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Anomalies in Quasifree Scattering from $p + {}^3$He Reactions*

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The reactions $He^3(p,pd)p$ and $He^3(p,2p)d$ have been studied at 35 MeV. A dominant feature of these data is p-p and p-d quasielectric scattering. A plane-wave impulse approximation gives a qualitative fit to the $He^3(p,2p)d$ data, but fails to explain the ${}^3He(p,pd)p$ data. Various possible explanations are discussed.

Many quasifree scattering (QFS) experiments have been performed and the data have been analyzed using the plane- or distorted-wave imulse approximation[1] with the aim of extracting nuclear structure information. In general, these methods have been moderately successful even at comparatively low energies. However, the inadequacy of the plane-wave impulse approximation (PWIA)[2] has been demonstrated[3] even in $p+d$ reactions, and it was pointed out[4,5] that contributions from higher-order terms could not be neglected.

Therefore, it seemed interesting to study the QFS in $He^3(p,2p)d$ and $He^3(p,pd)p$ experiments. If the mechanisms in the $p+He^3$ reaction were understood, one could try to extract crucial spectroscopic data of the $A=3$ system, for instance the He^3 ground-state wave function.

The target used was 99.99% pure isotopic He^3 gas, which was bombarded with the 35.0-MeV proton beam of the University of California at Los Angeles (UCLA) cyclotron. The reaction products were observed using two solid-state detector telescopes. The angular resolution of each arm was better than ± 2°. $E+\Delta E$ and ΔE signals from the detectors were stored in an online XDS-925 computer, thereby enabling us to measure both reactions, $He^3(p,2p)$ and $He^3(p,pd)$, simultaneously. The energy was determined accurate to better than ± 150 keV.

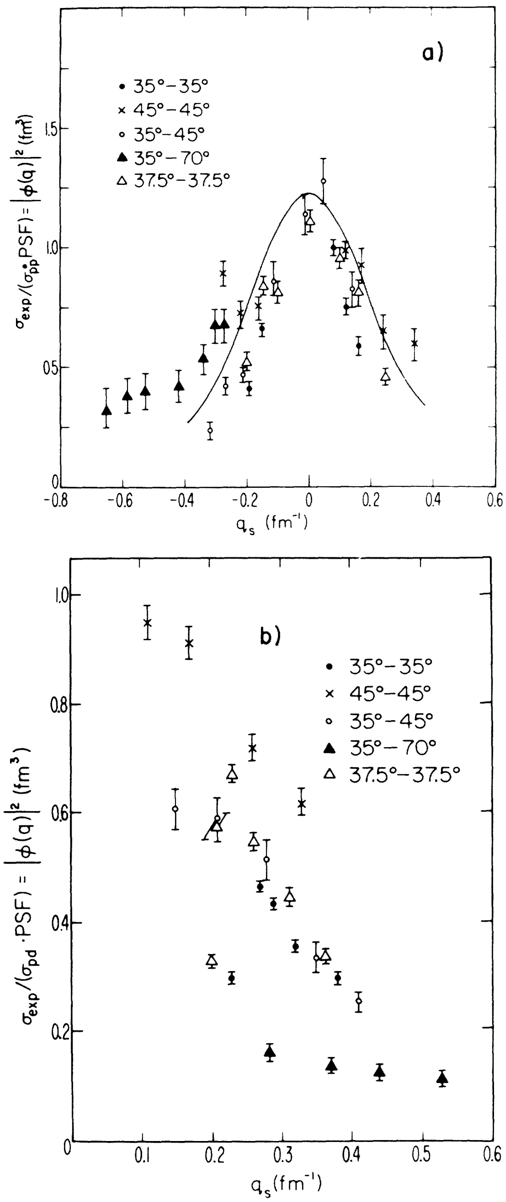

FIG. 1. (a) Experimental ^{3}He$(p, 2p)d$ cross section divided by the free pp cross section and phase-space factor, as a function of the momentum transfer. The curve is the result of a calculation using the overlap of the Irving-Gunn ^{3}He wave function with the Hulthén deuteron wave function. The curve has been normalized to the data. (b) Experimental ^{3}He$(p,pd)p$ cross section divided by the phase space and the free pd cross section.

Data were taken at various sets of angles. Only the data taken relevant to QFS will be reported here. Both $(p, 2p)$ and (p, pd) data exhibit broad QFS enhancements in the cross sections. Figure 1 shows the square of the p-d cluster momentum wave function $|\varphi(\vec{q}_s)|^2$ of He^3 extracted from the $(p, 2p)$ data [Fig. 1(a)] and (p, pd) data [Fig. 1(b)] using the PWIA.[6] σ_{free} was determined using the relative p-p or p-d energy in the final state. If the PWIA were correct, the same $\varphi(\vec{q}_s)$ should be obtained from the data taken at different angles and, since the He^3 wave function is predominantly an S state, the $|\varphi(\vec{q}_s)|^2$ should show a maximum at $\vec{q}_s = 0$. These features are clearly demonstrated for the reaction He$^3(p, 2p)d$ in Fig. 1(a); thus the calculation of $|\varphi(\vec{q}_s)|^2$ using the overlap of the Irving-Gunn He^3 wave function with the Hulthén deuteron wave function gives a good fit to the $(p, 2p)$ data.

