readily extended to such a case but a more complicated result is found.

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$s \leftrightarrow u$ Symmetry of s-Channel Helicity Amplitudes

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The $s \leftrightarrow u$ symmetry of s-channel helicity amplitudes is investigated *without the exchange mechanism being specified.* Conclusions are drawn concerning the possibility of phase assignment to scattering amplitudes at high energy. Difficulties are found in absorption models with $J_{\Delta\lambda} s^{\alpha}$ behavior.

Recently a number of models assigning a phase to s-channel helicity amplitudes at high energy have been proposed[1] *without the exact nature of the t-channel exchange mechanism* (such as fixed or moving poles, cuts, etc.) *being specified.* As is well known, such an assignment is indeed possible if an amplitude has definite $\nu \leftrightarrow -\nu$ symmetry $[\nu = \frac{1}{2}(s-u)]$ and ν^{α} behavior at high energy.[2] In this paper we use only C, T, P, and SU(2) [SU(3)] to find s-channel helicity amplitudes obeying the above mentioned symmetry.

Let us consider the s-channel reaction $ab \to cd$. Since $\nu \to -\nu$ correlates this reaction with $a\bar{d} \to c\bar{b}$ (u channel), it corresponds to the interchange of particles $\bar{d}$ and b in the t-channel reaction $\bar{d}b \to c\bar{a}$. Thus in order to obtain symmetry under $\nu \to -\nu$ for the s-channel helicity amplitudes, hereafter denoted by G, we shall first look for symmetry under $\bar{d} \leftrightarrow b$ in the t-channel helicity amplitudes, denoted by F.

We find that such symmetry exists for the t-channel helicity amplitudes in the following two cases: (a) Particles $\bar{d}$ and b belong to the same multiplet M of SU(2) (e.g., $p\pi^+ \to \Delta^{++}\pi^0$, $b=\pi^+$, $d=\pi^0$) or SU(3) (e.g., $pK^- \to \Lambda^0\pi^0$, $b=K^-$, $d=\pi^0$); (b) particles $\bar{d}$ and b belong to multiplets of SU(2) or SU(3) connected by change conjugation C, and particles c and $\bar{a}$ remain in their multiplets under C (e.g., $\pi^-p \to A_2^0 n$, $\pi^-p \to K^{*+}\Sigma^-$). For both cases the following symmetry holds for the t-channel helicity amplitudes:

$$F_{\lambda_c \lambda_a \lambda_d \lambda_b}{}^{i}(t,\nu) = (-1)^{\eta_i} F_{\lambda_c \lambda_a \lambda_b \lambda_d}{}^{i}(t,-\nu), \quad (1)$$

where i labels t-channel matrix elements reduced with respect to the internal symmetry [SU(2) or SU(3)], $\{\lambda\}$ denote the helicities of the particles in the t-channel, and $(-1)^{\eta_i} = \pm 1$ is a phase independent of helicities.

For case (a), Eq. (1) follows immediately from the statistics[3] and from the well-defined symmetry η_i of the irreducible representation $M \otimes M$ labeled by i. For case (b) one uses G and C invariance to prove Eq. (1). Consider for example, the t-channel reaction $\pi^+\rho^0 \to \bar{n}p$ (pure $I=1$). Using G parity and fermion anticommutation relations one easily obtains Eq. (1) with $\eta_i = +1$. A similar proof holds for all other class-(b) reactions in the SU(2) case. For b and $\bar{d}$ belonging to SU(3) octets one proves Eq. (1) [with i denoting the reduced amplitudes $\underline{1}$, $\underline{8}_{ss}$, $\underline{8}_{sa}$, $\underline{8}_{as}$, $\underline{8}_{aa}$, $\frac{1}{2}(\underline{10} \pm \underline{10}^*)$, or $\underline{27}$] by applying statistics and G parity to states of $I=0$ in $\underline{8} \otimes \underline{8}$.

Whenever Eq. (1) holds in the physical region of the t channel, we assume its validity for the whole analyticity domain of F in ν and t.

Let us now transform Eq. (1) from the t-channel helicity amplitudes F to the s-channel helicity amplitudes G, using the crossing relations of Cohen-Tannoudji, Morel, and Navelet.[4] The crossing as given in Ref. 4 reads[5]

$$G_{\lambda_c\lambda_d\lambda_a\lambda_b}{}^i(\nu,t)=(-1)^{\sigma+2s_b+2s_d}\exp[i\pi(\lambda_b-\lambda_c)]\sum_{\{\lambda'_i\}}d^{s_a}(\chi_a)_{\lambda_a}{}^{\lambda'_a}d^{s_b}(\chi_b)_{\lambda_b}{}^{\lambda'_b}\times d^{s_c}(\chi_c)_{\lambda_c}{}^{\lambda'_c}d^{s_d}(\chi_d)_{\lambda_d}{}^{\lambda'_d}F_{\lambda_c'\lambda_a'\lambda_d'\lambda_b'}(t,\nu), \tag{2}$$

