± 3.6 and -21.8 ± 4.4 (Table I). It has been argued in Ref. 1 that $1-R$ and $1-\gamma_\infty$ suffer from some uncertainties in their values. Taking these into account one may write finally the theoretical values of eQq as -18.6 ± 1.5 (tetrahedral) and -25.1 ± 2.3 (octahedral). These may be compared with the previously calculated coupling constants (believed to be in error as discussed above), -7 and -17 MHz for tetrahedral and octahedral sites, respectively, calculated by Nicholson and Burns[7] using a monopole approximation, and $Q(Fe^{57m}) = 0.20$ b.

One of the authors (R.R.S.) expresses his thanks to Dr. B. Dunlap for useful conversations.

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Shape of the Island of Superheavy Nuclei*

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On the basis of the Strutinsky shell-correction method applied to realistic diffuse-surface potentials we have calculated the shape of the island of superheavy nuclei. With respect to spontaneous fission, the island is a mountain ridge extending from 114 protons to about 124 protons. The descent from the mountain down to the sea of instability is rather gentle for decreasing neutron numbers below 184, but is more rapid on the other three sides.

In attempts to produce superheavy nuclei near the predicted closed shells at 114 protons and 184 neutrons, it is important to know the stability of these nuclei with respect to their various modes of decay. Because their stability is associated with shell closures, the size and shape of the island of superheavy nuclei depends strongly upon the single-particle potential felt by neutrons and protons near the Fermi surface. In practice this potential must be extrapolated from regions of known nuclei. We have therefore concentrated on techniques that permit more accurate extrapolations, both to the large deformations encountered in fission and to new regions of nuclei, in order to identify more clearly the extent of the island, especially with respect to spontaneous fission.

With this basic purpose in mind, we generate the spin-independent part of the single-particle potential by folding an effective two-nucleon interaction with a uniform sharp-surface pseudodensity corresponding to the given nuclear shape.[1,2] For both physical reasons and simplicity we take the effective two-nucleon interaction to be a Yukawa function with adjustable strength and range. In addition to this spin-independent part, the total potential felt by a nucleon includes also an invariant spin-orbit term and a Coulomb potential for protons.

For a spherical shape the folded Yukawa potential generated in this way is very close to a Woods-Saxon potential,[1,2] and either one of these potentials would be about equally satisfactory. The advantage of the folded Yukawa potential is that it generalizes in a more natural way for deformed shapes than does the Woods-Saxon potential. With such a folding procedure it is easy to generate a potential for all the conceivable shapes of interest in fission, including the transition at the scission point to a potential concentrated in each of the two (or more) fission fragments. The potential is very well behaved in the entire surface region and approaches zero at large distances.

A single-particle potential generated in this way contains a total of five parameters: a depth for neutrons, a depth for protons, a radius, an analog of the surface thickness, and the spin-orbit interaction strength. We attempted originally to determine these parameters by calculating the single-particle energies for the four doubly closed-shell nuclei ^{208}Pb, ^{48}Ca, ^{40}Ca, and ^{16}O, and adjusting the parameters to reproduce optimally the experimental single-particle energies. However, these attempts suggested that parameters obtained by adjusting to single-particle levels are difficult to extrapolate in a physical way. We have therefore decided to use single-particle parameters obtained from statistical (Thomas-Fermi) calculations that reproduce correctly the average trands throughout the periodic table of a wide variety of nuclear properties.[3] This permits the extrapolations to large deformations and to new regions of nuclei to be made with more confidence.

One we have generated a potential appropriate to a given shape, the next step is to solve the Schrödinger equation for the single-particle energies. We have done this both by means of the standard method of expanding the wave function in a set of deformed–harmonic-oscillator basis functions[4] and also by means of a finite-difference method. We had initially thought that the finite-difference method would be preferable for very large deformations, but this turned out not to be the case. We have therefore used the expansion method for most of our calculations, including the ones reported here.

From the calculated single-neutron and single-proton levels at a given deformation, shell and pairing corrections are calculated by means of the methods developed by Strutinsky.[5] These corrections are then added to the surface and Coulomb energies of the liquid-drop model[6] to obtain the total potential energy of deformation.

