Theorem on the Vanishing of Z_2: Evidence from Electron Scattering that the Proton Really Is a Composite Particle*

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We prove that the asymptotic vanishing of the longitudinal part of the total virtual photoabsorption cross section on protons implies the vanihsing of the wave-function renormalization constant Z_2. Present data indicate that $Z_2 \lesssim 0.1$ (consistent with zero) suggesting that the proton is indeed a composite particle.

The probability of finding the proton in its bare elementary state is usually identified with its wave-function renormalization constant Z_2 which plays a central role in conventional field theory. Several authors have suggested that the vanishing of this constant be used as a general criterion for distinguishing an elementary from a composite system.[1] That this criterion does indeed lead to a bound state has been explicitly verified in both the Lee and Zachariasen models.[1,2] In general when Z_2 vanishes, the field representing the particle explicitly decouples from the Lagrangian so that its properties are now completely determined (at least in principle) from those of the "fundamental" fields. The equivalence of this definition of compositeness with that used in S-matrix theory (e.g., the absence of both Castillejo-Dalitz-Dyson poles and Kronecker-delta l-plane singularities) has also been verifed.[3,4] Indeed the vanishing of the renormalization constants for all particles has been used as the basis of a field-theoretic formulation of the bootstrap idea.[4]

Nowadays, it is generally believed that the proton is a composite system either in the bootstrap sense or in the sense that it is built up from more elementary consituents. In either case one would expect Z_2 either to vanish or to be small. It is the purpose of the present paper to present a theorem which can be used to verify this expectation. Loosely speaking, the theorem states that, under certain technical assumptions (to be elucidated below), the vanishing of the longitudinal part of the total virtual photoabsorption cross section on protons (σ_L) in both the Regge and Bjorken limits implies the vanishing of Z_2. Recent experiments appear to be consistent with the asymptotic vanishing of σ_L although it is difficult to rule out a small constant value.[5] *In any case it would appear that experiment is telling us that the proton is indeed predominantly composite in nature.* It is interesting to note that recent popular models of the nucleon which were explicitly designed to explain the electron scattering data are composite in nature[6]; in particular Drell *et al.* used the vanishing of Z_2 as input for their field-theoretic model of the proton.

In order to state the theorem more precisely we introduce the following electromagnetic vertex function of the proton[7]:

$$\Gamma^\mu(p,p') = (\not{p} - M)\int d^4x\, e^{iq\cdot x}\langle 0|\,T[\psi(0)j^\mu(x)]\,|\not{p}\rangle, \tag{1}$$

where $q \equiv p' - p$. From general invariance arguments, Γ^μ must be of the form

$$\Gamma^\mu(p,p') = [(W+\not{p}')/2W][F_1(q^2,W)\gamma^\mu + F_2(q^2,W)i\sigma^{\mu\nu}q_\nu + F_3(q^2,W)q^\mu] + (W \to -W). \tag{2}$$

When $W = M$, F_1 and F_2 are just the conventional Dirac and Pauli form factors, respectively; in particular, $F_1(0,W) = 1$. It will prove convenient to introduce the analog of the charge form factor,

$$G_E(q^2,W) = F_1(q^2,W) + [q^2/(W+M)]F_2(q^2,W). \tag{3}$$

The longitudinal cross section σ_L is defined in the standard fashion; it is related to the conventional structure functions $W_{1,2}$ of inelastic electron scattering via the equation[5,6]

$$W_L(q^2,\nu) \equiv W_1 + \left(\frac{\nu^2 - q^2}{q^2}\right)W_2 = -\left(\frac{q^2 + 2M\nu}{2M}\right)\frac{\sigma_L(q^2,\nu)}{4\pi^2\alpha}. \tag{4}$$

Here ν is the energy of a virtual photon of mass q^2. By the Bjorken (or scaling) limit we mean $q^2 \to -\infty$ with $\omega \equiv -2M\nu/q^2$ fixed; by the Regge limit we mean $\nu \to \infty$ with q^2 fixed. We can now state the *theorem* more precisely: If (a) the $F_i(q^2,W)$ are asymptotically bounded in both variables, (b)

$$\lim_{q^2 \to -\infty}\left[G_E(q^2,M) + 2M\frac{\partial G}{\partial W}(q^2,M)\right] \neq 1,$$

and (c) $q^2\sigma_L \to 0$ in both the scaling and Regge limits, then $Z_2 = 0$.