where $\sigma=1$ if a and d are fermions and 0 otherwise, and χ_i are the crossing angles defined in Table XI of Ref. 4. G^i, as given in Eq. (2), is evaluated now in the limits $\nu\to\infty+i\epsilon$ and (by analytic continuation) $\nu\to-\infty+i\epsilon$, at fixed t. To compare these two limits we use Eq. (1) together with the equations

$$\lim_{\nu\to\infty+i\epsilon}\cos\chi_i=\lim_{\nu\to-\infty+i\epsilon}\cos\chi_i,\quad \lim_{\nu\to\infty+i\epsilon}\sin\chi_i=\lim_{\nu\to-\infty+i\epsilon}\sin\chi_i,$$

and $\chi_b=\chi_d+\pi$ at $|\nu|\to\infty$ (for $m_b\cong m_d$, valid for the cases treated here), and the well-defined properties of the d functions. We obtain for classes (a) and (b) the asymptotic relation (valid for $|\nu|\to\infty$)

$$G_{\lambda_c\lambda_d\lambda_a\lambda_b}{}^i(\nu,t)=(-1)^{\eta_i+2s_b+\lambda_b-\lambda_d}G_{\lambda_c,-\lambda_d,\lambda_a,-\lambda_b}{}^i(-\nu,t). \tag{3}$$

Therefore the combinations

$$H_\pm{}^i(\nu,t)=\tfrac{1}{2}[G_{\lambda_c\lambda_d\lambda_a\lambda_b}{}^i(\nu,t)\pm(-1)^\varphi G_{\lambda_c,-\lambda_d,\lambda_a,-\lambda_b}{}^i(\nu,t)] \tag{4}$$

(where $\varphi=\eta_i+2s_b+\lambda_b-\lambda_d$) have the definite $\nu\leftrightarrow-\nu$ symmetry

$$H_\pm{}^i(\nu,t)=\pm H_\pm{}^i(-\nu,t). \tag{5}$$

Thus, if one assumes $\nu^{\alpha_\pm(t)}$ behavior for $H_\pm{}^i$, respectively, it follows from the Phragmén-Lindelöff theorem and from real analyticity[2] that asymptotically

$$H_\pm{}^i(\nu,t)=f_\pm(\nu)\nu^{\alpha_\pm(t)}\exp[-\tfrac{1}{2}i\pi\alpha_\pm(t)], \tag{6}$$

where $f_\pm(t)$ are real functions of t.

The amplitudes G themselves have definite $\nu\leftrightarrow-\nu$ symmetry whenever [see Eq. (3)] (I) $\lambda_b=\lambda_d=0$; (II) $\lambda_a=\lambda_c=0$ (using parity conservation); or (III) $\lambda_a=\lambda_c$, and $-\lambda_b=\lambda_d$, and the s-channel reaction transforms into itself under time reversal. In all the above cases f_+ or f_- is identically zero [see Eqs. (3), (4), and (6)]. In other cases one obtains

$$G_{\lambda_c,\pm\lambda_d,\lambda_a,\pm\lambda_b}{}^i(\nu,t)=f_+\nu^{\alpha_+}\exp(-\tfrac{1}{2}i\pi\alpha_+)\pm if_-\nu^{\alpha_-}\exp(-\tfrac{1}{2}i\pi\alpha_-). \tag{7}$$

Let us discuss the consequences of Eq. (7) in the limit $\nu\to\infty$. For $\alpha_+\neq\alpha_-$ the term with $\alpha=\max(\alpha_+,\alpha_-)$ dominates in Eq. (7), hence

$$G_{\lambda_c\lambda_d\lambda_a\lambda_b}{}^i(\nu,t)=\pm G_{\lambda_c,-\lambda_d,\lambda_a,-\lambda_b}{}^i(\nu,t). \tag{8}$$

For $\alpha_+\neq\alpha_-$,

$$|G_{\lambda_c\lambda_d\lambda_a\lambda_b}{}^i(\nu,t)|=|G_{\lambda_c,-\lambda_d,\lambda_a,-\lambda_b}{}^i(\nu,t)|; \tag{9}$$

Eqs. (8) and (9) put severe restrictions on absorption models of helicity amplitudes for which $|\lambda_a-\lambda_b-\lambda_c+\lambda_d|\neq|\lambda_a+\lambda_b-\lambda_c-\lambda_d|$. In Shtokhamer, Berlad, and Eilam,[1] it is proposed that $|G|\propto J_{\Delta\lambda}\nu^\alpha$. An inconsistency of this assumption can be seen by considering the helicity amplitudes $G_{1,\pm1/2,0,\mp1/2}(\nu,t)$ for the reaction $\pi^-p\to\rho^0n$. One should have $|G_{1,+1/2,0,-1/2}|\propto J_0\nu^\alpha$, $|G_{1,-1/2,0,+1/2}|\propto J_2\nu^\alpha$. However, from Eqs. (8) or (9) it follows that $|J_0|\propto|J_2|$. Hence, since $J_0(R\sqrt{-t})=-J_2(R\sqrt{-t})$ at high $|t|$ only, one cannot hope for this model to be valid at small and intermediate $|t|$.[6]

