Spin-0 Kemmer Wave Functions and K_{l3} Form Factors

Bengt Nagel and Håkan Snellman
Department of Theoretical Physics, Royal Institute of Technology, S-100 44 Stockholm 70, Sweden
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The assertion in a recent Letter, stating the nonequivalence of the Kemmer and the Klein-Gordon spin-0 wave functions in the analysis of K_{l3} decays, is shown to be incorrect.

In a recent paper[1] it is claimed that by using five-component Kemmer wave functions instead of one-component Klein-Gordon wave functions to describe the spin-0 particles, one would get a qualitatively different phenomenological analysis of the form factors in K_{l3} decay. In particular, the scalar form factor $f_0(t)$ is predicted to have a zero at $t=t_0\equiv(m+\mu)^2$ [m and μ are the kaon and pion masses, respectively]. Since the five Kemmer components are just the scalar wave function and its four space-time derivatives, this "result" is astonishing. We shall show that it is in fact due to an unduly restricted choice of couplings of two Kemmer wave functions to form a vector.

The central point in the argument of Ref. 1 is that in the "Kemmer parametrization" of the hadronic vector-current matrix element

$$\langle\pi(p')|V_\lambda(0)|K(p)\rangle\sim\bar{u}_\pi(p')\{\beta_\lambda g_V(t)+[iq_\lambda/(m+\mu)]g_S(t)\}u_K(p),\quad q=p-p', \tag{1}$$

the form factors $g_V(t)$ and $g_S(t)$ are assumed to be smooth functions of t; in particular, $g_S(t)$ should not have a pole at $t=t_0$. Since $\bar{u}_\pi(p')u_K(p)=[(m+\mu)^2-t]/4m\mu$, this can be shown to imply that $f_0(t)$ has a zero at t_0. Here $f_0(t)=f_+(t)+[t/(m^2+\mu^2)]f_-(t)$, where $f_\pm(t)$ are the form factors in the ordinary parametrization

$$(p+p')_\lambda f_+(t)+(p-p')_\lambda f_-(t) \tag{2}$$

of the matrix element.

If $g_S(t)$ has a pole at t_0, then firstly, $f_0(t)$ does not have a zero at t_0, and secondly, the linear *Ansatz* $g_S(t)=g_S(0)(1+\gamma_S t/\mu^2)$ in $0\le t\le(m-\mu)^2$ cannot be maintained, so that the whole analysis of Ref. 1 is invalidated.

We give below the result of three different couplings of free Kemmer fields to a vector current $[\Gamma_\lambda=(i/3)(\beta_\lambda\beta_\nu\beta_\nu-\beta_\nu\beta_\nu\beta_\lambda)]$:

$$i\bar{\psi}_\pi(x)\beta_\lambda\psi_K(x)\sim i\bar{u}_\pi(p')\beta_\lambda u_K(p)=\tfrac{1}{2}(p_\lambda'/\mu+p_\lambda/m), \tag{a}$$

$$i\partial_\lambda[\bar{\psi}_\pi(x)\psi_K(x)]\sim-q_\lambda\bar{u}_\pi(p')u_K(p)=-q_\lambda([m+\mu]^2-t)/4m\mu, \tag{b}$$

$$\bar{\psi}_\pi(x)\Gamma_\lambda\psi_K(x)\sim\bar{u}_\pi(p')\Gamma_\lambda u_K(p)=\tfrac{1}{2}(p_\lambda'/\mu-p_\lambda/m). \tag{c}$$

(a) and (b) give rise to the two terms in (1), with constant $g_V(t)$ and $g_S(t)$, whereas (c) gives a combination of both, with $g_S(t)\sim(t-t_0)^{-1}$. Evidently, a suitable combination of (a) and (c) will correspond to the ordinary parametrization (2). Of course, there is no reason why only the induced couplings of types (a) and (b) should exist, and not (c); *hence the Kemmer approach is equivalent to the ordinary one.*

The only way in which the use of the Kemmer wave functions could give a definite prediction is if we make the *ad hoc* assumption that the relevant vector-current matrix element is approximated, in the region of interest, by, for example, the "simplest" Kemmer vector current $i\bar{\psi}_\pi(x)\beta_\lambda\psi_K(x)$, treated in the free-field approximation. This then gives constant form factors $f_\pm(t)=f_\pm(0)$ and a ratio $f_-(0)/f_+(0)=-(m-\mu)/(m+\mu)=-0.57$. It seems probable that these "predictions" could find experimental support by a judicious averaging of the present experimental data.

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[1]E. Fischbach, F. Iachello, A. Lande, M. M. Nieto, and C. K. Scott, Phys. Rev. Lett. 26, 1200 (1971).