The method that we use for describing the nuclear shape contains a total of five degrees of freedom,[7] one of which represents motion in the fission direction. Although the precise determination of the fission coordinate would require performing a dynamical calculation, during the early stages of fission the symmetric deformation coordinate y introduced by Hill and Wheeler[1,2,8] provides a fair approximation to the fission coordinate, and we have used this for the results reported here. To first order, y is related to the coordinates that describe spheroidal and Legendre-polynomial P_2 distortions by

$$y=\tfrac{2}{7}\epsilon=\tfrac{2}{7}\delta=\tfrac{3}{7}\alpha_2=\tfrac{3}{7}(5/4\pi)^{1/2}\beta\approx 0.270\beta.$$

Figure 1 summarizes our calculated fission barriers for superheavy nuclei. The positions of the nuclei corresponds to those in a chart of the nuclides, with the nuclei differing from each other by four neutrons and four protons. As indicated by the dashed curves, the liquid-drop contributions to the barriers are very small. In the region shown, stability against fission arises primarily from the negative single-particle correction for the spherical shape.

For the doubly closed-shell nucleus ^{298}114, the spherical single-particle correction is -10.3 MeV. As the nucleus deforms, the total potential energy (solid curve) increases until it reaches a maximum value of 2.7 MeV at the deformation $y=0.07$. Further deformation leads to a secondary minimum followed by a somewhat lower second peak. Beyond this peak the barrier drops rapidly. The difference in energy between the highest peak and the spherical shape is 13.0 MeV!

As neutrons are added beyond 184, the barrier height decreases drastically. Subtracting neutrons also lowers the barrier, but not as much as if the same number were added. When a small number of protons are added beyond 114, the barrier heights actually increase slightly.

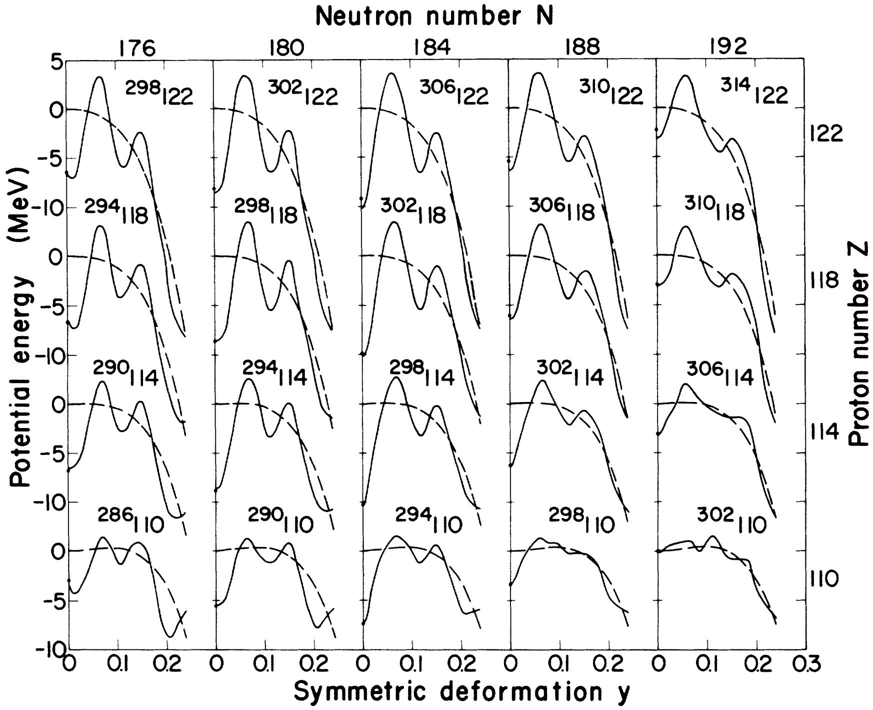

FIG. 1. Calculated fission barriers for superheavy nuclei. The dashed curves give the liquid-drop contributions and the solid curves the total potential energies.

When protons are subtracted, the barrier is again lowered.

For the nuclei near the top of Fig. 1, the peaks extend well above the liquid-drop background, and it is possible that mass-asymmetric (α_3) or axially asymmetric (γ) deformations could lower these barriers somewhat. However, the results of Möller[9] and Schultheis and Schultheis[10] suggest that the amount of this lowering is rather small. (We discount the results of Grumann *et al.*[11]—that superheavy nuclei undergo oblate fission through a barrier that is some 5 MeV lower than the barrier for prolate fission—since these barriers were calculated by simply summing single-particle energies, a procedure which is now known to be seriously inadequate.[12,13])

The present fission barriers have all been calculated using single-particle levels for the nucleus $^{298}114$. This represents a numerical approximation, because the levels themselves are somewhat different for the different nuclei. An idea of the accuracy of this approximation is gained by comparing the curves at zero deformation with the solid points, which have been calculated using the levels appropriate to the particular nucleus. This approximation is seen to be fairly good for nuclei close to $^{298}114$, but grows worse for nuclei far away.

