970).

[8]I. Fidone and G. Granata, Nucl. Fusion 11, 133 (1971).

[9]D. J. Grove, D. L. Dimock, E. Hinnov, J. C. Hosea, L. C. Johnson, E. B. Meservey, and E. L. Tolnas, in Proceedings of the Fourth European Conference on Controlled Fusion and Plasma Physics, Rome, Italy, August 1970 (to be published).

[10]I. Fidone, G. Granata, and J. Teichmann, Phys. Fluids 14, 737 (1971).

[11]R. Cano, I. Fidone, M. J. Schwartz, and B. Zanfagna, in Proceedings of the Tenth International Conference on Phenomena in Ionized Gases, Oxford, England, 1971 (to be published).

Electrostatic Confinement of an Alkali-Metal Plasma*

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The imposition of a hollow longitudinal beam of 2-kV electrons about a Q-machine potassium plasma column serves to deepen the radial potential well across that column. Transverse plasma flux is consequently decreased, while the peak ion density and end-plate collector current rise an order of magnitude. A calculation is made of the transverse flux of ions in the plasma column with energy sufficient to surmount the potential well.

Until quite recently, research aimed toward improving plasma confinement has concentrated largely on magnetic field geometries and schemes. Some attention has been given to electrostatic confinement; for example, a device has been proposed (and a forerunner built) to strip and confine heavy ions by immersing them in a relatively dense cloud of fast electrons which, in