trum.—In this case, $\epsilon_n = A(n+\frac{1}{2})^2$, where A is a constant, and each level has a degeneracy of $2(2n+1)$. A simple calculation by the partition-function method yields

$$\bar{g}(\epsilon) = 2/A + \tfrac{1}{6}\delta(\epsilon), \qquad (10)$$

and the shell correction can be trivially calculated. We again find excellent agreement between these results and the numerically calculated Strutinskii values.

These three simple solvable models lead us to believe that the Strutinskii method of shell correction yields essentially the same results as would be obtained by calculating $\bar{E}$ in a systematic semiclassical way. We thus have gained more confidence in the intuitive prescription of obtaining $\bar{E}$ as proposed by Strutinskii, noting that it is similar to a generalized Thomas-Fermi approximation.[7] In this paper we have only concentrated on models with discrete single-particle spectra. A more realistic case to consider would be that of a discrete spectrum followed by the continuum. Lin[11] has recently pointed out that the Strutinskii method does not seem to yield a unique γ-independent δE in this case. The investigation of this from the partition-function approach is currently being investigated.

We would like to thank Professor Nogami, Professor Preston, Professor Sprung, and Professor Warke for informative discussions.

*Research supported by the National Research Council of Canada.

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Production of Monochromatic γ Rays by Collimation of Coherent Bremsstrahlung

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The coherent bremsstrahlung from a 50-μm-thick crystal of silicon was collimated to $0.234(m_e c^2/E_1)$ rad with the incident energy of the electrons $E_1 = 1$ GeV. The energy spectrum shows a sharp spike with the signal-to-background ratio greater than 10 and the half-width $\Delta k = 15$ MeV at a photon energy $k = 185$ MeV. The spectrum is in good agreement with theory based on Überall's treatment. A maximum intensity of 2×10^4 photons/MeV sec has been obtained at the spike.

It has been proposed by Mozley and DeWire[1] that a monochromatic photon beam can be produced by collimating coherent bremsstrahlung from single crystals. Some numerical calculations on the energy spectrum have been carried out by Lutz and Bologna.[2]

The effect can be described kinematically as follows: In high-energy bremsstrahlung, the main contribution to the cross section comes from a small-angle process in which the angle between the directions of the photon and the incident electron (scattered electron) is less than $m_e c^2/E_1$ $(m_e c^2/E_2)$, where E is the energy of the electron. By using the energy-momentum conservation for this small-angle case, one can show that the longitudinal momentum transfer

q_l satisfies the relation

$$\delta \leq q_l \leq (1+\theta_c^2)\delta, \tag{1}$$

where $\delta = x/2E_1(1-x)$, $x=k/E_1$, and θ_c is the collimation angle divided by $m_e c^2/E_1$. For the coherent bremsstrahlung from crystals, the momentum transfer $\vec{q}$ is given in terms of the reciprocal-lattice vector. Equation (1) shows that the interference condition for the photon energy is $x_0 - \Delta x \leq x \leq x_0$, where $x_0 = 2E_1 q_l/(1+2E_1 q_l)$ and $\Delta x/x_0 = \theta_c^2(1-x_0)$.

Starting with Überall's treatment,[3] one obtains the following expression for the differential cross section for the collimated bremsstrahlung from single crystals:

$$\partial\sigma_{cr}/\partial k = N(Z^2e^6/k)\{[1+(1-x)^2][M^c(\delta)+M^i(\delta)] - \tfrac{2}{3}(1-x)[N^c(\delta)+N^i(\delta)]\}, \tag{2}$$

where

$$M^c(\delta) = \left\{4\int_\delta^1 q(q-\delta)^2 \frac{[1-F(q)]^2}{q^4}[1-\exp(-Aq^2)]\,dq + 4\right\} f(\theta_c, k),$$

$$N^c(\delta) = \left\{4\int_\delta^1 \left(q^3 - 6\delta^2 q \ln\frac{q}{\delta} + 3\delta^2 q - 4\delta^3\right)\frac{[1-F(q)]^2}{q^4}[1-\exp(-Aq^2)]\,dq + \tfrac{10}{3}\right\} f(\theta_c, k),$$

$$M^i(\delta) = \frac{(2\pi)^2}{a^3}\sum_{\vec{q}=\vec{b}} \exp(-Aq^2)D(\vec{q})4\delta\frac{q^2}{q_l^2}\frac{[1-F(\vec{q})]^2}{q^4},$$

$$N^i(\delta) = \frac{(2\pi)^2}{a^3}\sum_{\vec{q}=\vec{b}} \exp(-Aq^2)D(\vec{q})24\delta^2\frac{q^2(q_l-\delta)}{q_l^4}\frac{[1-F(\vec{q})]^2}{q^4}.$$

Here the units $c=\hbar=m_e=1$ are used. In Eq. (2), N is the total number of atoms in the crystal, Z is the atomic number, and $F(\vec{q})$ is the atomic form factor. The cross section depends on parameters characterizing the crystal structure, i.e., the Debye-Waller factor A, the lattice constant a, the reciprocal-lattice vector b, and the strength factor of the lattice point, $D(\vec{q})$.

