available experiments than $G_{PP(f)}{}^2$. As our analysis depends crucially on treating the Pomeranchuk trajectory as a Regge pole, verification of our results would help to clarify the nature of the Pomeranchuk singularity and shed light on related multiple-Pomeranchukon phenomena.[3]

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[6]A remark on the relation between (1) and (3) is in order. If the pole at $m_c{}^2$ is a stable particle pole, then (1) is exact; otherwise, it is only approximate since resonance poles have finite widths. Equation (3) is an asymptotic expansion which, in general, has a well-defined meaning only if the limit is taken away from the real axis. Just as in the case of two-body reactions, duality is used here to assert that these two modes of description are equivalent.

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[8]Details of the present work within the context of the dual-resonance model, along with that of Ref. 4, will be presented in a forthcoming paper.

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[10]Note that the factors $\beta_{a\bar{x}}/\sin\pi\alpha_{a\bar{x}}$, etc., associated with the external Regge residues and propagators are removed from R_2 as well as from R_4. Our definition of R_2 and R_4 is analogous to that of a "reduced" Regge residue in the ordinary two-body amplitude. It is these reduced Reggeon amplitudes which are real analytic in α. Our definition of the Reggeon amplitudes thus differs from that used by Wang and Wang in Ref. 9.

[11]The ω-angle dependence discussed in the literature (see for example, Refs. 2 and 4) comes from the dependence of the two-Reggeon–particle vertex G on K_1. For $s_{a\bar{1}} \cong 0$ or $s_{b\bar{2}} \cong 0$, one may ignore this added complication as discussed in Ref. 3.

[12]H. Harari, Phys. Rev. Lett. 20, 1395 (1968); P. G. O. Freund, Phys, Rev. Lett. 20, 235 (1968). It follows from (5) that the triple-Pomeranchukon contribution to inclusive cross sections is proportional to $\sin\{\pi[\alpha_P(0) - 2\alpha_P(t)]\} \times g_{PPP}(t)$, which will vanish at $t=0$ if (i) $\alpha_P(0) = 1$ and (ii) $g_{PPP}(t)$ does not contain the wrong-signatured nonsense fixed pole. This is to be contrasted with the case of Compton scattering where the total cross section is proportional to $\sin\{\pi[\alpha_P(0) - 2]\} g_{\gamma\gamma P}(0) \sim [\alpha_P(0) - 1]^{-1}$.

[13]In principle, (10) can be used to calculate the triple-Reggeon parameters g_{ijV} in terms of detailed resonance information, with arbitrary cutoffs. We have chosen N and Δ so as to exclude the η meson contribution. This can be justified if duality holds "locally." At any rate, since the η contribution is probably small, we thus believe that our results (12) are qualitatively sound.

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Discrete Scale Transformations and a Possible Lepton Mass Spectrum*

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A partial solution to the muon puzzle is offered. Discrete scale transformations are used to derive a mass spectrum for leptons which includes all the observed leptons. The difference between the electron and muonlike particles is demonstrated explicitly. Experimental tests of the theory are discussed including the detection of the predicted 22-GeV electron.

We address ourselves to the muon puzzle and offer a partial solution, suggesting that the electron and muon may be the lowest charged members of a family of leptons. We derive a lepton mass spectrum which includes all known leptons, as well as new ones, from a new kind of scale transformations which are discrete.

Consider the Lagrangian density for a Dirac field given by

$$\mathcal{L} = -\bar{\psi}\gamma_\mu \partial_\mu \psi - m\bar{\psi}\psi. \quad (1)$$

The Dirac field has dimension $l = -\frac{3}{2}$ in the absence of interactions. Hence the scale transformation

$$x_\mu \to x_\mu' = \lambda x_\mu$$

transforms $\psi(x)$ as

$$\psi(x) \to \psi'(x') = \lambda^{-3/2}\psi(\lambda x).$$

Instead of considering continuous scale transformations corresponding to continuous λ as is customary, we consider discrete transformations

$$x_\mu \to x_\mu' = \rho x_\mu,$$

where ρ is now not a variable parameter, but instead takes some fixed value. Hence the most general discrete scale transformation can be written in the form

$$x_\mu \to x_\mu' = \rho^n x_\mu \quad (n = \cdots, 3, 2, 1, 0, -1, -2, -3, \cdots),$$
$$\psi(x) \to \psi'(x') = \rho^{-3n/2}\psi(\rho^n x). \quad (2)$$