Figure 2 shows a contour map of the ground-state single-particle correction versus neutron and proton numbers. As we move away from the doubly closed-shell nucleus with 114 protons and 184 neutrons, the ground-state single-particle correction decreases in magnitude from -10 MeV for nuclei along the inner contour to -5 MeV for nuclei along the outer contour. The fission-barrier height decreases in a similar way. (These results are obtained from single-particle levels recomputed for each individual nucleus.) The portions of contour lines shown dashed correspond to nuclei calculated to be proton unstable. The dot-dashed curve gives the calculated line of β stability.

The present fission barriers are somewhat higher than most of those calculated previously by means of the Strutinsky shell-correction method, both for generalized harmonic-oscillator potentials[5,14-17] and for diffuse-surface potents[2,15,16] However, results qualitatively similar to ours are obtained in more recent calculations.[18,19] (Fission barriers for superheavy nuclei have also been calculated in Ref. 11 and by Mosel and Greiner[20] by the inadequate method of summing single-particle energies.)

In trying to reach the island of superheavy nuclei by means of a compound-nucleus reaction,

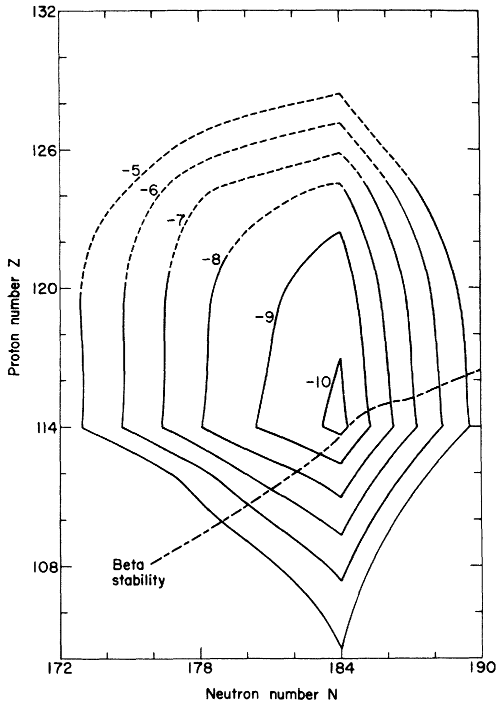

FIG. 2. Contour plot of calculated ground-state single-particle correction (in MeV) for superheavy nuclei. The portions of contour lines shown dashed correspond to nuclei calculated to be proton unstable. The dot-dashed curve gives the calculated line of β stability.

it is impossible to go directly to the center of the island, regardless of the target-projectile combination used. The results of Figs. 1 and 2 suggest that it is somewhat better to increase the proton number beyond 114 rather than decrease the neutron number below 184. If we aim for a compound nucleus containing at least 184 neutrons, then a quick examination of possible targets and projectiles from a chart of the nuclides tells us that the proton number must be at least 122. If we would be satisfied with 182 neutrons, then the proton number could be 120. These nuclei will have short α- and β-decay lifetimes, but it may nevertheless be possible to detect and study them. Alternatively, provided the spontaneous-fission lifetimes are sufficiently long, nuclei near $^{306}122$ could possibly be produced first, and then successively undergo α and β decay to more stable nuclei near the center of the island. A specific reaction of this type that has been suggested by Swiatecki[21] is

$$^{232}_{90}\mathrm{Th} + ^{76}_{32}\mathrm{Ge} \to ^{308}122^* \to \begin{cases} ^{304}122 + 4n \\ ^{302}120 + 2n + \alpha . \end{cases}$$

These results, which have been obtained by use of methods believed to be more reliable for extrapolations than previous methods, suggest that the fission barriers of superheavy nuclei near $^{298}114$ are even higher than previously supposed. With respect to spontaneous fission, the appearance of the island of superheavy nuclei is a mountain ridge extending north and south from 114 protons to about 124 protons. The descent from the mountain down to the sea of instability is rather gentle for decreasing neutron numbers below 184. However, for more than 184 neutrons, as well as more than 124 protons and less than 114 protons, the descent is more rapid. Nature has been very kind in extending the island in the direction of increasing proton number and decreasing neutron number, since it is this region that is most accessible experimentally.

We would like to acknowledge stimulating discussions concerning this work with many of our colleagues, particularly W. D. Myers and W. J. Swiatecki.

