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[16]See de Gennes, Ref. 5.

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Dislocation Inertial Model for the Increased Plasticity of the Superconducting State*

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A model is proposed to account for the increased plasticity found for superconductors entering the superconducting state. It is supposed that at low enough temperatures in superconductors, inertial effects permit dislocations to overshoot their static equilibrium positions against obstacles and exert greater forces on the barriers. Quantitative agreement is found with experimental results for the temperature dependence of the stress drop. There is evidence that inertial effects contribute to the flow stress of nonsuperconductors.

An increased plasticity of materials entering the superconducting state was discovered recently by Pustovalov, Startsev, and Fomenko[1-4] and by Kojima and Suzuki.[5] For constant strain-rate tests, the stress required to maintain the deformation rate drops. For creep measurements at constant stress, Soldatov, Startsev, and Vainblatt[6] found that the strain rate increases by factors of up to 250. For stress-relaxation experiments at constant strain, Suenaga and Galligan found that the stress drops suddenly.[7,8] The stress-change effects observed are typically of the order of 0.1% to 10%, but effects as large as 53% have been reported.[4] Since the first measurements, the dependence of the effect on crystal structure, purity, type of superconductor, temperature, strain hardening, magnetic field, strain rate, alloying, and crystal orientation have been explored.[9-14]

In addition to the great technical and theoretical interest in ordinary superconductors, the understanding of plastic flow in superconductors is of particular interest for discussions of plastic flow in pulsars.[15] Ideas which have been offered for the interpretation of the effect include a change in the electronic viscous drag,[4-6] a change in obstacle strengths,[6,9-12] and a change in the mobile dislocation density[10,14] on entering the superconducting state. The data and the interpretations have recently been reviewed by Alers, Buck, and Tittman.[16] None of the interpretations so far proposed seems to be free of serious difficulties. We give here a simple inertial model of dislocation motion in superconductors, and discuss in detail the predicted temperature dependence of the effect. A more extended treatment, showing that the model can give a quantitative account of the data so far available, will be published elsewhere.

The model we propose is entirely analogous to a loaded spring in a viscous medium and is easily understood by using Fig. 1. A dislocation line moving toward obstacles in position 1 of Fig. 1(a) meets the obstacles with a velocity v_0 at position 2. The static equilibrium position under an applied stress is position 3. If the viscous damping is larger than a critical value, the dislocation line approaches the static equilibrium position as in the solid line of Fig. 1(b). If the damping is less than critical, the dislocation line overshoots to position 4 and oscillates about the static-equilibrium line. In position 4, the force exerted by the dislocation line on the obstacle is greater than in the static-equilibrium case. Alternatively, one may say that a smaller stress is needed in the low-damping case to produce the same force on the obstacle.

If the string model,[17,18] which is used extensively in ultrasonic-attenuation and internal-friction measurements, is used to describe the dislocation displacement ξ, then ξ is a solution of

$$\frac{A\partial^2\xi}{\partial t^2}+\frac{B\partial\xi}{\partial t}-\frac{C\partial^2\xi}{\partial x^2}=b\sigma, \tag{1}$$

where $A=\rho b^2$ is the dislocation mass per unit length, ρ is the density, b is the Burgers vector, B is the viscous damping constant, and $C\cong Gb^2$ is the dislocation tension where G is the shear modulus. The dislocation displacement ξ is a function of time and position x along the dislocation line. The displacement can be expressed as the sum of

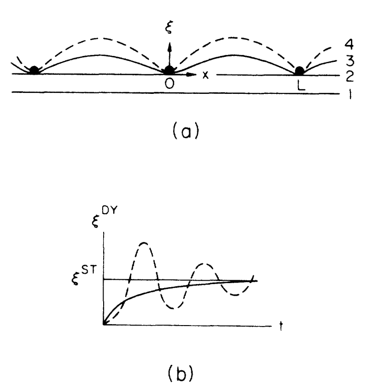

FIG. 1. (a) Schematic of the motion of the dislocation line. 1, dislocation line approaches pinning points; 2, dislocation line just touches pinning points; 3, static equilibrium position of dislocation line; 4, overshoot position of underdamped dislocation. (b) Displacement as a function of time for an underdamped dislocation (dashed curve) and for an overdamped dislocation (solid line).

a static displacement ξ^{ST} and a dynamic transient displacement ξ^{DY}. The solution can easily be found, but here we give only a greatly simplified treatment which contains, however, most of the essential features. The reader is referred to the more extended treatment mentioned earlier for details.

