

Partons and Deep-Inelastic Electron Scattering*†

Kenneth G. Wilson

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

(Received 6 July 1971)

A new parton model is defined. Two predictions are obtained for deep-inelastic $e-p$ scattering: (1) At an invariant momentum transfer $-Q^2$ of 1 GeV^2 , at least half the final states will contain a K meson. (2) For a value of $-Q^2$ somewhere between 8 and 30 GeV^2 the Bjorken scaling prediction will be violated and a large fraction of the final states will contain an antibaryon.

One of the questions raised by the recent Massachusetts Institute of Technology–Stanford Linear Accelerator Center (MIT–SLAC) experiments on deep-inelastic $e-p$ scattering¹ is whether these experiments provide information about possible constituents of the proton. To test this question some simple “parton” models of the proton have been constructed.² The purpose of this paper is to present a different kind of parton model to be called the “multiple-parton” model. The model described here is not well developed as yet; one cannot make quantitative calculations with it. It will be used to make some simple qualitative predictions about the final states in deep-inelastic scattering.

To define the multiple-parton model it is necessary to introduce a cutoff Λ . The limit $\Lambda \rightarrow \infty$ can be taken, but here the case of a fixed and finite Λ will be discussed. One can think of Λ as being 10^{20} eV or some equally ridiculous energy. The basic assumption of the multiple-parton model is this: As many quantities as possible depend only on Λ and not on the proton mass M or other strong-interaction mass parameters.³ Thus the mass of a parton is of order Λ . The size of a parton is of order Λ^{-1} . The rate at which virtual parton-antiparton pairs are produced is of order Λ per unit time. Due to this rapid rate of virtual-parton-antiparton pair production, there is a high density of virtual partons and antipartons inside the proton; the basic assumption now gives this density to be of order Λ^3 per unit volume. By “density” one means the equilibrium density, namely, the density for which the rate of virtual-parton pair annihilation equals the rate of virtual-parton pair production.

There must be strong binding potentials in the proton to cancel the rest masses of all the constituent partons. According to the basic assumption the range of these potentials must be of order Λ^{-1} and the strength of order Λ . Thus, most of the binding occurs between neighboring partons which are separated by a distance of order Λ^{-1}

due to the parton density being of order Λ^3 .

Now an oversimplified picture of the effects of the binding potentials will be described.⁴ Suppose that the partons bind in pairs (whether in parton-parton pairs or parton-antiparton pairs, or both, does not matter). One can then think of the parton as made up of these bound pairs rather than of the partons themselves. This is analogous to thinking about molecules as being composed of atoms rather than of electrons and nuclei. It is also analogous to thinking of nuclei as being made up of α particles instead of protons and neutrons. (The choice of having two partons in a bound state instead of four, or some more general picture, is part of the arbitrary oversimplification of this picture of parton binding.) The binding of a parton pair is very strong (of order Λ) which means these pairs will not be broken easily; for example, one would expect a virtual photon to be able to break up such a pair only if Q^2 is of order Λ^2 .

A real photon of high energy has very little chance of breaking up a parton pair. The reason is that high-energy interactions of real photons with protons seem to be mostly peripheral and cannot supply the strong forces needed to break apart a bound pair. However, photons with large Q^2 are expected from the Bjorken scaling theory⁵ to have a size of order Q^{-1} , and if Q is of order Λ it is perfectly possible for the photon to be absorbed by a single parton pair. If the photon also supplies enough energy then the pair can be broken apart. By a similar argument one can expect that parton pairs are unlikely to be broken apart in high-energy hadron-hadron collisions.

Not all of the binding potentials will go into the binding energies of parton pairs, for there will still be the potentials between partons of different pairs. There will also be potentials due to exchanges of bound pairs. These residual potentials will be strongest at short distances. The residual potentials can therefore bind pairs of parton pairs to form clusters containing four partons

each. Then two clusters of four partons bind to form a cluster of eight partons, etc., until one finally has the proton itself as a cluster containing about $(\Lambda/M)^3$ partons.