The summation over $\vec{q}$ is to be made under the condition (1). The effect of the collimation on the incoherent part is, to a good approximation, given by a factor defined by

$$f(\theta_c, k) = \frac{\int_0^{\theta_c}(\partial^2\sigma/\partial\theta\partial k)\sin\theta\,d\theta}{\int_0^{\pi}(\partial^2\sigma/\partial\theta\partial k)\sin\theta\,d\theta}, \tag{3}$$

where $\partial^2\sigma/\partial\theta\partial k$ is the differential cross section for the bremsstrahlung from an amorphous target. We used Schiff's formula[4] for $\partial^2\sigma/\partial\theta\partial k$ in our numerical analysis.

An experimental study on the collimation effect was first made on a silicon crystal by Kato *et al.*[5] although the result was obscured by the multiple scattering of electrons in the crystal target. Criegee *et al.*[6] recently observed the collimation effect by using thin crystals of silicon, but the effect was largely suppressed by the primary divergence of the electron beam in the synchrotron.

The experiment reported here was performed by using a 1.0-GeV electron beam of the synchrotron at the Institute for Nuclear Study, the University of Tokyo. The primary divergence of the electron beam was reduced by a set of scrapers distributed along the synchrotron orbit. The result obtained shows excellent agreement with theoretical predictions, and to our knowledge this is the first demonstration for the substantial monochromatization of the bremsstrahlung by the collimation method.

The major difficulty in getting monochromatic photons by collimation is due to the angular spread of the electron beam with respect to the crystal axes. Specifically, betatron oscillations, multiple Coulomb scattering in the crystal, and the multiple traverse of electrons through the crystal by repeated circulation are three sources of angular divergence.

To minimize the multiple Coulomb scattering, we prepared a crystal of 50 ± 10 μm thickness. The flat surface of the specimen was parallel to the (110) plane, and the specimen was set on its holder so that the electrons were incident in the (001) plane with a small angle with respect to [110] axis.

The crystal was placed as far from the central orbit of the synchrotron as possible so that the wall of the vacuum vessel might suppress the multiple traverse of electrons. To get rid of the difficulty due to betatron oscillations and the multiple traverse, we set three remote-controlled beam scrapers in straight sections of the synchrotron. The relative positions of the crystal and the scrapers were determined by considering the fact that the distance between adjacent straight sections is approximately equal to $\frac{1}{4}$ of the betatron wavelength. A scraper was set at about $\frac{1}{2}$ wavelength downstream from the crystal to control the vertical divergence of the beam. Two scrapers were placed at about $\frac{3}{4}$ and $\frac{5}{4}$ wavelengths downstream from the crystal to reject the horizontally deflected electrons.

The photon beam from the crystal target passed through the first collimator of 3 mm diam at a distance of 260 cm and the second collimator of 3 mm diam placed at a distance of 1254 cm. The energy spectrum of the bremsstrahlung was measured by an electron-positron pair spectrometer with a momentum resolution of $\Delta p/p \simeq \pm 2.5\%$. The details of the spectrometer are described in Ref. 5.

The primary beam divergence was independently checked by replacing the crystal target with a 50-μm-thick aluminum plate and exposing x-ray films to the photon beam. The grain-density distribution of the films was measured by a photometer, and the angular distribution of the photon beam thus obtained was compared with the theoretical formula based on Lanzl and Hanson's treatment.[7] The beam divergence estimated from this analysis is $(1.0 \pm 0.5) \times 10^{-4}$ rad.

In the measurement of the energy spectrum of the coherent bremsstrahlung, we adjusted the crystal orientation to $\pm 1.0 \times 10^{-4}$ rad, the position of the scrapers to ± 0.05 mm, and the position of the second collimator to ± 0.1 mm.

Figure 1(a) shows a typical example of the observed spectrum. The spectrum shows a sharp spike at $k = 185$ MeV with the half-width $\Delta k = 15$ MeV. The beam intensity at the spike is about 2×10^4 photons/MeV sec. To compare the observed spectrum with the formula (2), we made a Monte Carlo calculation with the following assumptions on the beam characteristics: (1) a two-dimensional Gaussian distribution for the incident angular divergence; (2) a uniform distribution for the interaction region in the crystal; and (3) a two-dimensional Gaussian distribution for the multiple Coulomb scattering with the standard deviation given by Rossi and Greisen's theory.[8] Assumption (2) provides the effective thickness for multiple Coulomb scattering. In fitting to the experimental data, we also included the effect of the momentum resolution of the spectrometer. The best fit was achieved by