Now to the proof of the theorem. The vertex function (1) satisfies the Ward-Takahaski identity,

$$q^\mu \Gamma_\mu(p, p')u(p) = \not{q}u(p), \tag{5}$$

which, in terms of the F_i, reads

$$(W-M)F_1(q^2, W) + q^2 F_3(q^2, W) = W - M. \tag{6}$$

Bincer[7] has proven that the F_i are analytic in the cut W plane, and so, using assumption (a), we can write once-subtracted fixed-q^2 dispersion relations in the variable W. Consider the function $G_E(q^2, W)$, Eq. (3); from its definition, G_E has a kinematical pole at $W = -M$ as well as a dynamical cut starting at $W = M + \mu$ (where μ is the pion mass). Although the residue of the pole is known [see (iv) below], we choose rather to write a dispersion relation for the function $[(W+M)/(W+M)^2]G_E(q^2, W)$. We thus obtain the following representation:

$$G_E(q^2, W) = G_E(q^2, M) + 2M\left(\frac{W-M}{W+M}\right) G_E'(q^2, M) + \frac{(W-M)^2}{(W+M)} \frac{1}{\pi} \int_{M+\mu}^{\infty} \left[\frac{(W'+M)\,\mathrm{Im}G_E(q^2, W')}{(W'-M)^2(W'-W)} + (W' \to -W')\right] dW', \tag{7}$$

where the prime on G_E indicates differentiation with respect to W. Using this representation together with Eq. (6) and assumption (a) leads to the following equation:

$$\mathfrak{F}(q^2) \equiv 1 - G_E(q^2, M) - 2MG_E'(q^2, M) = \lim_{W\to\infty} \frac{W}{\pi} \int_{M+\mu}^{\infty} \left[\frac{(W'+M)\,\mathrm{Im}G_E(q^2, W')}{(W'-M)^2(W'-W)} + (W' \to -W')\right] dW'. \tag{8}$$

This is equivalent to Bincer's Eq. (23).

We can determine an upper bound on $\mathrm{Im}G_E(q^2, W')$ by applying the Schwarz inequality to the imaginary part of Eq. (1). A lengthy but straightforward calculation leads to the following result[8]:

$$|(W+M)\,\mathrm{Im}G_E(q^2, W)|^2 \leq \frac{16\pi^2 W^2 (W-M)^2(-q^2)\rho_1(W^2) W_L(q^2, W^2)}{(W-M)^2 - q^2}, \tag{9}$$

where we have introduced the spectral function of the proton, $\rho_1(W^2)$. By using this in Eq. (8) we obtain the inequality

$$|\mathfrak{F}(q^2)| \leq 4(-q^2)^{1/2} \lim_{W\to\infty} W \int_{(M+\mu)^2}^{\infty} \frac{\rho_1^{1/2}(W'^2) W_L^{1/2}(q^2, W'^2)\, dW'^2}{(W'-W)(W'-M)[(W-M)^2 - q^2]^{1/2}} \tag{10}$$

Now, from the Lehmann spectral representation one can derive the well-known sum rule for Z_2:

$$Z_2^{-1} = 1 + \int_{(M+\mu)^2}^{\infty} \rho_1(W^2)\, dW^2. \tag{11}$$

Provided $\sigma_L \to 0$ in the Regge limit [assumption (c)] we can use this representation together with the Schwarz integral inequality on (10) to derive the inequality

$$|\mathfrak{F}(q^2)|^2 \leq 16(Z_2^{-1} - 1)(-q^2) \int_{(M+\mu)^2}^{\infty} \frac{W_L(q^2, W^2)\, dW^2}{(W-M)^2[(W-M)^2 - q^2]}. \tag{12}$$

Transforming to the "scaling" variable ω allows us to express (12) in the convenient form

$$Z_2^{-1} - 1 \geq \frac{|\mathfrak{F}(q^2)|^2}{16\int_1^\infty d\omega\, W_L(q^2, \omega)/\omega(\omega - 1)}, \tag{13a}$$

or

$$Z_2^{-1} - 1 \geq \frac{\pi^2\alpha\, |\mathfrak{F}(q^2)|^2}{-4\int_1^\infty (d\omega/\omega) q^2 \sigma_L(q^2, \omega)}. \tag{13b}$$

The proof of the theorem can now be completed by taking the limit $q^2 \to -\infty$ of these inequalities and by using assumptions (b) and (c), for in that case the right-hand side diverges and we deduce that $Z_2 = 0$, q.e.d.