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New Five-Quasiparticle Isomeric State in ^{177}Hf†

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A new high-spin isomer ^{177m_2}Hf has been produced via the reactions ^{176}Yb($\alpha, 3n$)^{177}Hf and ^{186}W($p, p\alpha 5n$)^{177}Hf. It is interpreted as a $K^\pi = \frac{37}{2}^-$ five-quasiparticle state. Seven γ rays attributed to the decay of this new isomer, and the γ radiation from 1.1-sec ^{177m_1}Hf in equilibrium, decayed with a (51.6 ± 1.6)-min half-life. These γ rays result from de-excitation of an isomeric level in ^{177}Hf at 2739.7 keV to the rotational band members of the $K^\pi = \frac{23}{2}^+$ isomeric state.

A new high-spin isomer, ^{177m_2}Hf ($T_{1/2} = 51.6$ min), was observed and interpreted as a $K^\pi = \frac{37}{2}^-$ five-quasiparticle state. This isomer represents the highest spin state reported to date having an appreciable half-life ($T_{1/2}$ greater than a few microseconds) and is the first five-quasiparticle state observed in deformed nuclei. A number of other quasiparticle isomeric states are known[1-4] in the hafnium region, i.e., two-quasiparticle $K^\pi = 8^-$ isomers of ^{176}Hf, ^{178}Hf, ^{180}Hf, and ^{182}Hf; three-quasiparticle isomers of ^{177}Lu ($K^\pi = \frac{23}{2}^-$), ^{177}Hf ($K^\pi = \frac{23}{2}^+$), and ^{179}Hf ($K^\pi = \frac{25}{2}^-$)[5,6]; and a four-quasiparticle isomer of ^{178}Hf ($K^\pi = 16^+$).[7] These two-, three-, and four-quasiparticle states have configurations involving two or more of the following Nilsson states: $\frac{7}{2}-[514]_n$, $\frac{9}{2}+[624]_n$, $\frac{7}{2}+[404]_p$, and $\frac{9}{2}-[514]_p$.

Sources of ^{177m_2}Hf were produced by irradiation of isotopically enriched $^{186}WO_3$ (97%) and $^{176}Yb_2O_3$ (96%) with 96-MeV protons and 46-MeV α particles, respectively making use of the reactions ^{186}W($p, p\alpha 5n$)^{177}Hf and ^{176}Yb($\alpha, 3n$)^{177}Hf. Cross sections for the production reactions at these energies were estimated to be ~50 and ~400 μb for protons and α particles, respectively. The radiochemical separation of Hf from irradiated tungsten and ytterbium targets was identical to the one used in Ref. 4. The sources of hafnium obtained were highly decontaminated from neighboring elements. In one case, the Hf source was isotopically mass separated. The samples were counted with Ge(Li) detectors which had been calibrated for energy and efficiency with International Atomic Energy Agency standard sources. Analysis of the γ-ray spectra was performed by means of a modified version of the computer code BRUTAL. The program CLSQ[9] was used for γ-ray decay curve resolutions.

In Table I are listed the γ-ray energies and relative intensities for ^{177m_2}Hf. We observed 25 additional lines and assigned them to transitions of the $K^\pi = \frac{23}{2}^+$ three-quasiparticle isomer of ^{177}Hf which was in equilibrium. The decay of 1.1-sec ^{177m_1}Hf has been studied[10,11] quite well from the β decay of 161-d ^{177m}Lu ($K^\pi = \frac{23}{3}^-$), and the γ-ray energies and relative intensities measured in the present study were within the experimental errors of those previous measurements. The averaged γ-ray half-life of those lines listed in Table I and the additional 25 lines from ^{177m_1}Hf in equilibrium were determined to be 51.6 ± 1.6 min. The new activity was assigned as an isomer of ^{177}Hf based on the following considerations: (a) chemical identification with hafnium, (b) observation of ^{177m_1}Hf (1.1 sec) in equilibrium, and (c) isotopic mass separation and identification with $A = 177$.

In Fig. 1 is shown the proposed isomeric decay scheme of ^{177m_2}Hf. The decay of ^{177m_1}Hf (1.1 sec) has been omitted for simplicity; its well-known decay scheme can be found in Refs. 1, 10, and 11. The isomeric state at 2739.7 keV was established from energy-sum and relative γ-ray intensity considerations. The γ rays of 277.1, 294.9, 311.3, 326.6, 572.3, 606.5, and 638.0 keV fit well for the expected crossover and cascade transitions of the rotational band members