Can an Electromagnetic Violation of Time-Reversal Invariance be Observed in Low-Energy Nuclear Processes?

C. F. Clement
Theoretical Physics Division, Atomic Energy Research Establishment, Harwell, Didcot, Berkshire, England

and

Leon Heller*
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87544
(Received 28 April 1971)

Starting from a T-odd Lagrangian involving the nucleon, meson, and electromagnetic fields interacting at a point, we calculate the one-meson–exchange diagram and obtain the longest-range part of a two-nucleon electromagnetic operator. Even if this interaction were present with "maximal" strength, we would not expect T-invariance violation to have been seen in any of the existing $E2$-$M1$ interference experiments including ^{36}Cl which has the smallest gap between model and experiment.

Following the suggestion that the electromagnetic interaction of hadrons may not be invariant under time reversal,[1] several experiments have been performed to look for this effect. The results of these experiments are summarized in a fine review by Henley.[2] Although two medium-energy reciprocity experiments[3,4] involving a photon appear to show a violation of time-reversal invariance, none of the low-energy nuclear-physics experiments do. We shall be concerned primarily with γ-ray transitions in nuclei and will show that the present experimental upper limit[2] of approximately 3×10^{-3} on the T-invariance–violating relative phase in $E(2)$-$M(1)$ interference is larger than what would be expected even if the T-invariance–violating interaction were "maximal."

When exploring the question of where electromagnetic T-invariance violation can be introduced theoretically, there is an obstacle coming from the fact[5] that the matrix elements of the electromagnetic current between any pair of states which contain just one physical nucleon each cannot be odd under time reversal if they are even under space inversion. One possible place to introduce T-invariance violation is at an $NN\gamma$ vertex where one of the nucleons is off shell, and this has been studied by Huffman[6] who used it to calculate a T-invariance–violating nucleon-nucleon potential generated by the exchange of one photon and one π meson. An estimate was made of the amount of reciprocity violation to be expected in direct nuclear reactions,[7] and this turned out to be somewhat smaller than the experimental upper limits.[8]

Another possible place to introduce T-invariance violation is at an $NN^*\gamma$ vertex, and this has been used[9] to estimate reciprocity violation in $\gamma+d\rightarrow n+p$. The experimental situation is described in Ref. 4.

We introduce T-invariance violation in a manner which appears to have a better chance of showing up in low-energy nuclear physics, namely, at a four-particle vertex, $NNM\gamma$ with M a meson, as shown in Fig. 1(a). This will be done via an explicit term in the Lagrangian which

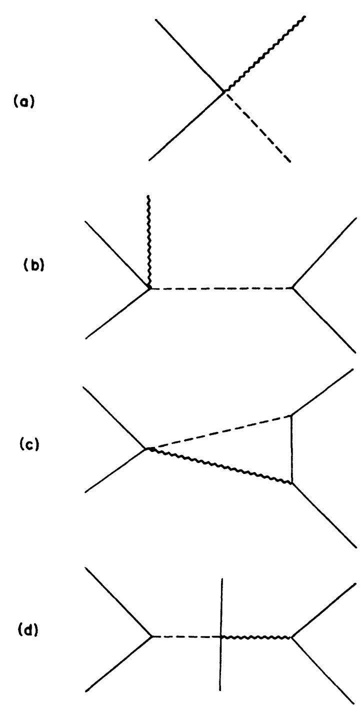

FIG. 1. (a) The basic T-invariance–violating interaction, (b) the two-body electromagnetic operator, (c) contribution to a T-invariance–violating nucleon-nucleon potential, (d) contribution to a three-nucleon T-invariance–violating potential.