The situation is markedly different for the reaction He$^3(p, pd)p$. As seen in Fig. 1(b), data taken at different angular sets do not yield the same $|\varphi(\vec{q}_s)|^2$. In Figs. 2(a) and 2(b) the data were compared with the PWIA calculation using a Hulthén-type wave function (curve A). It is clear that this calculation does not fit the (p, pd) data. Similar results are obtained if one uses the Irving-Gunn [curve B, Fig. 2(b)] or Irving wave function, or the He^3 wave function which exhibits the effects of the hard core. All the data show that the peak in the cross sections is shifted by ~1–3 MeV from the PWIA predictions, and, further, indicate a structure in the neighborhood of the QFS enhancement.

We have considered various explanations for these anomalies: (1) Shifts of the QFS peaks have been observed in other reactions and it has been suggested[7] that momentum transfer to the system, before the QFS, due to the long-range Coulomb force can explain these shifts. Such an explanation works for the data of Ref. 7, but does not explain the He$^3(p, pd)p$ data. In fact, an attractive long-range force would be required to reproduce the observed shift [Fig. 2(a), curve B]. Although the inclusion of a long-range attractive force in the initial state is questionable, it does improve the fit to the shape of the He$^3(p, pd)p$ data and does not alter the shape of the calculated He$^3(p, 2p)d$ spectra; however, it does introduce serious discrepancies in the relative magnitude of the He$^3(p, 2p)d$ QFS cross section observed at different angles.

(2) The study[8] of the reaction H$^3(p, pd)n$ where we observe a similar shift in QFS peak indicates that the anomaly is not due mainly to Coulomb interaction in the final state.

(3) The effect of the p-p final-state interaction on the QFS was investigated by modifying the PWIA cross section by the multiplicative factor

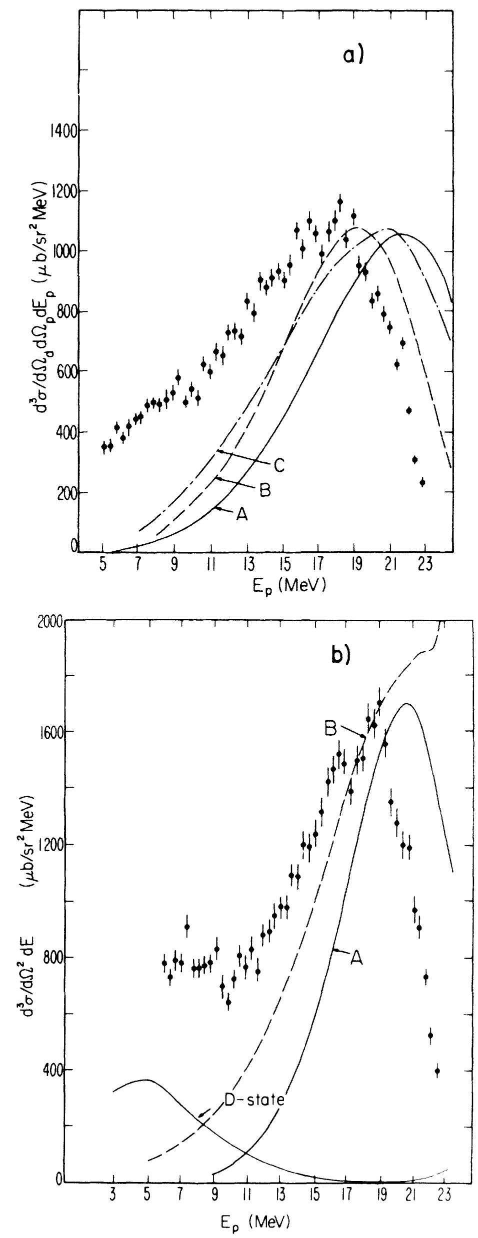

FIG. 2. (a) $^3He(p,pd)p$ cross section at $\theta_p=\theta_d=35°$. Models: curve *A*, Hulthén-type wave function $(e^{-ar}-e^{-br})/r$, $a=0.4203$ fm^{-1}, $b=1.33$ fm^{-1}, using the radial cutoff $R=3.9$ fm; curve *B*, same as *A* with the long-range attraction interaction in the initial state corresponding to the momentum transfer of 100 MeV/*c*; curve *C*, same as *A* with FSI enhancement (see text). All calculations have been normalized; normalization factor $N=0.83$, $N=0.5$, and $N=2.5$, respectively. (b) $^3He(p,pd)p$ cross section at $\theta_p=\theta_d=45°$. Models: curve *A*, Hulthén-type wave function same as in (a), $N=1$; curve *B*, overlap of the Irving-Gunn ^{3}He wave function with the Hulthén deuteron wave function, $N=0.29$; a *D*-state ^{3}He Hulthén-type wave function assuming $P_d=15\%$ and no radial cutoff is also shown.