We now assume that one can think of the proton as being made up of parton clusters of arbitrary size. That is, the properties of the proton should be determined if one knows the properties of parton clusters containing 2^l partons, for any given l , and the binding potentials between these clusters. To characterize the properties of the parton clusters one can make a simple scaling assumption. Let the parton clusters containing 2^l partons have a size Λ_l^{-1} . Then assume that the properties of these clusters depend only on Λ_l and not on any other mass (neither the proton mass M nor the original cutoff Λ). So the mass of a cluster is of order Λ_l , the rate of production of pairs of clusters is of order Λ_l^{-1} , the density of clusters is of order Λ_l^3 , etc. With this scaling assumption one can easily imagine that there will be a scaling law for deep-inelastic electron scattering, namely a photon with momentum transfer Q will interact with a parton cluster of mass of order Q ; since the properties of this cluster are independent of Q except as a scale factor, the deep-inelastic cross sections should only involve Q as a scale factor. Exact scaling puts a restriction on the binding potentials: Given that clusters of adjacent sizes have a size ratio $\Lambda_l^{-1}/\Lambda_{l+1}^{-1}$, then the mass ratio must be exactly Λ_l/Λ_{l+1} .

In order that the proton be of finite size the scaling must break down for $\Lambda_l \sim M$. It will be assumed that this breakdown is caused by potentials of strength M which violate the scaling condition. These potentials are negligible for small clusters, but for large-sized clusters they should cancel the scale-invariant potentials and prevent binding of clusters with a size larger than the physical proton.

The most interesting question to study is the problem of the internal quantum numbers of the parton clusters. First we make the assumption that parton clusters cannot have smaller masses than the physical particles which have the same quantum numbers. For example, clusters carrying baryon number ± 1 cannot have a mass smaller than $930 \text{ MeV}/c^2$, and clusters carrying strangeness ± 1 cannot have a mass smaller than $490 \text{ MeV}/c^2$. However, clusters with the quantum numbers of the pion can have a mass as small as $140 \text{ MeV}/c^2$. If quarks exist, quarklike clusters must have a mass greater than the physical

quark mass. This mass will be assumed to be much larger than the proton mass. It will continue to be assumed that the binding potentials are of strength Λ_l for clusters of size Λ_l^{-1} . This suggests that the largest scale for which the proton can be built up of clusters is the scale with $\Lambda_l \sim m_\pi$; on this scale the proton can consist of a protonlike cluster coupled with one or two pionlike clusters. In this case the binding potentials need only to compensate the masses of the pionlike clusters, which they are strong enough to do. On this scale it is not possible for the proton to contain a pair of K -type clusters or a pair of baryon- and antibaryonlike clusters because the binding potentials are not strong enough to compensate for the masses of the extra clusters. For scales $\Lambda_l \gg M$ there is enough binding energy available to bind both K - and baryon-types of clusters. On this scale it is reasonable to expect all types of clusters to be present with comparable densities. Alternatively, one might try to argue on grounds of simplicity that only baryonlike clusters occur for $\Lambda_l \gg M$, with the mesons being bound states built from these clusters. (For very, very large Λ_l one might have quarklike clusters, but this possibility will not be explored here.)

It will be assumed here that a scaling region is a range of Λ_l over which the properties of the clusters are unchanged except for the change in the scale Λ_l itself. In a range of Λ_l for which the relative densities of different types of clusters are changing, the virtual-photon absorption cross sections at corresponding values of Q^2 will be changing in a nonscaling manner.

Given this general picture of the proton, what can one say about deep-inelastic scattering? Let us discuss deep-inelastic electron scattering as a function of Q^2 as Q increases. For small Q , say $Q \lesssim m_\rho$, the photon should be large enough so that it sees only the simple structure of a baryon cluster coupled to pionlike clusters. In this case the final state should be a nucleon plus mesons; through final-state interactions, ρ 's and N^* 's can also be produced. For somewhat larger Q the photon should be small enough so that there can be K -type clusters of the same size inside the proton. Once one is considering small enough lengths so that there is enough binding energy available to bind K -type clusters, there should be a reasonably high density of K -type clusters. This is due to $SU(3)$ symmetry, which would suggest that the density of K -type clusters should be about equal to the density of π -type clusters once

the K - π mass difference can be ignored. Given that there are K -type clusters of size Q^{-1} , one would expect photons of momentum transfer $-Q^2$ to be able to knock out such a cluster, resulting in a K meson in the final state. The K would be accompanied by a Σ , Λ , or \bar{K} meson. Final-state interactions would probably result in several pions accompanying the K , either as separate particles or as decay products of K^* 's or Y^* 's or both. It seems unlikely that the K meson would be annihilated by final-state interactions. If the K clusters are equally as dense as π clusters, then roughly half the deep-inelastic events would include a K meson.

