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Vortex-Free Landau State in Rotating Superfluid Helium

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Evidence is presented that below a rather large angular velocity, ω_{c1}, there is a vortex-free state in rotating locked helium, which we call the Landau state. This is similar to the Meissner state in superconductors. We further show that all the properties of the rotating helium have counterparts in irreversible type-II superconductors and find that ω_{c2} (analogous to H_{c2}) greatly exceeds our high speeds of rotation.

An important aspect of the two-fluid theory of superfluid helium as proposed by Landau[1] is that the superfluid component can flow only in an irrotational manner (i.e., $\nabla\times\vec{v}_s=0$, where $\vec{v}_s$ is the velocity of the superfluid component). The realization of stable states where $\nabla\times\vec{v}_s=0$ everywhere in the superfluid has been extremely difficult since for small velocities[2] the superfluid has a tendency to form quantized vortex lines at which $\nabla\times\vec{v}_s$ becomes singular. In this paper we report on direct observations of the stable Landau state ($\nabla\times\vec{v}_s=0$ everywhere) at rather high angular velocities for superfluid helium contained in a superleak (geometry formed by packed fine powder). It is well known that the London condition for superconductors [$\nabla\times(\vec{v}_s+q\vec{A}/mc)=0$, where $\vec{v}_s$ is the velocity of the superelectrons, $\vec{A}$ is the vector potential, and q/m is the ratio of the electron's charge to its mass] is analogous to the Landau condition for superfluid helium.[3] Thus the Meissner state of superconductors which is a consequence of the London condition is often compared with the Landau state of superfluid helium. Our experiments suggest a more precise comparison between superconductors and superfluid helium.

We have previously reported[4] on a new method of measuring the velocity of persistent currents in superfluid helium contained in a superleak. The method makes use of the fact that the speed of the wave mode (fourth sound) which propagates in clamped superfluid helium depends upon the velocity of the superfluid relative to the normal fluid (taken to be at rest) and at low temperature is given by[5]

$$C_4=C_4^{(0)}\pm(\rho_s/\rho)v_s, \tag{1}$$

where $C_4^{(0)}$ is the stationary velocity of fourth sound, ρ_s/ρ is the superfluid fraction, and v_s is the velocity of the persistent current. In Eq. (1) the plus and minus signs refer to propagation with and opposed to the persistent current. Experimentally one gets v_s from a measurement of the splitting [implied by Eq. (1)] of the resonance

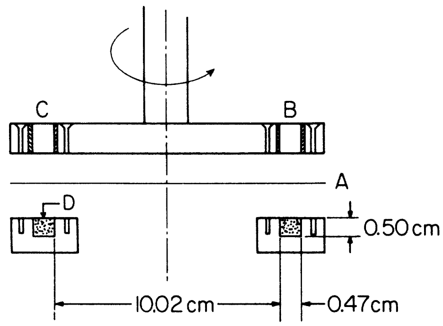

FIG. 1. Schematic diagram of torus resonator. The relative velocity of the normal fluid and superfluid is determined by measuring the frequency difference of fourth-sound modes which run around the resonator in both directions. A is a Mylar diaphragm metalized on the lower surface. B and C are the electrodes of the capacitative transducers of which A is the sensitive element. D is the channel of the torus filled with packed powder.

frequency of the angularly dependent fourth-sound modes.

The experiments which we discuss here differ from those described earlier[4] in two important aspects. Instead of the tube we now use a torus of square cross section shown schematically in Fig. 1. A second and much more important difference is that now through the use of slip rings and brushes we make measurements while the resonator (and hence the normal-fluid component which is locked to it by its viscous interaction with the superleak) is rotating. In this case the splitting $\Delta\omega$ determines the frequency of rotation of the superfluid relative to the rotation system, or

$$\omega_n - \omega_s = \frac{\rho\Delta\omega}{2\rho_s m}, \qquad (2)$$

where ω_n and ω_s are the rotational frequencies of normal and superfluid, and m is the order of the acoustic harmonic—i.e., the number of wavelengths in the circumference. The powder which is packed into the resonator to form the superleak is Al_2O_3, whose grains have typical diameters of 170 – 320 Å.