We take ρ to be the ratio of muon and electron masses ($\rho = m_\mu/m_e \simeq 207$). No attempt is made to explain the number ρ; instead ρ is regarded as a fundamental constant. (We do not rule out the possibility that ρ can be explained later in terms of fundamental coupling constants such as α, κ, etc.)

The transformation (2) takes

$$\mathcal{L} \to \mathcal{L}' = \rho^{-4n}[-\bar{\psi}'\gamma_\mu \partial_\mu'\psi' - (m\rho^n)\bar{\psi}'\psi'], \quad (3)$$

where the primes denote the transformed quantities. $\mathcal{L}'$ is now of the same form as $\mathcal{L}$ with m replaced by $m\rho^n$. This result is interpreted as follows: The existence of a Dirac field whose quanta have mass m implies the existence of other Dirac fields with quanta of masses $m_n = m\rho^n$. (n is a positive or negative integer.) In other words, we demand form invariance under our scale transformations.

We do not restrict ρ to be positive. Then for *odd* values of n with *negative* ρ, our requirement above leads to negative masses $m_n = m\rho^n$. However, if we define the transformation for $\rho < 0$ to be

$$x_\mu \to x_\mu' = \rho^n x_\mu \quad (\rho < 0),$$
$$\psi(x) \to \psi'(x') = \rho^{-3n/2}\gamma_5{}^{|n|}\psi(\rho^n x), \quad (4)$$

then it takes $\mathcal{L}$ given by (1) into the Lagrangian density for another Dirac field of mass $m|\rho|^n$, because the transformation (4) reduces to (2) when $\rho = -\sigma$ ($\sigma > 0$) for even n, and it reduces to

$$x_\mu \to -\sigma^n x_\mu,$$
$$\psi(x) \to \psi'(x') = -\sigma^{-3n/2}\gamma_5\psi(-\sigma^n x), \quad (5)$$

when n is odd. This changes $\mathcal{L}$ to

$$\mathcal{L}^1 = -\sigma^{4n}[-\bar{\psi}'\gamma_\mu\partial_\mu'\psi' - (m\sigma^n)\bar{\psi}'\psi'].$$

The factor γ_5 in front of $\psi(-\sigma^n x)$ in (5) mixes the field components of the spinor. *Hence even and odd fields differ by the factor* γ_5. If we assume that when $n = 0$, $\mathcal{L}$ is the Lagrangian density for the electron field, *then all fields corresponding to even n are electronlike and carry the electron lepton number, while all odd fields are muonlike and carry the muon lepton number.* Apart from the dilation factor σ, the transformation

$$\psi(x) \to \psi'(x') = -\sigma^{3/2}\gamma_5\psi(-\sigma x),$$

which takes electron field into the muon field, is similar to a parity transformation.

The charge on a lepton is defined to be

$$Q = \tfrac{1}{2}e(1 + n/|n|),$$

which is the analog of the Gell-Mann–Nishijima relation for leptons. Particles with negative n then have zero charge, and the leptons in the series with negative n can be interpreted as neutrinos. The electromagnetic interaction of leptons takes the form $\frac{1}{2}e(1+n/|n|)\bar{\psi}\gamma_\mu\psi A_\mu$.