The solution of Eq. 1 contains a damping factor $\exp(-Bt/2A)$ for the transient ξ^{DY}. The force F on the obstacle is given by

$$F=2C\xi_x(1+\xi_x^2)^{-1/2}\cong 2C\xi_x \tag{2}$$

for small enough displacements. The exact form for ξ depends upon the initial velocity of the dislocation. For velocities which are small compared to sound velocities it is easy to show that the initial velocity of the dislocation can be neglected in the expression for the displacement. It is then apparent from Fig. 1, that the ratio of the maximum dynamic force F^{DY} to the force F^{ST} exerted on the obstacle in the static case is obtained when $t=\pi/\omega_0$, where ω_0, the lowest resonant frequency of the dislocation segment length L, is given by

$$\omega_0=\pi(C/A)^{1/2}L^{-1}\simeq\pi v_s L^{-1}, \tag{3}$$

with v_s the shear-wave velocity. Then we find that

$$F^{DY}/F^{ST}=1+\gamma\exp(-Z), \tag{4}$$

where $\gamma\sim 1$ and

$$Z=B\pi/2A\omega_0. \tag{5}$$

Under the assumption that plastic flow proceeds when the force on the obstacle exceeds a critical force, the ratio of the stress required for the underdamped case to that required for the static or overdamped case is

$$\sigma^{DY}/\sigma^{ST}=[1+\gamma\exp(-Z)]^{-1}. \tag{6}$$

The ratio of the stress in the superconducting state σ_s to that in the normal state σ is then given by

$$\frac{\sigma_s}{\sigma}=\frac{1+\gamma\exp(-Z_n)}{1+\gamma\exp(-Z_s)}, \tag{7}$$

where Z_n and Z_s are values of Z corresponding to values of B appropriate for the normal and superconducting states. Then

$$\varphi=\frac{\sigma-\sigma_s}{\sigma}=\frac{\gamma[\exp(-Z_s)-\exp(-Z_n)]}{1+\gamma\exp(-Z_s)}. \tag{8}$$

As the temperature $T\to 0$, $B\to 0$ (neglecting radiation damping) and $Z_s \to 0$, so that

$$\varphi \to \varphi_0 = [\gamma/(1+\gamma)][1-\exp(-Z_n)]. \quad (9)$$

As $T \to T_c$, the critical temperature, $Z_s \to Z_n$ and $\varphi \to 0$. According to Eq. (9), the maximum percentage effect is given by $\gamma/(1+\gamma) \approx 50\%$ for $\gamma \sim 1$. [A more detailed analysis gives $\gamma/(1+\gamma) \approx 40\%$.]

For $Z_n \ll 1$, Eq. (8) becomes, by using Eq. (5),

$$\varphi = \frac{\gamma}{(1+\gamma)}(Z_n - Z_s) = \frac{\gamma}{(1+\gamma)} \frac{\pi B_n}{2A\omega_0}\left(1 - \frac{B_s}{B_n}\right). \quad (10)$$

If it is assumed that the drag B is proportional to the normal electronic density $\rho_n(T)$, then one has

$$\varphi = \frac{\gamma}{1+\gamma} \frac{B_n L}{2Av_s} \rho_s(T), \quad (11)$$

where $\rho_s(T)$ is the superconducting electron density as a function of temperature, and Eq. (3) has been used. The predicted temperature dependence of the precentage stress drop would then be that of $\rho_s(T)$, provided that $Z_n \ll 1$ or that the percentage change in stress is small. With the representative values of $\gamma \sim 1$, $B_n \sim 2\times10^{-5}$ cgs units, $A \sim 10^{-14}$ g/cm, and $\omega_0 \sim 10^{10}$ sec^{-1}, Eqs. (10) and (11) give $\varphi \sim 0.05\rho_s(T)$. If the percentage drop is large, then the temperature dependence is more complicated and stronger, dropping to low values at temperatures below T_c, according to Eq. (8).