The first prediction is, therefore, that for sufficiently large Q^2 at least half of all deep-inelastic scattering events will include a K meson in the final state.⁶ How large must Q^2 be? In the present picture the relative density of π clusters and K -type clusters cannot change in a scaling region. Since the MIT-SLAC experiments show scaling in the range 1 to 8 GeV² for Q^2 , then the relative density must be constant in this range.⁷ It is hard to imagine K -meson production becoming important only above 8 GeV². Therefore the best guess is that K -meson production is important already for $Q^2 \sim 1$ GeV².

As Q increases still further, Q becomes large enough for there to be antibaryon clusters inside the proton. According to the picture described above, at sufficiently short distances one would perhaps see only baryon and antibaryon clusters and not mesonlike clusters.⁸ There should be a breakdown of scaling associated with the transition from mesonlike clusters to baryonlike clusters. It seems unlikely that this transition could take place for $Q^2 \lesssim 1$ GeV², so this transition must occur for $Q^2 > 8$ GeV². Accompanying this transition there would be a large increase in antibaryon production.

The second prediction is, therefore, that the scaling seen in the MIT-SLAC experiments from 1 to 8 GeV² will break down at some Q^2 above 8 GeV². At the value of Q^2 where breakdown occurs, a large fraction of the events should include an antibaryon in the final state.⁹ As a guess, this breakdown will occur between 8 and 30 GeV².

These predictions would be remarkable, if true, because they are very different from anything seen in hadron-hadron collisions.

The author is grateful to Professor Abdus Salam, the International Atomic Energy Agency, and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. The author thanks Professor H. E. Stanley for a discussion.

*Work supported by the National Science Foundation.

†An invited paper presented at the Topical Seminar on Electromagnetic Interactions, International Centre for Theoretical Physics, Trieste, 21-26 June 1971.

¹E. D. Bloom *et al.*, in Proceedings of the Fifteenth International Conference on High-Energy Physics, Kiev, U. S. S. R., 1970 (Atomizdat., Moscow, to be published).

²R. P. Feynman, Phys. Rev. Lett. **23**, 1416 (1969); for further references see H. T. Nieh and J.-M. Wang, Phys. Rev. Lett. **26**, 1139 (1971).

³This assumption is motivated by the result of Gell-Mann and Low that quantum electrodynamics at short distances depends only on a cutoff mass if a cutoff is introduced, and not on the physical electron mass. See M. Gell-Mann and F. E. Low, Phys. Rev. **95**, 1300 (1953); see also K. Wilson, Phys. Rev. D **3**, 1818 (1971), and references cited therein.

⁴The model of parton binding described here is similar in spirit to Kadanoff's theory of scaling in ferromagnets near the critical point. See L. Kadanoff, Physics (Long Is. City, N. Y.) **2**, 263 (1966); see also K. Wilson, Cornell University Reports No. CLNS 133 and No. CLNS 149, 1971 (unpublished).

⁵J. D. Bjorken, Phys. Rev. **179**, 1547 (1969).

⁶A similar prediction was made in N. Cabibbo, G. Parisi, M. Testa, and A. Verganelakis, Lett. Nuovo Cimento **4**, 569 (1970).

⁷In discussing scaling, it will be assumed that the scaling variable $\omega = 2M\nu/Q^2$ (with ν being the energy loss) is $\gtrsim 4$ in order to be well above kinematic thresholds.

⁸Krisch has suggested that strong interactions also show distinct effects from π , K , and antibaryon components in the proton. See A. Krisch, in *Lectures in Theoretical Physics*, edited by W. E. Brittin and L. Marshall (University of Colorado Press, Boulder, Colo., 1965), Vol. VII B, p. 274.

⁹The model of S. S. Shei and D. M. Tow, Phys. Rev. Lett. **26**, 470 (1971), has antibaryons in many of the final states.