Figure 2 shows the results at $T = 1.34$°K of measurements made on a torus resonator whose inside and outside diameters are 10.02 and 10.99 cm. The resonator and He II are initially at rest. We note the following:

(1) $A \leftrightarrow B$.—If the speed of the resonator never exceeds that at B, namely 1.22 cps, the superfluid frequency ω_s is completely characterized

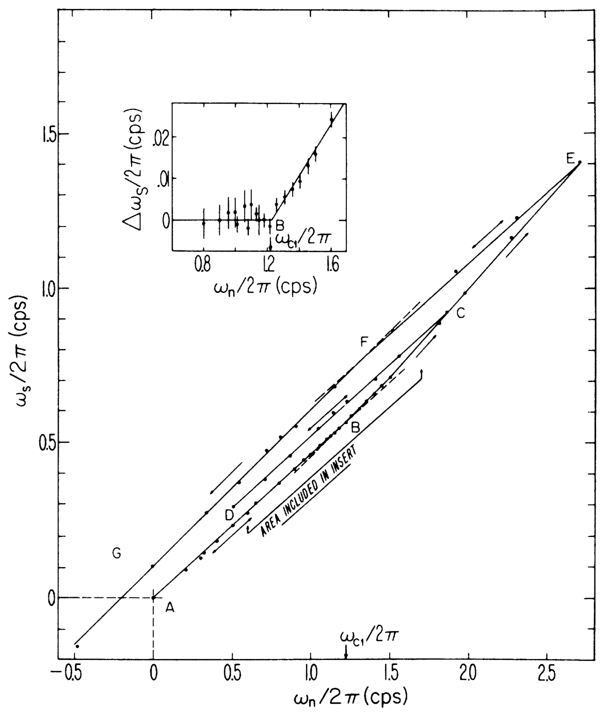

FIG. 2. A plot of the angular velocities of the normal fluid and superfluid when the resonator is rotated starting from rest. The sequence is in the following order: *ABCDCEFG*. Changes along *AB*, *CD*, and *EF* are reversible and these line segments are accurately parallel. *AB* is the Landau state. $\omega(B) = \omega_{c1}$ is the angular velocity at which vortices first enter the helium. The insert shows the data in the neighborhood of ω_{c1}. The difference between the measured value of ω_s and that along the extended line *AB* is plotted against ω_n. It is apparent that the departure from line *AB* occurs abruptly at ω_{c1}.

by the value of the rotation frequency ω_n within experimental accuracy (which at B is 1% of the speed of rotation) and is given by $\omega_s = 0.46\omega_n$; $\omega_n(B)$ is called ω_{c1} for reasons which will become clear.

(2) $B \rightarrow C$.—If ω_n is increased beyond ω_{c1} the slope of the curve increases. This part of the curve is irreversible. If ω_n is reduced at C, then the curve traversed is $C \rightarrow D$.

(3) $C \leftrightarrow D$.—This line is accurately parallel to *AB*. Processes here are reversible just as in *AB* provided that $\omega_n(D)$ is not too different from $\omega_n(C)$. After reaching D the speed ω_n was increased and point C was regained.

(4) $C \rightarrow E$.—Further increases in speed give the segment *CE* which lies on the line determined by B and C. If ω_n at E is very much greater than that shown, there will be curvature in the graph before E is reached. At very high speeds the val-

ue of $\omega_n - \omega_s$ saturates at a value 2.87 cps and the graph has a slope of 1.0. If at E (however large ω_n) the speed is reduced, the curve traversed is EF.

(5) $E \rightarrow F$.—This line is accurately parallel to both AB and DC, and increases and decreases of speed produce reversible results. Further decreases below ω_n at F produce the line FG.

(6) $F \rightarrow G$.—This is accurately parallel to BE. If the speed is reduced far beyond G (this may require reversing the direction of rotation) there is curvature in the graph, the slope increasing.

In the insert in Fig. 2 we show the data of $ABCE$ with ω_n as the abscissa and the departure of ω_s from the values on the line AB as the ordinate. The purpose here is to magnify departures from the line AB. It is apparent that this occurs abruptly at $\omega_{c1}/2\pi = 1.22$ cps.

