Magnetic Moments of the $\frac{7}{2}^-$ Mirror States in ^{37}Ar and ^{37}K†

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The magnetic moments of the $\frac{7}{2}^-$ states at 1.610 MeV in ^{37}Ar and 1.380 MeV in ^{37}K were measured by differential spin-precession methods. The moments, together with the remeasured halflives, are $\mu = -1.33(5)\mu_N$ and $T_{1/2} = 4.6(2)$ nsec for ^{37}Ar, and $\mu = +5.2(3)\mu_N$ and $T_{1/2} = 10.5(5)$ nsec for the ^{37}K state. The prediction of the Sachs mirror theorem is shown to hold, and systematic trends in $\frac{7}{2}^-$-state moments of $1f_{7/2}$-shell nuclei are examined.

The early development of the shell model was dependent, in part, on the desire to explain nuclear magnetic moment values. Although the moments rarely fall on the Schmidt limits,[1] the fact that most moments lie inside the lines and near them at the closed shell is an important indication of the model's validity. Explanation of the deviations of moments from the simple predictions should be an equally important means for checking more detailed calculations.

One aspect of magnetic moments that makes them especially useful as input to theory is that general assumptions in some cases allow the moments of two or more nuclei to be separated into model-independent and -dependent quantities. For example, following the ideas of Sachs,[2,3] the magnetic moments of an isospin multiplet may be written as the sum of isoscalar and isovector parts. The predominant model-dependent corrections are contained in the isovector part and thus cancel in the sum of the moments of a mirror pair of states.

In recent years, the number of experimentally measured moments has rapidly increased. Recently, Leonardi and Rosa-Clot[4] and Sugimoto[5] have pointed out some interesting regularities for light odd-mirror nuclei ($A \leq 39$). This paper reports on the first measurements of the magnetic moments of a mirror pair of $\frac{7}{2}^-$ states which may be described as single-particle excitations into the $1f_{7/2}$ shell, and on attempts to extend the study of magnetic moment regularities to this shell.

^{37}Ar measurements.—Measurements of the magnetic moment of the excited $\frac{7}{2}^-$ state in ^{37}Ar were performed at the Berlin 5.5-MeV Van de Graaff accelerator. A pulsed proton beam of 6 nsec pulse width and a period of 1 μsec impinged on a thick KCl target (enriched in ^{37}Cl) mounted between the pole tips of an electromagnet providing external magnetic fields of up to 47 kOe.

The decay and precession of the angular distribution of the 1.610-MeV γ rays from the reaction ^{37}Cl$(p,n)^{37}$Ar* were observed differentially in time with apparatus similar to that described by Bleck *et al.*[6] The modulation spectrum from one measurement is shown in Fig. 1. The external magnetic field was calibrated by observing the spin precession of the excited $\frac{5}{2}^+$ state in ^{19}F whose magnetic moment is well known.

Independent measurements were made using magnetic fields of from 16.6 to 47 kOe. The mean value from all measurements is $\mu = -1.33(5)\mu_N$. The half-life of $T_{1/2} = 4.5(2)$ nsec, as determined in this work, is in agreement with the measurements of Goosman and Kavanagh[7] who report $T_{1/2} = 5.15(70)$ nsec. Additional n-γ correlation experiments were performed at the University of

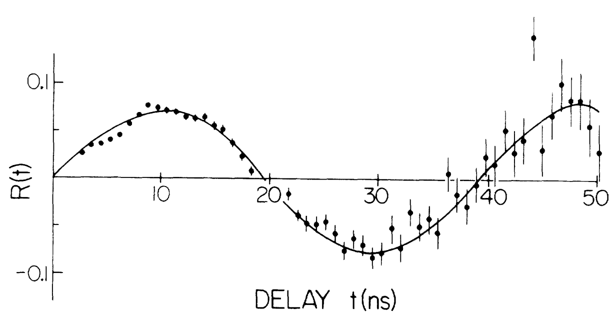

FIG. 1. Modulation spectrum for the ^{37}Ar measurement.

$$R(t) = \frac{I(t, -50°) - I(t, 40°)}{I(t, -50°) + I(t, 40°)},$$

where $I(t, \theta)$ is the number of γ rays detected as a function of time at the angle θ. The solid line represents a least-squares fit of the data by $R(t) = a + b \cos 2(\theta - \omega_L t)$.

Wisconsin. One result was that within the time resolution of 3 nsec no other long-lifetime ^{37}Ar levels were observed. Another result was that the sign of A_2, the angular-correlation coefficient, was positive, allowing for the unique determination of the sign of the magnetic moment from the starting phase of the modulation.

^{37}K measurement.—Measurements of the magnetic moment of the excited $\frac{7}{2}^-$ state in ^{37}K at 1.380 MeV were made at the University of Wisconsin tandem electrostatic accelerator. The level was populated through the reaction ^{40}Ca$(p, \alpha)^{37}$K* with an incident beam energy of 13.0 MeV. The target consisted of approximately 1.5 mg/cm^2 of natural Ca evaporated onto a 5-mg/cm^2 gold backing foil. An electromagnet that was calibrated by means of the precession of the $\frac{5}{2}^+$ state in ^{19}F provided an external magnetic field of 8.2 kOe.