—except for the coupling constant—is unique for the π meson provided the number of derivatives is kept to the minimum. By this procedure we avoid the high-energy requirement associated with the N^*, and the off-shell factors of (pion mass/nucleon mass)^2 in Huffman's method.[6] In fact the initial and final nucleon can be *on shell* and still give rise to T-invariance violation. There will also be a direct T-invariance-violating contribution to the photomeson production process.[10]

In nuclear processes at energies below meson production, the meson is virtual and attached to a second nucleon as in Fig. 1(b). There are still two distinct possibilities according to whether the photon is (A) real or (B) virtual. In the former case we insert the radiation field into Fig. 1(b) and obtain a *two-body* electromagnetic operator. In the latter case we could, as in Fig. 1(c), attach the photon to one of the same nucleons as the meson, thereby obtaining a T-invariance-violating nucleon-nucleon potential. This procedure would be similar to that of Ref. 6 except that no virtual nucleon is involved (on the T-invariance-violation side).

There is the additional possibility of attaching the virtual photon to a third nucleon as in Fig. 1(d) and taking advantage of the long range of the Coulomb potential by summing over all the third nucleons. Thus we expect the dominant T-invariance-violating potential to be *three-body* in nature. The summation over the third nucleon reduces it to an effective two-body potential with coordinates referred to the center of mass of the other charged nucleons. Again we expect this result to be very general.[11]

Initially one might expect larger T-invariance-violating effects in electromagnetic (EM) transitions than in nuclear wave functions. This is because the extra photon vertex in a nuclear potential diagram introduces the extra small factor $e/\hbar c$ in addition to the small T-invariance-violating vertex in both diagrams. However, our summation over Z third nucleons mostly removes this disadvantage in heavy nuclei by replacing e by Ze. On the other hand, the forms of our results appear to show that we can expect little or no coherence when we take matrix elements of the sum over pairs of nucleons. We should therefore look for relatively large violations where the normal nuclear processes are greatly inhibited. This is difficult to do with nuclear reactions, and the optimum place to look may be at EM transitions which are inhibited owing to the smallness of the matrix element of the normal one-body operator.

Before proceeding to the details, it is necessary to discuss the isospin properties of that part of the interaction which is odd under T. For an interaction $NNM\gamma$, with M a scalar or pseudoscalar meson, it is necessary to make the Lagrangian the third component of an isovector if T invariance is to be violated (see below). For a vector meson the T-invariance-violating interaction can be isoscalar or isotensor. Two processes limit the size of isovector *matrix elements* which violate T invariance (or C invariance), one from the failure to observe[12,1] $\eta^0 \to \pi^0 e^+ e^-$, and the other[3] from $\pi^- + p \to \gamma + n$. To decide just what these limits imply for the size of the *coupling constant* in a particular T-invariance-violating isovector interaction requires good models for the processes and detailed calculations which have not yet been performed. Since this isospin question is not clear-cut we have not rejected any possibilities and now show the results for a pseudoscalar meson. Detailed formulas for all three meson types will be given elsewhere. The Dirac matrices of Bjorken and Drell[13] are used and we take units with $\hbar = c = 1$.

Since the pseudoscalar isovector meson field is taken to interact strongly with the nucleon field through the pseudoscalar coupling

$$\mathcal{L}_1(x) = G\bar{\Psi}(x) i\gamma^5 \vec{\tau}\cdot\vec{\varphi}(x)\Psi(x), \tag{1}$$

time-reversal invariance of $\mathcal{L}_1$ requires that $\varphi_1 \pm i\varphi_2$ and φ_3 change sign under the operation. The same is true for the equivalent pseudovector coupling. A T-odd interaction involving the nucleon, meson, and electromagnetic fields which is Lorentz and gauge invariant and conserves parity and electric charge has a unique (quadrilinear) form if the number of derivatives is kept to a minimum:

$$\mathcal{L}_2(x) = \lambda m_0^{-2}\bar{\Psi}(x)[\vec{\tau}\times\vec{\varphi}(x)]_3 i\gamma^5\sigma^{\mu\nu}\Psi(x)F_{\mu\nu}(x), \tag{2}$$

where the mass m_0 has been introduced to make the coupling constant λ dimensionless and γ^5 is Hermitean. The isospin factor is odd under charge symmetry as well as T. The equation of motion for the nucleon field has the form of a Dirac equation $H(x)\Psi(x) = i\partial\Psi(x)/\partial t$ with the extra term $H'(x)$ coming from $\mathcal{L}_2(x)$,

$$H'(x) = -\lambda m_0^{-2}[\vec{\tau}\times\vec{\varphi}(x)]_3\, i\gamma^0\gamma^5\sigma^{\mu\nu}F_{\mu\nu}(x). \tag{3}$$