$(F_0\cos\delta_0+G_0\sin\delta_0)^2/\sin^2 kr$.[9] In the QFS region the *p-p* relative energies range from 4 to 14 MeV and the effective-range approximation cannot be used. Curve *C* in Fig. 2(a) shows the results of such calculations. One sees that inclusion of the *p-p* final state interaction (FSI) as a factor which modulates the QFS process accounts only for a fraction of the energy shift, which is to be expected in view of the large *p-p* relative energies. The *p-d* FSI has not been taken into account.

(4) Various approaches could be used for calculating the $p+d\to p+d$ vertex. Since this vertex is further off the energy shell than in the corresponding case of the reaction D(*p*, 2*p*), we investigated to what extent the inadequate description of the *p-d* vertex contributes to these anomalies. We have extracted σ_{pd} from the PWIA and these σ_{pd} are inconsistent even for $\vec{q}_s=$ const. We interpret this as an indication that the anomalies are not due mainly to the *pd* vertex.

(5) The structure in the ^{3}He(*p*, *pd*) spectra can be due to the *D*-state component of the ^{3}He wave function, resonances in the $p+d$ ststem, and/or the contributions of other reaction mechanisms. Calculation of the *D*-state contribution was done using a Hulthén-type wave function with four exponential terms [Fig. 2(b)]. The structure is much too narrow to be generated by the *D* state. A kinematic analysis of the spectra in terms of $A=3$ resonances has been done, but the interpretation of the structure requires a better understanding of the reaction mechanism.

The study of the angular dependence of the reaction ^{3}He(*p*, *pd*) demonstrates a drastic deviation from QFS [e.g., Fig. 1(b)] and indicates the importance of other mechanisms.

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Reggeon Amplitude and Duality Sum Rule*

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Based on the dual amplitude analysis of the "ditriple" Regge limits in inclusive hadronic reactions, the general properties of the Reggeon-Reggeon amplitude are conjectured and a duality sum rule is suggested. From this sum rule, predictions are made on the two-Reggeon–particle and triple-Reggeon couplings for diffraction dissociations.

We present in this Letter a bootstrap calculation of the Regge parameters of inclusive processes by relating them to those of exclusive reactions through the finite-energy sum rule for a *four-Reggeon* scattering amplitude; that is, we relate the triple-Reggeon vertices[1] to two-Reggeon–particle couplings.[2] We apply our method in particular to diffraction-dissociation phenomena.[3]

Our method is motivated by the observation that the two-particle production cross section in the "ditriple" Regge limit[4] can be obtained in two ways: In the inclusive reaction $a+b\to x_1+x_2+$anything, near the resonant pole c in the missing mass squared $M^2=(p_a+p_b-p_1-p_2)^2$, the cross section takes the form

$$sE_{x_1}E_{x_2}\frac{d\sigma}{d^3p_1\,d^3p_2}\equiv f\propto|T_{23}|^2\delta(M^2-m_c{}^2), \tag{1}$$

where T_{23} denotes the scattering amplitude for $a+b+x_1+x_2+c$. At high energy with $s_{a\bar{1}}$ and $s_{b\bar{2}}$ fixed, T_{23} has a double-Regge representation [see Fig. 1(a)]

$$T_{23}=\beta_{a\bar{1}}\frac{(-\alpha_{ab\bar{2}})^{\alpha_{b\bar{1}}}}{\sin\alpha_{b\bar{1}}}G\frac{(-\alpha_{ab\bar{1}})^{\alpha_{b\bar{1}}}}{\sin\pi\alpha_{b\bar{2}}}\beta_{b\bar{2}}, \tag{2}$$

where $\alpha_{i\ldots j}=\alpha' s_{i\ldots j}+\alpha_{i\ldots j}(0)$, $\alpha'=1$, β_{ij} is the "reduced" Regge residue, and G is the "reduced" two-

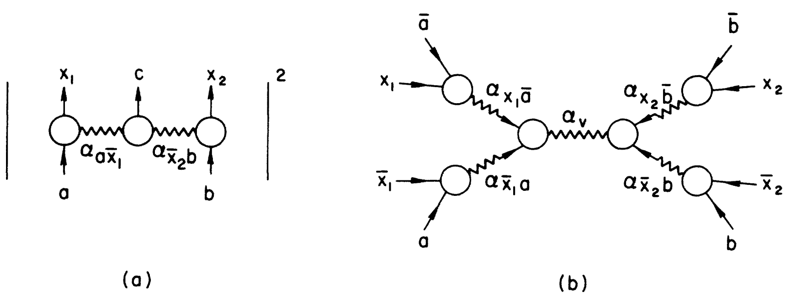

FIG. 1. Two modes of describing the "ditriple" Regge region: (a) resonance saturation, (b) Regge asymptotic extrapolation.