The temperature dependence of the stress change can be compared to the temperature dependence of the London penetration depth λ_L through the relation $\rho_s(T) = \Lambda(0)/\Lambda(T) = [\lambda_L(0)/\lambda_L(T)]^2$, where Λ is the London parameter. Some difficulties arise in a detailed comparison because of uncertainties in both the stress and penetration-depth temperature dependences. The earliest data for $[\lambda(0)/\lambda(T)]^2$ for most materials tended to support the $1-t^4$ ($t = T/T_c$) relation given by the Gorter-Casimir[19] two-fluid model. More recent and more accurate data show departures from the simple $1-t^4$ relation which tend to be in closer agreement with the BCS[20] predicted temperature dependence. The deviations are usually in such a direction as to give smaller values of ρ_s at a given temperature. Uncertainties in the temperature dependence of the penetration depth arise from several sources. The theoretical temperature dependence of the measured penetration depth depends upon the ratio of the coherence length ξ_0 to the penetration depth λ, being closer to $1-t^4$ for large ξ_0/λ.[20] The measured penetration depth is not simply related, in general, to the London penetration depth λ_L. Most of the measurements to date are not absolute measurements, and assumptions have to be made to obtain absolute values. Different temperature dependences can also be expected if the energy gap or its temperature dependence differ from that given by the BCS theory. Also, some evidence has been obtained for anisotropic effects. Pb is a strong-coupling superconductor, which may not follow the BCS predictions. All in all, it appears that one should expect a temperature dependence lying somewhere between the Mühlschlegel[21] calculation of $\Lambda(0)/\Lambda(T)$ based on the BCS theory (dashed line of Fig. 2) and a $1-t^4$ curve (solid line).

In Fig. 2 the data of Suenaga and Galligan[8] for Pb (open symbols) and of Alers, Buck, and Tittman[9] and Hutchison and Pawlowicz[12] for In (closed symbols) are shown. The Mühlschlegel BCS curve is close (except at low temperatures) to the $1-t^2$ dependence claimed by Alers, Buck, and Tittman and by Hutchison and Pawlowicz (but denied by Pustovalov and Fomenko[13]), but the data have here been renormalized to $(\Delta\sigma)_0 = 15$ g/mm^2 for Alers, Buck, and Tittman, and 9.4 g/mm^2 for Hutchison and Pawlowicz. The agreement with the Mühlschlegel curve is fairly good.

The Pb data appear to be in closer agreement with the $1-t^4$ curve. However, the difference between the tin and lead data shown in Fig. 2 may be somewhat exaggerated. The lead data points are as given by Suenaga and Galligan. The tin data points were normalized to give the best fit with the BCS curve. But both could be renormalized to give values lying closer together. However,

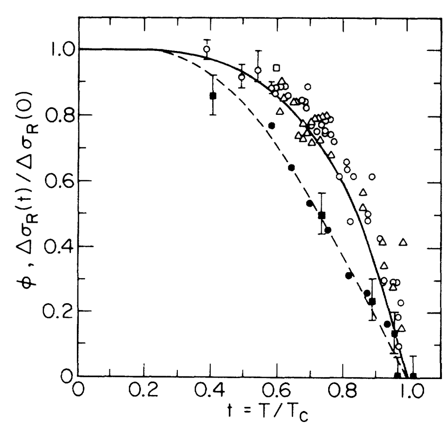

FIG. 2. The temperature dependence of $\Delta\sigma$ for Pb (open symbols) according to Ref. 8 and for In (closed symbols) according to Ref. 9 (squares with error bars) and to Ref. 12 (circles). The dashed line is Mühlschlegel's calculation of the superconducting electron density according to the BCS theory for weak superconductors, and the solid line is $1-t^4$.

the lead data would still appear to have a stronger temperature dependence than do the tin data. More measurements at lower temperatures would be required to determine the temperature dependence more accurately. The penetration-depth measurements of Gaparovic and McLean[22] for lead and of Dheer[23] for tin are both fitted by a $1-t^4$ dependence, but the thin-film measurements of Jaggi and Sommerhalder[24] for tin lie between the $1-t^4$ and Mühlschlegel curves. Taking into account the uncertainties in both the penetration-depth and stress measurements, the temperature dependence given by $\rho_s(T)$ seems to be in reasonable agreement with experiment. If more accurate measurements should show a discrepancy, then the simplifying assumption used in Eq. (11) would have to be dropped, and a more detailed examination of $B_s(T)$ would be required. A more critical test of the model than a comparison with penetration depths is more likely to be a test of the predicted change in the temperature dependence for large $\Delta\sigma$ given by Eq. (8).