In the full field-theory Hamiltonian density the term *of order* λ is precisely expression (3) sandwiched between $\Psi^+(x)$ and $\Psi(x)$.[14]

Evaluating Fig. 1(b) gives the longest-range part of the two-body electromagnetic operator. In the nonrelativistic limit in coordinate space this becomes

$$V(\vec{r}_1, \vec{r}_2) = \frac{\lambda F}{2\pi m_0^2\mu}(\vec{\tau}_1\times\vec{\tau}_2)_3\{[\vec{\sigma}_1\cdot\vec{E}(\vec{r}_1)](\vec{\sigma}_2\cdot\nabla_1) - [\vec{\sigma}_2\cdot\vec{E}(\vec{r}_2)](\vec{\sigma}_1\cdot\nabla_2)\}\frac{e^{-\mu r}}{r}, \tag{4}$$

where we have included the contribution with vertices 1 and 2 interchanged. The pseudovector coupling constant F and meson mass μ have been used and the gradient operators act only on the function shown, not on the wave function. $\vec{E}$ is the electric field vector, and $r \equiv |\vec{r}_1 - \vec{r}_2|$.

We first consider the radiation of a real photon and obtain the EM transition operator by taking $\vec{E}$ in Eq. (4) to be the radiation field, and perform the multipole expansion in the long-wavelength limit. Also because of the short range in r of the operator, we keep only the leading term in an expansion in r/R_{12}, where $\vec{R}_{12} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2)$. Details will be given elsewhere. We give here explicit expressions for the lowest-multipole operators. For the pseudoscalar-meson case which we are considering, the ratio of successive multipoles for the T-invariance-violating operators is the same as for the normal operators. We have

$$Q_{1M}'(E1) = \frac{1}{3\pi}\frac{\lambda}{m_0^2}\frac{F}{\mu}\sum_{i<j}(\vec{\tau}_i\times\vec{\tau}_j)_3\frac{d}{dr_{ij}}\left(\frac{\exp(-\mu r_{ij})}{r_{ij}}\right)\{\vec{\sigma}_i\cdot\vec{\sigma}_j Y_{1M}^*(\vec{r}_{ij}) + (\tfrac{5}{3})^{1/2}[\vec{Y}_1(\vec{r}_{ij})\times[\vec{\sigma}_i\times\vec{\sigma}_j]^2]_M^{1*}\}, \tag{5}$$

$$Q_{1M}'(M1) = -(\tfrac{2}{3}\pi)^{1/2}\frac{\lambda}{2\pi m_0^2}\frac{F}{\mu}K\sum_{i<j}(\vec{\tau}_i\times\vec{\tau}_j)_3\frac{d}{dr_{ij}}\left(\frac{\exp(-\mu r_{ij})}{r_{ij}}\right)\Big\{\tfrac{2}{3}R_{ij}\vec{\sigma}_i\cdot\vec{\sigma}_j[\vec{Y}_1(\vec{R}_{ij})\times\vec{Y}_1(\vec{r}_{ij})]_M^{1*}$$
$$-\frac{1}{4\pi}r_{ij}[\vec{\sigma}_i\times\vec{\sigma}_j]_M^{1*} + 2R_{ij}\sum_{L'}[5(2L'+1)]^{1/2}\begin{Bmatrix}L' & 2 & 1\\ 1 & 1 & 1\end{Bmatrix}[[\vec{Y}_1(\vec{R}_{ij})\times\vec{Y}_1(\vec{r}_{ij})]^{L'}\times[\vec{\sigma}_i\times\vec{\sigma}_j]^2]_M^{1*}\Big\}, \tag{6}$$

where the square brackets[15] denote vector coupling, a 6-j symbol is used, and K is the photon energy.