The predicted lepton spectrum consists of an infinite number of charged and neutral members (see Fig. 1). The existence of leptons with the

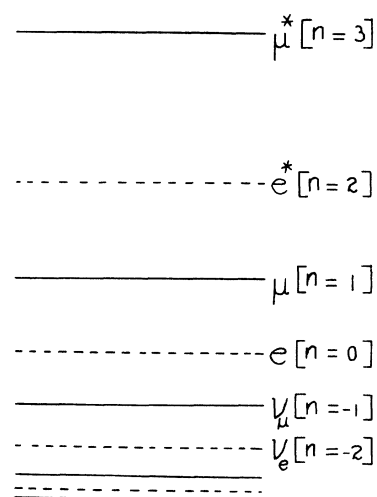

FIG. 1. The predicted lepton mass spectrum. (Not to scale. It is amusing that the masses would have equal spacing on a log scale.)

masses obtained from our theory would not lead to any inconsistency regarding the magnetic moments of the electron or muon.[1]

The number n specifies a lepton completely and is a "universal quantum number" for leptons. The masses of leptons are given by $m_n = m_e \rho^n$. The leptons with zero or even n are electronlike, those with odd n are muonlike, leptons with positive n are charged, and those with negative n are neutral (neutrinos). The lepton just above the muon, denoted by e^*, is an electronlike particle of mass 22 GeV; the next lepton μ^* is muonlike of mass 4554 GeV, and so on. Also, according to this theory, the heaviest neutrino is muonlike and has a mass 2.5×10^{-3} MeV. The neutrino just below it is electronlike and has a mass of 12.1 eV. The current experimental upper limit for the mass of muon neutrino is 1.6 MeV and for the mass of the electron neutrino is 60 eV. The finite masses of the neutrinos lead to the possibility that there are time delays in the arrival of photons and different neutrinos from an event such as a supernova explosion.

Apart from mass, all neutrinos of the same type may behave as identical particles. However, there is nothing at present to rule out the possibility that each charged lepton is coupled to different neutrinos of the same type with different coupling constants. Yet another possibility is that the basic field with $n = 1$ is not the electron field but is a doublet of the electron and the electron neutrino which has zero mass. Then our scale transformation can be easily seen to generate a spectrum of leptons which is the same as before except that now there are only two neutrinos, ν_e and ν_μ, both with mass zero. An extremely accurate determination of the masses of the neutrinos can decide which of these alternatives is realized in nature.

In the present scheme, the muon number is neither additive nor multiplicative, but there exists an operator which takes one from an electron to a muon. In order to make a statement about processes like $\mu^+ + e^- \to \mu^- + e^+$ (which is allowed by a multiplicative muon number), we have to assume something about interactions, e.g., if we assume form invariance for the four-fermion interaction, then $\mu^+ + e^- \to \mu^- + e^+$ is related to $e^- + \nu_\mu \to e^- + \nu_\mu$ and is thus of order G^2 (in the matrix element).

Decay modes of charged leptons.—Heavy charged leptons can decay into (i) leptons lower in the series, (ii) mesons and leptons, (iii) baryons and antibaryons and leptons, (iv) γ rays and leptons via electromagnetic decay, and (v) leptons and intermediate bosons (if the intermediate boson exists). The decay rate of e^* for the mode $e^* \to \mu + \bar\nu_\mu + \nu_e$ can be calculated using the formula for the muon decay rate with the muon mass replaced by the e^* mass. The mean partial lifetime for the above modes is about 6×10^{-18} sec. The mean life for the decay mode[2] $e^* \to \pi + \nu$ is about 10^{-10} sec. The hadronic decay modes are probably suppressed[3] because of the influence of form factors.

A possible form of the interaction[4] of e^* and e with the electromagnetic field is

$$\lambda(e/m)\bar\psi_1 \sigma_{\mu\nu}\psi_2 F_{\mu\nu}, \tag{6}$$

where ψ_1 and ψ_2 are the e^* and e fields, m is the mass of the e^*, and λ is a constant. The above interaction can give rise to the decay $e^* \to e + \gamma$ with a mean partial lifetime

$$\tau \simeq \pi m^2/e^2\nu^3\lambda^2,$$

where ν is the frequency of the emitted photon and m is the mass of the e^*. For $\lambda = 1$, this yields $\tau \simeq 4\times10^{-17}$ sec.

Can the neutrinos decay?—An interesting question is whether the neutrinos can undergo decay down the series. If the usual four-fermion interaction is assumed to exist between various neutrinos, then the decay $\nu_1 \to \nu_2 + \nu_3 + \bar\nu_4$ (the heaviest muon neutrino is denoted by ν_1) occurs with a mean life time of 10^{10} yr. Hence, if the interaction between various neutrinos is the usual Fermi interaction, then they are practically stable. But, such properties of neutrinos might have astrophysical consequences.