We make the following assertions:

(1) In the region AB the superfluid component flow has zero circulation. There is no vorticity; $\nabla \times \vec{v}_s = 0$ everywhere. (2) At ω_{c1} vortices begin to enter the helium and are pinned on arrays of powder grains. (3) In the reversible regions CD and EF, all changes in the superfluid component are due to changes in the circulation-free motion, the pinned quantized vortices remaining fixed. (4) The region FG is characterized by either a reduction in the number of vortices or the introduction of vortices of opposite sign. We have some evidence that the latter is a preferable interpretation and tentatively adopt it.

The reasoning which leads to the first assertion is along the following lines. A well-known result of classical hydrodynamics is that in imparting a velocity $\vec{v}$ to a sphere immersed in an initially stationary, inviscid, incompressible, and irrotational fluid one also imparts a momentum of the same sign to the fluid which is the product of $\vec{v}$ and one half the mass of the fluid displaced by the sphere.[6] (For a cylinder moving in a direction perpendicular to its length the numeric is one.) This result also holds for a sphere set into rotation in a circular channel. In this case the circulation being initially zero must remain zero, and we thus have angular momentum but no circulation. The powder grains occupy 35% of the volume. If the grains were spherical and each grain produced this impulse we should have $[(0.5 \times 0.35)/0.65]\omega_n = 0.27\omega_n$. We find $0.46\omega_n$. The difference is not surprising considering the dense packing of the grains and the departure from sphericity of the grain shape. We have measurements on a resonator in which the powder occupies a smaller percentage of the volume (22%) and, as is to be expected, the superfluid velocity is a smaller fraction of ω_n. Moreover ω_{c1} is lower, about 1.0 cps, occurring approximately at the same value of ω_n-ω_s in the two systems. In any event it should be noted that this hydrodynamical effect is completely reversible as is the experimentally observed behavior of the He II in the region under discussion.

The motivation for assertions 2, 3, and 4 comes from the generally accepted fact that when He II is not executing pure potential flow, this is due to the appearance of quantized vortex lines (or rings).

The above line of reasoning was first used by Mehl and Zimmerman[7] to explain their observation on persistent currents and the period of vibration of a torsion pendulum whose inertial member contained He II.

Continuing the discussion of the region AB we note that in the Meissner region of superconductors there are no quantized flux lines—only circulation-free current which cancels the applied field. In He II the analogous current (which is the velocity of the superfluid relative to the normal fluid) tries to keep the rotation out of the superfluid and is a volume current. This last point reflects the fact that in He II the penetration depth is infinite. In this comparison the angular velocity of the normal-fluid component takes the place of the applied magnetic field and the importance of making measurements while the apparatus is in rotation becomes abundantly clear.

In Fig. 3 are plotted the results of measurements which cover a much greater range of angular velocities in the superleak with smaller solid volume mentioned earlier. The curves $\omega_n - \omega_s$ vs

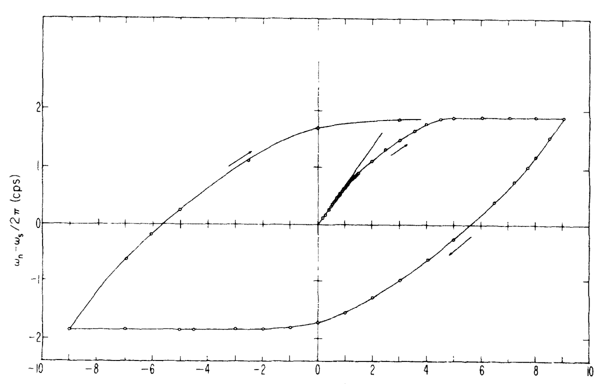

FIG. 3. A plot of $\omega_n - \omega_s$ for complete cycling of ω_n starting from rest for a more lightly packed superleak than that of Fig. 2 (volume of Al_2O_3 powder is 22% of total volume). The Landau region occurs below 1.0 cps.