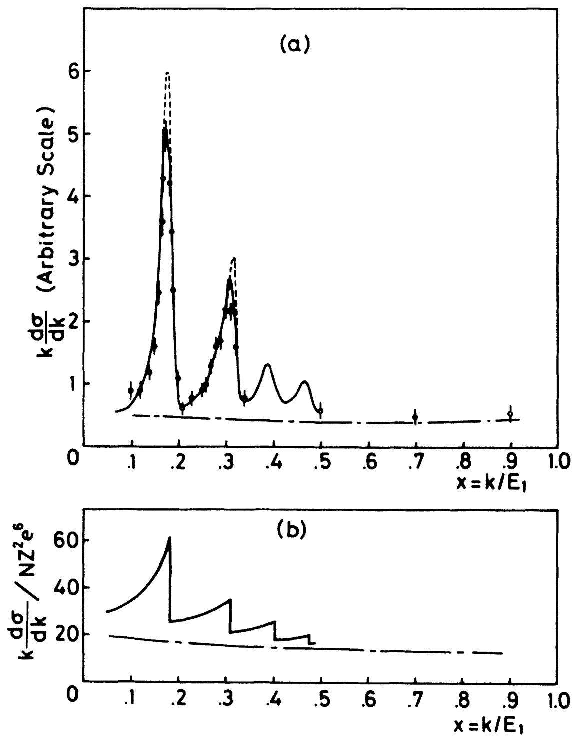

FIG. 1. (a) A typical example of the energy spectrum of collimated bremsstrahlung from a 50-μm-thick silicon crystal. The primary energy of the electron is $E_1 = 1000$ MeV and the collimation angle is $\theta_c = 0.234 m_e c^2/E_1$. The electrons are incident in the (001) plane at an angle 9.19×10^{-3} with the [110] axis. The full curve shows our best fit by the Monte Carlo calculation discussed in the text. The dashed curve is obtained if one omits the effect of the finite resolution of the spectrometer. The dotted-dashed curve represents the continuous background due to the thermal vibration of the crystal lattice. (b) A calculated spectrum for the uncollimated bremsstrahlung. The kinematical conditions are the same as those for Fig. 1(a). The beam divergence is assumed to be zero.

assuming the primary beam divergence to be 0.7×10^{-4} rad, which is consistent with the aforementioned result of x-ray–film photometry. The full curve in Fig. 1(a) shows our best fit. The dashed curve is the original pattern of the calculated spectrum in which the resolution effect of the spectrometer is not included. In Fig. 1(b) is shown a calculated spectrum for uncollimated bremsstrahlung. The kinematical conditions for Fig. 1(b) are the same as those for Fig. 1(a).

As a concluding remark, we want to mention the possibility of extending the collimation method to higher energies. In principle, the method is more favorable at higher energies since the signal-to-background ratio increases with increasing electron energy. The opening angle of the bremsstrahlung, however, decreases as $1/E_1$; and, correspondingly, the tolerable angular divergence for the electron beam decreases. Fortunately, the beam divergence in and from the accelerator as well as multiple Coulomb scattering in the target also decrease approximately as $1/E_1$ with beam energy. Hence, if the technical problem of the beam control is solved, the method can be used with higher-energy electron accelerators.

We wish to acknowledge the valuable cooperation of Professor S. Yamaguchi and the synchrotron crew. We are indebted to Professor K. Kohra, Dr. S. Kikuta, and Dr. T. Matsushita for preparing a thin crystal of silicon, and to the people of the machine shop of the Institute for Nuclear Study for mechanical design and construction of the beam scrapers. This work was financially supported by Toyo Rayon Foundation for the Promotion of Science and Technology.

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Measurement of the Up-Down Asymmetries in the β Decay of Polarized Λ Hyperons (Argonne–Chicago–Ohio State–Washington University Collaboration)*

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We report results from a counter and optical spark-chamber–spectrometer experiment on the β decay of polarized Λ hyperons. A sample of 218 decays, constituting approximately one third of the total data, has been identified. The measured up-down asymmetries from a selected subsample of 173 events are $A_\nu = 0.67 \pm 0.18$, $A_e = 0.14 \pm 0.17$, and $A_p = -0.55 \pm 0.18$. When interpreted in the framework of a $V-A$ theory with no second-class currents ($g_2 = 0$), they confirm the sign of the form-factor ratio g_1/f_1 as given by the Cabibbo model, but favor a somewhat smaller magnitude.

We have performed an experiment on the decay $\Lambda^0 \to pe^- \bar{\nu}$ at the Argonne National Laboratory zero-gradient synchrotron using optical spark-chamber and counter techniques. Our objective was to study the form of the weak interaction in this decay by measuring the up-down asymmetries of neutrinos, electrons, and protons with respect to the Λ spin.

Polarized Λ hyperons were produced in the reaction $\pi^- p \to \Lambda^0 K^0$ using (1025 ± 3)-MeV/c π^- (just below ΣK threshold) incident on a liquid-hydrogen target.[1] The e^- and p momenta for each $\Lambda \to pe\nu$ event were measured with a magnetic spectrometer. We determined the plane of production