Remarks.—(i) A sufficient (but *not* necessary) condition for the dispersion relation (7) to converge is that $\rho_1(W^2)\sigma_L(q^2, W^2) \to 0$ in the Regge limit, which is a relatively weak requirement. On the other hand a sufficient (but *not* necessary) condition for a once-subtracted dispersion relation for F_3 to converge is that $W^2\rho_1(W^2)\sigma_L(q^2, W^2)$

$\to 0$ in the Regge limit, which is a much stronger requirement. However, it should be noted (see Ref. 8) that in our inequalities only states with the quantum numbers of the nucleon contribute to σ_L, and it is quite possible that this partial cross section has a more convergent asymptotic behavior than the complete cross section. In any case we choose to view once-subtracted dispersion relations as an independent assumption.

(ii) Assumption (a) applied to Eq. (6) implies that $F_1(q^2, \infty) = 1$; this is the content of Eq. (8). Partial support for this, as well as for assumption (a) itself, can be obtained by investigating the application of Bjorken's "theorem" to Eq. (1).[9] Any reasonable model for the commutator $[\psi(0), j^\mu(0, \vec{x})]$ does indeed lead to $F_1 \to 1$ and is consistent with once-subtracted dispersion relations for the F_i.

(iii) Empirically,

$$\lim_{q^2 \to -\infty} G_E(q^2, M) = 0;$$

it is likewise very reasonable to assume that[10]

$$\lim_{q^2 \to -\infty} G_E'(q^2, M) = 0.$$

In that case

$$\lim_{q^2 \to -\infty} \mathfrak{F}(q^2) = 1.$$

Experiment also seems to support the Bjorken conjecture that the scaling limits[5,6]

$$\lim_{q^2 \to -\infty} W_1(q^2, \omega) \equiv F_1(\omega),$$

$$\lim_{q^2 \to -\infty} \nu W_2(q^2, \omega) \equiv F_2(\omega)$$

exist. These clearly imply the existence of

$$\lim_{q^2 \to -\infty} W_L(q^2, W) = F_L(W).$$

F_L can be expressed in terms of the more popular functions F_2 and R (the ratio of longitudinal to transverse cross sections) by the equation

$$F_L = \tfrac{1}{2}\omega F_2 R/(1+R). \qquad (14)$$

The inequality (13) can now be written in a form more useful for estimating an upper limit for Z_2 in the case where $R \neq 0$:

$$\frac{Z_2}{1-Z_2} \leq 8\int_1^\infty \frac{d\omega}{(\omega-1)} F_2(\omega)\left(\frac{R}{1+R}\right). \qquad (15)$$

Although R is consistent with zero in the scaling limit (it can be parametrized by the function $-q^2/\nu^2$), it can alse be fitted by a constant ~ 0.18.[6] Using the data for $F_2(\omega)$ and assuming the integral in (15) still converges, we can estimate an upper limit for Z_2. We find $Z_2 \leq 0.1$. This is a remarkably small upper limit and supports the view that the proton is predominately composite.

(iv) A slightly alternative way of proving the theorem is to write dispersion relations for $G_E(q^2, W)$, taking into account the pole at $W = -M$. Equations (13) can again be derived with the only difference being that $\mathfrak{F}(q^2)$ is replaced by the combination $1 - G_E(q^2, M) - (q^2/2M)F_2(-q^2, M)$. Condition (b) of the theorem can therefore be replaced by the assumption that

$$\lim_{q^2 \to -\infty} [G_E(q^2, M) + (q^2/2M)F_2(q^2, -M)] = 0.$$

Empirically we know that

$$\lim_{q^2 \to -\infty} F_2(q^2, M) = 0$$

and so the assumption is a reasonable one.[10]

(v) The theorem can also be proved for the case of scalar or pseudoscalar particles. The result is actually somewhat neater than the one considered here in that the function $\mathfrak{F}(q^2)$ turns out to be $1 - F(q^2)$, where $F(q^2)$ is the particle's on-shell elastic form factor.

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[1]See, e.g., J. C. Houard and B. Jouvet, Nuovo Cimento 18, 466 (1969); M. T. Vaughan, R. Aaron, and R. D. Amado, Phys. Rev. 124, 1258 (1961); A. Salam, Nuovo Cimento 25, 224 (1962); S. Weinberg, Phys. Rev. 130, 776 (1963), and 137, B672 (1965). In this last reference Weinberg shows that the empirical values of the triplet n-p scattering length together with the effective range leads to a small value of Z_2 for the deuteron ($Z_2 < 0.2$).

[2]See, e.g., R. Acharya, Nuovo Cimento 24, 870 (1962); I. S. Gerstein and N. G. Deshpande, Phys. Rev. 140, B1643 (1965).

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[6]For example, J. D. Bjorken and E. Paschos, Phys. Rev. 185, 1975 (1969); S. D. Drell, D. J. Levy, and T. M. Yan, Phys. Rev. 187, 2159 (1969).