Pustovalov, Startsev, and Fomenko[4] have reported an average change of 30% for the critical resolved shear stress of lead single crystals with tension axes close to $\langle 110\rangle$ and of 20% for crystals of somewhat different orientation. For polycrystalline samples of comparable microstructure, the yield stress was found to decrease by 34 to 41%. According to Eq. (8), the temperature dependence for these specimens would be predicted to be stronger than that of $\rho_s(T)$. However, measurements of the temperature dependence have so far only been reported for cases where the percentage change was less than 10%.

For those cases where $\varphi(T)$ follows $\rho_s(T)$, one must have $Z_n \ll 1$ according to Eq. (8). This means that the dislocations are also underdamped in the normal state. In such cases, a substantial fraction of the flow stress in the normal state is also of inertial origin. This suggests that part of the normal flow stress, also for nonsuperconductors, is dynamic in origin and should carry the temperature dependence of B. The effect of phonon damping may then be felt at somewhat higher temperatures. As the phonon density increases, the inertial effect should decrease, giving a contribution to the flow stress which tends to increase the flow stress with increasing temperature. There appears to be experimental evidence for this in the work of Startsev, Pustovalov, and Fomenko[2] who found anomalous maxima in the flow stress of different lead specimens at temperatures between the superconducting temperature and liquid nitrogen temperature.

In the more detailed account published elsewhere, we compare the quantitative predictions of the simplified inertial model with experiment for the dependence of the effect on other variables, taking into account more specific expressions for the dislocation displacement, dislocation radiation losses, and the background stresses.

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Zero-Bias Anomaly in Irradiated Pb-GaAs Tunnel Junctions, and the Mott Transition*

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Pb-GaAs Schottky barriers irradiated progressively by 10-MeV electrons or fast neutrons were found to exhibit anomalous behavior identical to that produced by varying the initial doping. The results are consistent with the Hubbard model of the Mott transition.

In a recent Letter, Wolf, Losee, Cullen, and Compton[1] have reported the observation of an anomalous zero-bias resistance peak in Schottky-barrier tunnel junctions on Si:B which appears as the semiconductor impurity concentration approaches the critical Mott concentration N_c.[2,3] They suggest that the peak is due to a gap or sharp minimum, at the Fermi energy, in the density of states in the semiconductor, in accord with the Hubbard[4] model of the Mott transition. Here, we present results of an extensive study, carried out independently of the above authors' work, of the effect of high-energy (10-MeV) electron and fast-neutron bombardment on the zero-bias anomaly in p-GaAs-Pb Schottky-barrier tunnel junctions. Our results strongly suggest that the zero-bias anomaly can indeed be explained in terms of the Hubbard-Mott model for the impurity band in heavily doped semiconductors. They show that the changes in the anomaly produced by the irradiation result solely from the reduction of the free carrier concentration through trapping of carriers by defect states introduced into the GaAs by the irradiation. The obvious advantage of these irradiation experiments is that one can study the effect on the anomaly of a change in carrier concentration in the *same* junction. An explanation of the anomaly in terms of two-step tunneling through real intermediate states in the barrier[5,6] is possible although not as likely as one involving the Mott transition, as will be shown below and in more detail in a later publication.[7]

The junctions were prepared by the vacuum deposition of Pb onto p-GaAs single-crystal wafers that were chemomechanically polished in a dilute solution of Br (0.05–0.1% by volume) in methanol. Typically, seven junctions were made on a single wafer using Kodak Thin Film Resist to define the junction area (approximately 1.2×10^{-4} cm^2). Samples with selected carrier concentrations were irradiated with progressively increasing doses of 10-MeV electron pulses or by fast reactor neutrons.

Prior to irradiation, highly doped junctions ($p \geq 8\times10^{18}$ cm^{-3}) showed a good Pb superconducting gap, a well-resolved Pb phonon density of states, and the GaAs LO-phonon structure at +36 mV, thus showing them to be good tunnel junctions.[7] The zero-bias anomaly appeared as a resistance peak with a magnitude at $T \leq 4.2$°K of ~15% relative to the extrapolated background R_b, with a half-width of ~12 mV, as evident from Fig. 1(a). The smaller and narrower (~1%, half-width 1 mV) resistance dip superimposed upon the broader resistance peak [Fig. 1(a)] was attributable to tunneling with spin-flip via localized magnetic states in the barrier.[8,9]

The effect of both electron and neutron irradiations on the tunneling characteristics was essentially the same. Progressively larger doses of radiation increased the relative size of the re-