The decay and precession of the angular distribution of the 1.380-MeV γ rays were observed differentially in time by means of a particle-γ coincidence apparatus. The α particles were detected near 180° by an annular surface-barrier detector. Using suitable bias, the α particles corresponding to the population of the 1.380-MeV state appeared above the proton edge in the particle pulse-height spectra.

Since only $M = \pm\frac{1}{2}$ substates are populated in the reaction ^{40}Ca(p, α), the angular-correlation coefficients can be explicitly calculated if the assumption is made that the γ decay is predominantly $M2$.[8] This allowed for a unique determination of the sign of the magnetic moment. The modulation spectrum for this measurement is shown in Fig. 2.

The mean value of all measurements is $\mu = +5.2(3)\mu_N$. The half-life of $T_{1/2} = 10.5(5)$ nsec, as determined in this work, is in agreement with the measurement of $T_{1/2} = 9.6(1.4)$ nsec of Goosman and Kavanagh.[9]

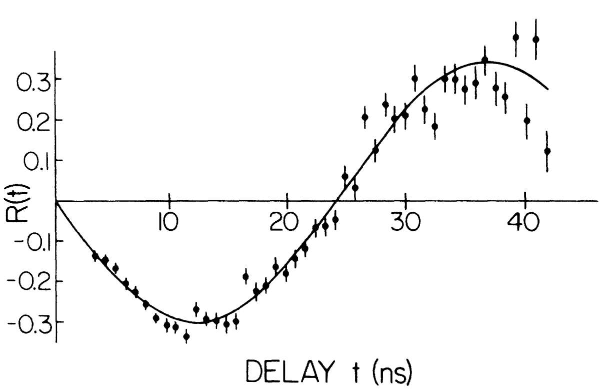

FIG. 2. Modulation spectrum for the ^{37}K data where $R(t) = \dfrac{I(t, -135°) - I(t, 135°)}{I(t, -135°) + I(t, 135°)}$.

Discussion and conclusions.—If one assumes the absence of mesonic exchange currents, the magnetic moment of a nuclear state can be obtained by calculating the expectation value of the operator

$$\vec{\mu} = \sum_{i=1}^{p} [\vec{l}(i) + \mu_p \vec{\sigma}(i)] + \sum_{i=1}^{n} \mu_n \vec{\sigma}(i), \tag{1}$$

where μ_p and μ_n are the free-particle moments of the proton and neutron, respectively. With the help of the isospin formalism this expression can be separated into an isoscalar and an isovector part,[10]

$$\vec{\mu} = \vec{\mu}_0 + \vec{\mu}_3. \tag{2}$$

It follows under the assumption of charge independence of nuclear forces that, for a particular set of states which comprise an isospin multiplet, the isoscalar parts are identical and the isovector parts are proportional to T_3. Thus, for a pair of mirror states,

$$\mu_0 = \tfrac{1}{2}[\vec{\mu}(T_3) + \vec{\mu}(-T_3)]. \tag{3}$$

The results of this work for the moments of the mass-37 isospin doublet yield the values $\mu_0 = 1.94(15)\mu_N$ and $\mu_3 = 3.27(15)\mu_N$. Within the jj-coupling model, the isoscalar part μ_0, for a pure j^n configuration with $J = j$, is simply the mean of the Schmidt values for a neutron and a proton in the j orbit,[10] $\mu_0(j = \frac{7}{2}) = 1.94\mu_N$. The experimentally determined value is in excellent agreement with that prediction.

In order to look for overall trends, a phenomenological analysis of the measured moments of nuclei with $J^\pi = \frac{7}{2}^-$ can be carried out. This is analogous to the analyses of Leonardi and Rosa-Clot[4] and Sugimoto[5] for the lower shells. Table I [11] lists the available experimentally determined magnetic moments for odd-A nuclei with $J^\pi = \frac{7}{2}^-$, along with the moments of the conjugate nuclei calculated from Eq. (3) with $\mu_0 = 1.94\mu_N$. A plot of these data versus A appears in Fig. 3. As the isotone series are merely reflections through μ_0 of the corresponding isotope series, only the situation for the latter will be discussed.

It is obvious from Fig. 3 that the experimental values are lower than the Schmidt values and that the isotope series exhibits a quenching towards the middle of the shell. A better overall agreement (even within the single-particle model)

TABLE I. Experimentally determined magnetic moments for $\frac{7}{2}^-$ states of odd-A nuclei together with the moments of the conjugate nuclei calculated from Eq. (3).