These operators are to be compared with the normal operators

$$Q_{1M}(E1) = \textstyle\sum_i e_i r_i Y_{1M}^*(\vec{r}_i),$$

$$Q_{1M}(M1) = \left(\frac{3}{4\pi}\right)^{1/2}\frac{1}{2M}[e_i\vec{l}_i + \mu_i\vec{\sigma}_i]_M^{1*}.$$

The T-invariance-violating operators only affect neutron-proton pairs and can initiate transitions with $\Delta S = 2$. The $M1/E1$ ratio is of the order KR, where R is some typical nuclear size, whereas the normal ratio is $(MR)^{-1}$. Other things being equal the maximum violation occurs in high-energy strongly inhibited $M1$ transitions, whereas for low-energy transitions $E1$ is favored, the crossover point being about 1 MeV for light nuclei.

For a pair of particles, Q' gives the strongest transition between a relative S and a relative P state, and an estimate of the space matrix element is obtained by integrating $(d/dr)[r^{-1}\exp(-\mu r)]$ between such states normalized over the nuclear volume. This introduces a factor $\mu^{-2}R^{-4}$, where R is the nuclear radius. To obtain a numerical estimate of the magnitude of the ratio of a T-odd two-nucleon matrix element to the T-even single-nucleon matrix element requires a statement about the magnitude of the coupling constant λ/m_0^2. We take $\lambda = e$ and m_0^2 equal to the product of the meson mass and nucleon mass.[16] At best this must be considered a rough guess which gives for the ratio of corresponding electric matrix elements

$$(\lambda F/m_0^2\pi\mu^3R^4)/eR \approx 0.3/R^5, \tag{7}$$

where $\mu = 140$ MeV, $F^2/4\pi = 0.08$, and R is in femtometers. For magnetic transitions the ratio is

$$(\lambda FK/m_0^2\pi\mu^3R^3)(e/M)^{-1} \approx 6\times10^{-3}E_\gamma/R^3, \tag{8}$$

where E_γ is the transition energy in MeV. The corresponding ratios for the vector mesons are much smaller.

From the estimates in Eqs. (7) and (8), we find that the 7.79-MeV γ ray in ^{36}Cl has the *largest* expected effect of all the transitions which have been measured,[2] the $E2$ ratio being 3×10^{-4} and the $M1$ being 8×10^{-4}. Furthermore the experimental[17] upper limit on the T-invariance-violating $E2$-$M1$ relative phase in ^{36}Cl is the *smallest* of all those which have been measured, being

$(0.8\pm 2.3)\times 10^{-3}$. Even in this case it is seen that *the normal matrix element would have to be suppressed or the T-invariance–violating matrix element enhanced compared with our estimates, for T-invariance violation to have been observable at the accuracy of the experiment.*[17] Anyone proposing to do another experiment of this type should put the appropriate radius and γ energy into Eqs. (7) and (8) and compare those estimates with the expected accuracy to see if the results are likely to be more significant than for ^{36}Cl.

We would like to thank Dr. A. M. Lane and Dr. J. K. Perring for useful discussions and Dr. N. W. Tanner for stimulating our interest in the subject. One of us (L.H.) wants to thank Dr. W. M. Visscher, Dr. M. Bolsterli, Dr. E. Lomon, and Dr. F. E. Low for helpful conversations and Dr. R. R. Silbar for valuable suggestions regarding the manuscript, and to express his appreciation to Dr. Lane and Dr. A. B. Lidiard for the warm hospitality of the Theoretical Physics Division at Harwell during 1969-1970.

*Work performed while a visitor at Harwell, and supported by the U. S. Atomic Energy Commission.

[1]J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. 139, B1650 (1965).

[2]E. M. Henley, Annu. Rev. Nucl. Sci. 19, 367 (1969).