[7] A. Bincer, Phys. Rev. 118, 855 (1960).

[8] Such a calculation has also been carried out in a paper which is closely related to this one, F. Cooper and H. Pagels, Phys. Rev. D 2, 228 (1970). My result (9) differs from theirs by a factor of 2; however it should be noted that the right-hand side of their inequalities (2.15) appear to be inconsistent with the right-hand side of their (2.7), (2.10), and (2.11) by precisely this factor of 2. The factor is inessential to the proof of the theorem but raises the upper bound on Z_2 from 0.1 to 0.2 [see (iii) below].

[9] J. D. Bjorken, Phys. Rev. 148, 1467 (1966).

[10] We have unsuccessfully attempted to prove that under quite general assumptions (such as those required by the existence of a Deser-Gilbert-Sudarshan representation) the asymptotic vanishing of $G_E(q^2,M)$ implies the asymptotic vanishing of $G_E'(q^2,M)$. A similar remark applies to $F_2(q^2, -M)$ *vis-à-vis* $F_2(q^2,M)$.

Production of Intermediate Bosons in Strong Interactions*

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From data on muon pairs and by use of the hypotheses of conservation of vector current and scale invariance, we estimate the cross section for production of weak vector mesons $W^\pm$ at energies pertinent to Brookhaven National Laboratory, the CERN intersecting storage rings, and the National Accelerator Laboratory. With existing data, these estimates imply a lower limit of 4.5 GeV/c^2 for the $W^\pm$ mass.

Several searches for the weak intermediate boson (W) have been carried out using the reaction

$$p + Z \to W + \text{anything}, \tag{1}$$

with the decay of the W into muons as the signature.[1,2] Failure to observe a muon signal from any source other than the decay of pions or kaons has led to limits on the product of the cross section and branching ratio of $\sigma_W B < 2\times10^{-34}$ cm^2. A recent experiment[3] which measured both the intensity and polarization of muons produced by the interaction of 28-GeV protons finds $\sigma_W B < 6\times10^{-36}$ cm^2.

A process very similar to Reaction (1),

$$p + Z \to \mu^+ + \mu^- + \text{anything}, \tag{2}$$

has recently been observed at Brookhaven National Laboratory (BNL) in 30-GeV proton-uranium collisions.[4] The considerable similarity between these two processes holds out the hope of determining the W-production cross section from direct measurement of the μ-pair (or e-pair) process. After summation over all possible final hadronic states, the differential cross sections for each of these processes can be written[5]

$$\frac{d\sigma_{W^\pm}(Z)}{d^3q} = \frac{G M_W{}^2 (2\pi)^3}{2\sqrt{2} q_0 v}(\delta^{\mu\nu} + q^\mu q^\nu/M_W{}^2)\int e^{+iq\cdot x} d^4x \langle (pZ)^{\text{in}}|[V_\mu{}^\pm(x) + A_\mu{}^\pm(x)][V_\nu{}^\mp(0) + A_\nu{}^\mp(0)]|(pZ)^{\text{in}}\rangle \tag{3}$$

and

$$\frac{d\sigma_{l^+l^-}(Z)}{d^3q\,dm} = \frac{4\alpha^2(2\pi)^3}{3q_0 m v}(\delta^{\mu\nu} + q^\mu q^\nu/m^2)\int e^{+iq\cdot x} d^4x \langle (pZ)^{\text{in}}| J_\mu{}^\gamma(x) J_\nu{}^\gamma(0)|(pZ)^{\text{in}}\rangle, \tag{4}$$

where $V_\mu{}^\pm$, $A_\mu{}^\pm$, and $J_\mu{}^\gamma$ are the weak vector, weak axial-vector, and electromagnetic currents, G the Fermi constant, q the four-momentum of the W, and v the magnitude of the relative velocity between the incident proton and the target Z. Equation (4) gives the cross section for the production of either electron or muon pairs, $l^+l^- = e^+e^-$ or $\mu^+\mu^-$. In computing this lepton-pair cross section [Eq. (4)], we have neglected the lepton mass, integrated over all configurations of lepton momenta with fixed total leptonic four-momentum q, and defined m as $\sqrt{(-q^2)}$. Although not explicitly shown, averages of beam- and target-particle spins are to be carried out in Eqs. (4) and (5).

The following remarks can be made about the connection[6,7] between these two cross sections provided by conserved vector current theory (CVC):

(i) Since a Lorentz-invariant pseudoscalar cannot be formed out of three four-momenta, the odd-