ω_n bear a strong resemblance to the magnetization $M(H)$ in irreversible type-II superconductors (high-critical-field materials[8] and filamentary superconductors[9]). In these superconductors the magnetization is *strictly* reversible only up to a field H_{c1}. As the field increases above H_{c1}, flux lines appear but the magnetization continues to increase. If from a state with a field larger than H_{c1} the field is then reduced, the magnetization decreases with a slope which is initially equal to that of the Meissner line. This hysteresis effect is due to the pinning of quantized flux lines in these superconductors. In view of these remarks and our experimental observations as displayed in Fig. 3, we are led to conclude that the behavior of He II contained in a rotating superleak is analogous to the behavior of an irreversible type-II superconductor in an external field and ω_{c1} plays the role of H_{c1}.[10]

In addition, when an irreversible type-II superconductor with pinning forces is cooled down below T_c under a constant applied magnetic field H_a, the magnetization remains zero as long as H_a is kept constant.[11] In a similar way we have cooled the rotating superleak below T_λ and observed that $\omega_n = \omega_s$ as long as ω_n is not varied.

In superconductors the magnetization is found to vanish at the upper critical field H_{c2}. If a comparable effect exists in He II, then from our observation that $\omega_n - \omega_s$ shows no signs of diminishing even at the highest speeds of rotation we conclude that ω_{c2} is very much greater than our high speeds of rotation and may be so high as to be unobservable—this would be the case, for example, if the linear speed exceeds $C_4^{(0)}$.

The exclusion of the first vortex line has been directly observed in simply connected geometries in the experiments of Hess and Fairbank[2] and of Packard and Sanders.[12] They observe

$$\omega_{c1} \cong (\hbar/mR^2)\ln(R/a), \tag{3}$$

where R is the radius of the vessel and a the radius of the vortex core. We find from measurements on a smaller resonator, of one third the radius, that ω_{c1} is approximately 3 times larger. Thus (3) does not apply for our geometry. Our observations suggest that ω_{c1} is on the order of $\hbar/mRd$, where d is the size of the channels through which He II flows, i.e., the pore size. Fetter[13] finds that in geometries roughly similar to those used in this experiment the free energy will be minimized by the appearance of vortices only when the speed of rotation exceeds $\omega_{c1} \sim \hbar/mRd$. It should be noted that in superconductors H_{c1} is also size dependent.[14]

The torus has an area of 15.2 cm^2 and at 1.2 cps there would have to be 2.28×10^5 unit quantized vortex lines to reduce $\omega_n - \omega_s$ to zero at ω_{c1}. Thus what is remarkable in our experiment is the magnitude of the effect—the Landau region for He II contained in a superleak is extensive.

A basic defect in our argument is that we have no direct experimental evidence that vortex states are absent below ω_{c1} and obviously this is difficult to obtain. Our argument rests on a logical interpretation of experimental data. We have a great deal of data not presented here which fit this interpretation and which will be presented in a future article dealing with other interesting phenomena in rotating helium. We would especially emphasize that all results mentioned here are completely reproducible from experiment to experiment.

We have profited greatly from a conversation with Friederich H. Busse and a conversation with Y. B. Kim.

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$$C_4 = C_4^{(0)} \pm v_s \left\{ \frac{\rho_s}{\rho} + \left[S\left(\frac{\partial}{\partial T}\frac{\rho_n}{\rho}\right)\left(\frac{\partial S}{\partial T}\right)^{-1} \right] - \rho\left(\frac{\partial}{\partial P}\frac{\rho_n}{\rho}\right)\left(\frac{\partial \rho}{\partial P}\right)^{-1} \right\},$$

where S, P, and T are the specific entropy, pressure, and temperature, respectively.

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[10] London (Ref. 3, p. 83) has pointed out that rotation in superconductors can play the role of an externally

applied field. This suggests a comparison of rotating He II with a rotating irreversible type-II superconductor which, however, we have not attempted since there is insufficient literature on the subject.

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Observations of Nonlinear Landau Damping of Ion-Acoustic Waves

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Nonlinear Landau damping and growth have been observed for ion-acoustic waves. The measured nonlinear wave-wave coupling coefficients are in reasonable agreement with theory. The forced beat oscillations between the two injected waves, which cause nonlinear Landau damping, were also observed.