[3]For the reaction $\pi^- + p \to \gamma + n$ and the comparison with its inverse, see P. A. Berardo, R. P. Haddock, B. M. K. Nefkens, L. J. Verhey, M. E. Zeller, A. S. L. Parsons, and P. Truoel, Phys. Rev. Lett. 26, 201, 205 (1971), and references therein.

[4]For the reaction $n + p \to d + \gamma$ and the comparison with its inverse, see D. F. Bartlett, C. E. Friedberg, K. Goulianos, I. S. Hammerman, and D. P. Hutchinson, Phys. Rev. Lett. 23, 893, 1205(E) (1969), and references therein; also B. L. Schrock, R. P. Haddock, J. A. Helland, M. J. Longo, K. K. Young, S. S. Wilson, and D. Cheng, Bull. Amer. Phys. Soc. 16, 635 (1971), who find no evidence for T-invariance violation.

[5]G. Salzman, Phys. Rev. 99, 973 (1955), Ref. 1.

[6]A. H. Huffman, Phys. Rev. D 1, 882 (1970).

[7]A. H. Huffman, Phys. Rev. D 1, 890 (1970).

[8]These are summarized in Ref. 2.

[9]S. Barshay, Phys. Rev. Lett. 17, 49 (1966).

[10]There is a phenomenological discussion of this process in N. Christ and T. D. Lee, Phys. Rev. 148, 1520 (1966). The experimental situation is described in Ref. 3.

[11]The situation is, in fact, more complicated. We have not evaluated the contribution of Fig. 1(c), but if the electric field is the important quantity, a rough estimate of the ratio of the contributions of Fig. 1(c) to 1(d) might be $(er^{-2})/(ZeR^{-2})$, where R is the size of the nucleus and r is the two-nucleon meson potential range. This ratio favors the three-body force by a small factor, particularly on heavy nuclei. Figure 1(d) will certainly provide the long-range part of the potential.

[12]M. Roos *et al.*, Phys. Lett. 33B, 1 (1970).

[13]J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[14]The electric vector differs from the momentum canonically conjugate to $\vec{A}(x)$ by a term of order λ.

[15]This is the notation of A. de-Shalit and I. Talmi, *Nuclear Shell Theory* (Academic, New York, 1963).

[16]This can be considered to be "maximal" T-invariance violation.

[17]J. Eichler, Nucl. Phys. A120, 535 (1968).

Some Remarks on the $(\underline{3}, \underline{3}^*) \oplus (\underline{3}^*, \underline{3})$ Breaking of Chiral Symmetry*

Riazuddin† and S. Oneda
Center for Theoretical Physics, Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 7 May 1971)

We calculate the $\eta' \to \eta\pi\pi$ decay rate, assuming that SU(3)⊗SU(3) symmetry is of the Goldstone-Nambu type and that the symmetry-breaking Hamiltonian belongs to the $(\underline{3}, \underline{3}^*) \oplus (\underline{3}^*, \underline{3})$ representation of SU(3)⊗SU(3). It turns out that the rate is anomalously small and indicates that the pure $(\underline{3}, \underline{3}^*) \oplus (\underline{3}^*, \underline{3})$ symmetry breaking is unlikely.

It appears reasonable to regard the SU(3) ⊗SU(3) symmetry of strong interactions to be of the Nambu-Goldstone type.[1,2] That is, one adopts[2] a picture of strong interactions in which the strong-interaction Hamiltonian is written as

$$H = H_0 + \epsilon H' = \int H_0(\vec{x}, 0)d^3x + \epsilon \int H'(\vec{x}, 0)d^3x, \qquad (1)$$

where H_0 is invariant under SU(3)⊗SU(3) symmetry and $\epsilon H'$ breaks the symmetry and takes care of corrections not only to the hypothesis of partial conservation of axial-vector current (PCAC) and soft-pion theorems but also to SU(3) itself. In the limit $\epsilon \to 0$, the vacuum is taken to be an SU(3) singlet but not an SU(3)⊗SU(3) singlet, and the symmetry manifests itself through SU(3) multiplets and an octet of massless pseudoscalar mesons. The pseudoscalar mesons ac-