If $\alpha_+\neq\alpha_-$ then $\mathrm{Im}G_{1,1/2,0,-1/2}=\pm\mathrm{Im}G_{1,-1/2,0,1/2}$ [Eq. (8)]. Therefore the proposal of Harari, $\mathrm{Im}G_{[\lambda]}\propto J_{\Delta\lambda}\nu^\alpha$, is also inconsistent. However the model of Harari is satisfactory for our purposes in the case of $\alpha_+\neq\alpha_-$. Using the assumptions

$$\mathrm{Im}G_{1,1/2,0,-1/2}(\nu,t)\propto J_0(R\sqrt{-t})\nu^\alpha,\quad \mathrm{Im}G_{1,-1/2,0,1/2}(\nu,t)\propto J_2(R\sqrt{-t})\nu^\alpha$$

and Eq. (7) one easily obtains

$$f_+=-\frac{J_0(R\sqrt{-t})+J_2(R\sqrt{-t})}{2\sin\pi\alpha/2},\quad \pm f_-=\frac{J_0(R\sqrt{-t})-J_2(R\sqrt{-t})}{2\cos\pi\alpha/2}.$$

[The functions $J_{\Delta\lambda}(R\sqrt{-t})$ may still be multiplied by factors of the form e^{At}]. We find that the helicity amplitudes G are no longer proportional to $J_{\Delta\lambda}$ as expected from naive absorption models,[7] and they may diverge whenever $\sin\pi\alpha = 0$.

To summarize: In the context of absorbtion models with phase, the assumption which does not lead to inconsistency is $\mathrm{Im}G_{[\lambda]} \propto J_{\Delta\lambda}$ together with $\alpha_+ = \alpha_-$. However, according to our treatment here, (1) there is only a limited class of reactions [types (a) and (b)] for which $\nu \to -\nu$ symmetry may exist at high ν for s-channel helicity amplitudes, to which an asymptotic phase may be assigned. Thus reactions like $\pi N \to \rho\Delta$ are excluded from this category. (2) Even for reactions of types (a) and (b) the above-mentioned symmetry does not hold for all individual amplitudes as for example $G_{1,-1/2,0,1/2}$ in $\pi N \to \rho N$.

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Rigorous Inequalities from Positivity and Duality in the Zero-Width Approximation*

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It is shown that in dual-resonance models with no ghosts the least massive spin-J meson which couples to baryons must have $I=0$ and $P=C=(-1)^J$ for $J \geq J_0$ (number of subtractions at $t=0$), e.g., $M_\omega \leq M_\rho$, $M_f \leq M_{A2}$, etc. If degeneracy occurs, e.g., $M_\omega = M_\rho$, then the $I=0$, $P=C=(-1)^J$ coupling must dominate. Inequalities are found among couplings of arbitrary-spin baryons to leading mesons. For ω-nucleon coupling, $g\gamma^\mu + (if/2m)\sigma^{\mu\nu}q_\nu$, with $M_\omega < M_\rho$ one finds that $g^2 \geq (fM_\omega/2m)^2$. With ω-ρ degeneracy, $g_\omega^2 - g_\rho^2 \geq (M_\omega/2m)^2|f_\omega^2 - f_\rho^2|$. Bounds on SU(3) mixing angles and baryon masses are also found.

We present a theorem proven by one of us (D.C.) which applies to a wide class of zero-width dual resonance scattering amplitudes. We apply it first to the sample example of $K\bar{K}$ scattering and then to the much more general problem of arbitrary-spin, equal- or unequal-mass baryon-antibaryon ($B_1\bar{B}_2$) scattering. All of our results follow from model-independent features of duality in the zero-width approximation. Thus, we avoid the problem of assessing the significance of predictions of a specific model such as the Veneziano model[1] which suffers from problems with ghosts[2] and tachyons,[2] or the naive quark model which is beset with difficulties[3] for processes involving baryons. Our results permit direct experimental tests of duality provided the width-dependent corrections are small. All presently available data which we have examined are consistent with the predictions of the theorem.

The statement of the theorem, which can easily be proved,[4] is as follows: If (a) $M(s,t)$ is an analytic function whose only singularities are simple poles at $s=s_i>0$ and $t=t_j>0$, $i,j=0,1,2,\cdots$, where s_i and t_j are real constants, (b) $M(s,t)$ is polynomially bounded away from its poles as $|s| \to \infty$ for fixed t, (c) residues of poles in s are polynomials in t, and (d) with the possible exception of a finite number of these residues, the co-