Nucleus	$\mu_{exp.}$	$\mu_{calc.}$	Nucleus	$\mu_{exp.}$	$\mu_{calc.}$	T
^{37}K	5.2(3)		^{37}Ar	-1.33(5)		1/2
^{41}K	4.41(5)[a]		^{41}Ti		-0.53	3/2
^{39}Sc		5.18	^{39}Ar	-1.3(3)		3/2
^{41}Sc		5.47	^{41}Ca	-1.59		1/2
^{43}Sc	4.62		^{43}Ti		-0.76	1/2
^{45}Sc	4.76		^{45}Cr		-0.87	3/2
^{47}Sc	5.34		^{47}Fe		-1.46	5/2
^{43}V		5.21	^{43}Ca	-1.32		3/2
^{45}V		3.97	^{45}Ti	-0.095[b]		1/2
^{49}V	4.46(5)		^{49}Fe		-0.58	3/2
^{51}V	5.15		^{51}Ni		-1.27	5/2
^{49}Co			^{49}Ti	-1.10		5/2
^{51}Co			^{51}Cr	-0.94[b]		3/2
^{55}Co	4.3(3)[c]		^{55}Ni		-0. (3)	1/2

[a]See Ref. 12.
[b]Sign undetermined—assumed negative.
[c]Sign undetermined—assumed positive.

might be obtained by using effective single-particle moments. However, the use of effective single-particle values will not give any structure,

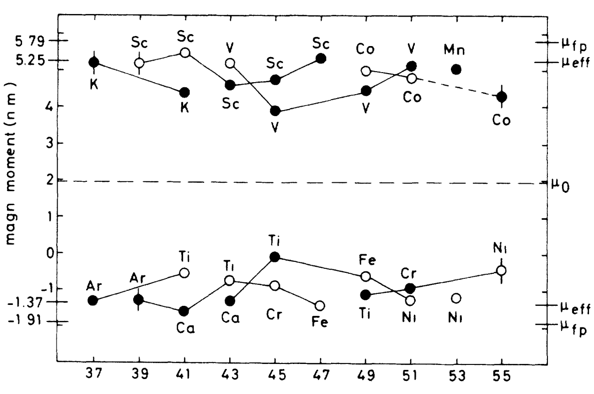

FIG. 3. Plot of the magnetic moments for $J^\pi=\frac{7}{2}^-$ states versus the mass number. The heavy lines are drawn to guide the eye. Circles represent experimentally measured moments and crosses represent moments calculated using Eq. (3). The line labeled μ_{fp} shows the Schmidt prediction using free-particle values for μ_p and μ_n and the line labeled μ_{eff} shows the Schmidt prediction using the effective values for μ_p and μ_n.

and therefore more detailed model assumptions are necessary in order to reproduce the apparent regularities.

One model approach that (even within the restriction of pure j^n configurations) qualitatively fits the observed structure is that of Bayman, McCullen, and Zamick.[13] The basic assumption is that of configuration mixing of $f_{7/2}$ particles outside an inert core and an effective two-body interaction derived from the level structure of ^{42}Sc. In order to improve the agreement of the magnetic moments calculated in this way with the experimental values, the authors used effective single-nucleon g factors that they determined by a least-squares analysis of the measured moments. From their results, $g_p = +1.50$ and $g_n = -0.39$, we deduce $\mu_{0,\text{eff}} = +1.95\mu_N$, equal to the value deduced from the Schmidt values and from the experimental values from this paper.

In addition, the effective single-particle moments of the $f_{7/2}$ neutron and proton are in excellent agreement with the experimental values for the magnetic moments of the $\frac{7}{2}^-$ states of ^{37}Ar and ^{37}K, respectively. This leads to the conclusion that the experimental results are consistent with a description of the $\frac{7}{2}^-$ states as single-particle states relative to a closed ^{36}Ar core that, so far as the influence on the quenching of the single-particle magnetic moments of the $f_{7/2}$ nucleons is concerned, behaves like the ^{40}Ca core.

A striking feature of the calculations of Bayman, McCullen, and Zamick[13] is the fact that the systematic trend for the magnetic moments of the Sc isotopes is well reproduced. The analysis of the calculated wave functions shows that the quenching is related to the fraction of configurations with seniority $\nu = 3$. This leads to the conclusion that the observed structure for all isotope series, at least qualitatively, may be due to different admixtures of higher seniorities.

This phenomenological analysis is an attempt to understand the systematic trends observed in the data consistent with as few assumptions as possible concerning model dependence. It is encouraging that the reasonably straightforward model of inert core plus configuration mixing of extra-core nucleons within one shell provides a qualitative explanation of these trends.

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Shell Correction in the Independent-Particle Model*

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We formulate a method for obtaining a semiclassical expression for the single-particle density of states from the partition-function approach. In some simple solvable models, the semi-intuitive Strutinskii prescription is demonstrated to yield essentially identical results for shell correction and the proposed semiclassical procedure. This gives us greater confidence in the Strutinskii method.

The binding energy of nuclei as a function of mass number A and proton number Z displays a smooth behavior which is well reproduced by the liquid-drop (LD) mass formula.[1,2] However, there remains some small systematic structure superimposed on this smooth trend which has an important bearing on the stability of heavy and superheavy nuclei. This structure is due to the grouping of the single-particle levels into shells, an effect not included in the LD model which is based on the average statistical distribution of these levels. These deviations from the smooth trend are termed shell corrections to the LD mass formula, and various theoretical methods[3,4] have been proposed for calculating them. In particular, Strutinskii[4] has formulated a prescrip-