Experimental detection of e^*.—All experiments so far carried out using either electron synchrotrons or colliding beams exclude the possibility of having a lepton of mass less than 1 GeV. Colliding electron-positron beams can produce the heavy electron via $e^+ + e^- \to e^{*+} + e^{*-}$. The total cross section[5] for the above process is approximately 10^{-34} cm^2. Although the cross section is small, it may be possible to detect the characteristic 11-GeV γ rays produced through electromagnetic decay. The e^* can also be produced in e-p or γ-p scattering. The differential cross section for this process has been calculated by Gutbrod and Schildknecht.[6] The e^* produced this way can be detected through the electrons and muons with very high transverse momenta that are produced by its decay. Another way to produce e^* is the high-energy neutrino reaction $\nu_e + e \to \nu_e + e^*$.

Possible evidence for the e^* in cosmic rays.—According to Kaufman and Mongan,[7] the cosmic-ray flux measurements obtained by the calorimeters on the satellites Proton I and Proton II imply the existence of a particle of mass of about 19 GeV. The new particle is probably produced by p-p or C-p collisions. It is hard to determine the lifetime of the particle from cosmic-ray data. A lifetime of 10^{-18} sec (which is the estimated lifetime of the e^*) does not exclude the possibility that the above particle is the e^*.

The best (i.e., the one facing the least number of difficulties) explanation for the "X process" in the Utah effect in cosmic-ray muons[8] seems to be that suggested by Bjorken.[9] The model requires all of (a) $p+p \to X+\bar{X}+$hadron, (b) decay of the X^- into a left-handed μ^-, and (c) the possibility of muon shower formation through $\mu+N \to X+\bar{X}+\mu+$hadrons. If X is taken to be the e^*, then all of these are fulfilled since $e^{*-} \to \mu^- + \nu_e + \bar{\nu}_\mu$ produces predominantly left-handed μ^-'s and is fast enough. This model also explains the muon showers observed[10] as being due to (c) above. Furthermore, an analysis of the Utah data on this basis leads to a mass of X between 10 and 30 GeV.[11] The problem of obtaining a sufficiently large cross section for the process (a) remains.

The electron at 22 GeV may also account for extensive air showers and muon-free air showers, provided there is a mechanism for production of the e^* with a cross section[12] around 10^{-29} cm^2. The electromagnetic decay will account for muon-free air showers and the leptonic decay mode will acount for horizontal air showers.

Possibility of detecting the heavy muon μ^* in cosmic rays.—The μ^* has a mass of 4554 GeV (4.5 TeV). The partial lifetimes of the pure leptonic decay modes $\mu^* \to e + \nu_\mu + \bar{\nu}_e$ and $\mu^* \to \nu_\mu + \bar{\nu}_\mu$ are about 10^{-29} sec. The mesonic decay modes have partial lifetimes of the order 10^{-17} sec. For an electromagnetic decay mode due to an interaction of the form (6), the partial lifetime is of the order 10^{-19} sec. The total width of the μ^* is close to 100 TeV. Although the estimated width of μ^* is abnormally large, certain anomalous properties of cosmic-ray muons may be manifestations of such a heavy muon. There are some experimental indications of an unusually large rate for energy loss of cosmic-ray muons in the TeV region.[8] The major contribution to the energy loss of cosmic-ray muons in the above energy region comes from bremsstrahlung. If the μ^* exists, the muon propagator is modified due to the presence of a virtual μ^* and the cross section of the bremsstrahlung process is enhanced as a result of this resonance effect. This can explain at least part of the increased energy loss of muons of energies of a few TeV.

In conclusion then, we present an admittedly speculative theory of leptons which can, nevertheless, be tested rigorously by measuring neutrino masses and by proving or disproving the existence of heavy electrons and muons of predicted masses. A more detailed version of this work will be published elsewhere; there the alternate schemes mentioned earlier, the possible schemes of interactions, and the analysis of available cosmic-ray data will be treated in detail.

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