Nonlinear Landau damping (or growth) is one of the important processes in nonlinear wave kinematics.[1-3] It is often treated as a fundamental mechanism for energy transfer between waves and particles in theories of weak plasma turbulence. Recently, experimental observations[4,5] which support the existence of nonlinear Landau damping have been reported for particular waves (the electron-cyclotron harmonic waves and the Trivelpiece mode[6]). In this Letter, we report the first experimental observation, to the authors' knowledge, of nonlinear Landau damping and growth of ion-acoustic waves in collisionless plasma. Quantitative measurements of the coupling coefficients have also been made.

In a plasma with a high electron-to-ion temperature ratio, T_e/T_i, the linear Landau damping of ion-acoustic waves is weak. When two waves (ω_1, k_1) and (ω_2, k_2) propagate, plasma nonlinearity produces waves at the sum and the difference frequencies (and wave numbers). Since the group velocity $d\omega/dk$ of ion-acoustic waves is small at high frequencies, difference-frequency waves, $(\omega_1-\omega_2, k_1-k_2)$, are nonresonant, i.e., they do not satisfy the dispersion relationship $\epsilon(k,\omega)=0$. Because the velocity of the beat wave, $\Delta\omega/\Delta k = (\omega_1-\omega_2)/(k_1-k_2)$, can be close to the ion thermal velocity, the nonresonant wave can strongly interact with the ions. By this process, the higher-frequency wave transfers its energy to the lower-frequency one.

Based on the one-dimensional[7] collisionless Boltzmann equation for ions, third-order perturbation theory[3] gives the following wave kinetic equations for the two original waves:

$$\partial|\psi_{\omega_1}|^2/\partial x + 2\gamma_1|\psi_{\omega_1}|^2 = \alpha_1|\psi_{\omega_1}|^2|\psi_{\omega_2}|^2, \qquad (1)$$

$$\partial|\psi_{\omega_2}|^2/\partial x + 2\gamma_2|\psi_{\omega_2}|^2 = \alpha_2|\psi_{\omega_1}|^2|\psi_{\omega_2}|^2, \qquad (2)$$

with nonlinear coupling coefficients

$$\alpha_1 = \frac{\pi}{4}\left(\frac{\partial\epsilon}{\partial k_1}\right)^{-1}\frac{\omega_{pi}{}^6 k_2{}^2\Delta k}{(k_1\omega_2-k_2\omega_1)^4}\left(\frac{T_e}{T_i}\right)^2\int dv\,\delta(\Delta\omega-\Delta k v)\frac{\partial f_0}{\partial v}$$
$$\times\left|\frac{1}{\epsilon(\Delta k,\Delta\omega)}\left\{\frac{k_1W'(\omega_1/k_1v_T)-k_2W'(\omega_2/k_2v_T)}{\Delta k}-Z'\left(\frac{\Delta\omega}{\Delta k v_T}\right)\right\}-\frac{\Delta k^2 v_T{}^2}{\omega_{pi}{}^2}\right|^2 \qquad (3)$$

and

$$\alpha_2(k_2,k_1;\omega_2,\omega_1) = \alpha_1(k_1,k_2;\omega_1,\omega_2). \qquad (4)$$

Here ψ is the dimensionless wave potential $e\varphi/\kappa T_e$, γ the linear Landau damping rate, W the real part of plasma dispersion function Z, $\Delta k = k_1-k_2$, and $\Delta\omega = \omega_1-\omega_2$. The other notations are standard. In the derivation of the above equations, the damping (or growth) of the waves has been assumed to be small and the sum-frequency wave, whose contribution to the coefficients α_1 and α_2 is very small, has been neglected. It was also assumed that $\epsilon(\Delta k,\Delta\omega)\neq 0$, so that Eqs. (1)–(4) cannot be used when ω_1 and $\omega_2 \ll \omega_{pi}$, where resonant wave-wave coupling dominates. Note that the set of Eqs. (1)–(4) satisfies the Manley-Rowe relation[8] when linear Landau damping can