contribute considerably to the next $\frac{1}{2}^-$ (E_x=1.4 MeV) and $\frac{5}{2}^-$ (E_x=1.53 MeV) states. In the last column, the sum of the absolute squares of the coefficients is given; this number should be unity if the basis is sufficient. The values obtained are systematically larger than 1 for the lower $\frac{3}{2}^-$, $\frac{1}{2}^-$, $\frac{5}{2}^-$, and $\frac{7}{2}^-$ states and smaller than 1 for the higher $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states. The deviations of the lower states are assumed to be due to an underestimation of the single-particle widths.

This is the first time that an extensive analysis of inelastic scattering of polarized protons to excited collective states has been performed in the region of IAR's. The description of the direct background scattering should be refined by DWBA calculations and by extending the experiments to more scattering angles. However, the results reported here indicate that important nuclear structure information will be obtained from inelastic scattering of polarized protons.

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Electromagnetic Test Fields Around a Kerr-Metric Black Hole*

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Weak electromagnetic perturbations (test fields) in the exterior of a Kerr-metric black hole are studied. It is shown that (i) the only physically acceptable, time-independent perturbation is an axisymmetric field which corresponds to adding charge to the source and (ii) all axisymmetric normal modes are stable which, assuming completeness, guarantees stability for arbitrary axisymmetric perturbations. The proof of the nonexistence of time-independent, nonaxisymmetric test fields actually holds for perturbations associated with any physical (e.g., gravitational) field.

Present theoretical evidence within general relativity suggests strongly that when a rotating star collapses completely it leaves behind a black hole whose exterior geometry is the Kerr metric.[1-4] If so, the study of electromagnetic (em) perturbations of the Kerr metric relates directly to the history of the em field of a star which undergoes gravitational collapse and to the em information about the collapse which would be received by a distant observer. Also, the study of em perturbations might point out a path to be followed in attacking the more difficult problem of gravitational perturbations.

In this Letter we prove two theorems concerning physically acceptable (specific definition given below) weak em perturbations[5] of the exterior of a Kerr black hole:

First, the only time-independent em perturbation is axisymmetric and corresponds to the addition of charge to the black hole. An immediate corollary is that when a star collapses the higher multipoles of its (weak) em field must be attenuated.[6] The proof of the nonexistence of time-independent, nonaxisymmetric perturbations is very general and is valid for perturbations associated with any physical field. The proof thus extends Carter's theorem[4] that a Kerr black hole has no time-independent, axisymmetric, gravitational perturbations besides those which correspond to passage along the Kerr sequence, i.e., to changing the mass and angular momentum of the black hole.

Second, all axisymmetric normal modes (axisymmetric modes with real squared frequency and with physically acceptable behavior at the event horizon and at infinity) of the em field in the Kerr geometry are oscillatory in time; no modes grow exponentially. Consequently, under the usual assumption that these modes form a complete set, all axisymmetric em perturbations are stable.

In sketching the proofs, we make use of the wave equation derived recently, which governs weak em fields in the Kerr geometry.[7] In Boyer-Lindquist[8] coordinates (t, r, θ, φ), the wave equation reads

$$0=\left(1+\frac{a^2}{r^2}\cos^2\theta\right)\Box\Omega_1+\frac{2M(r+ia\cos\theta)}{r^2(r-ia\cos\theta)^2}\Omega_1$$
$$=\frac{1}{r^2}\frac{\partial}{\partial r}\left[(r^2-2Mr+a^2)\frac{\partial\Omega_1}{\partial r}\right]+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Omega_1}{\partial\theta}\right)+\frac{1}{r^2}\left(\frac{1}{\sin^2\theta}-\frac{a^2}{r^2-2Mr+a^2}\right)\frac{\partial^2\Omega_1}{\partial\varphi^2}$$
$$-\frac{4aM}{r(r^2-2Mr+a^2)}\frac{\partial^2\Omega_1}{\partial\varphi\partial t}-\frac{1}{r^2}\left[\frac{(r^2+a^2)^2}{r^2-2Mr+a^2}-a^2\sin^2\theta\right]\frac{\partial^2\Omega_1}{\partial t^2}+\frac{2M(r+ia\cos\theta)}{r^2(r-ia\cos\theta)^2}\Omega_1. \tag{1}$$

Here $\Box$ is the d'Alembertian; M is the mass of the black hole; $a\leqslant M$ is the angular momentum per unit mass; and Ω_1 is a certain null-tetrad component of the em field tensor $F_{\mu\nu}$, namely,

$$\Omega_1=\tfrac{1}{2}(r-ia\cos\theta)(r^2+a^2\cos^2\theta)^{-1}[(r^2+a^2)F_{tr}-(i/\sin\theta)F_{\theta\varphi}-aF_{r\varphi}+iaF_{t\theta}\sin\theta]. \tag{2}$$

A physically acceptable Ω_1 must satisfy certain boundary conditions at the event horizon, $r=r_+\equiv M+(M^2-a^2)^{1/2}$, and at infinity. For large r Eq. (2) yields

$$\Omega_1\sim-\tfrac{1}{2}r(E^r+iB^r)\rightarrow\Omega_1\leqslant O(r^{-1}).$$

Near $r=r_+$ it is convenient to focus attention first upon an axisymmetric perturbation with time dependence $e^{i\omega t}$; nonaxisymmetric perturbations will be considered subsequently. For an axisymmetric mode with $\omega^2\leqslant 0$, Eq. (1) implies that near $r=r_+$

$$\Omega_1\sim Ae^{\alpha r^*}+Be^{-\alpha r^*}\ \text{if}\ \omega^2<0,$$
$$\sim Cr^*+D\ \text{if}\ \omega^2=0.$$

Here $A,\cdots,D$ are functions of θ and t, $\alpha\equiv(1+a^2/r_+^2)|\omega|$, and $dr^*\equiv dr/(1-2Mr^{-1}+a^2r^{-2})$ so that $r^*\rightarrow-\infty$ as $r\rightarrow r_+$. For a physically acceptable perturbation $B=C=0$ since an in-falling charge would experience an infinite em force at $r=r_+$ if Ω_1 blows up there.

It is useful to factor out the solution $\Omega_1=(r-ia\cos\theta)^{-1}$ which corresponds to adding charge to the black hole.[9] In terms of the quantity

$$\Phi_1\equiv-(r-ia\cos\theta)\Omega_1, \tag{3}$$

Eq. (1) becomes, for an axisymmetric mode with $e^{i\omega t}$ time dependence,

$$0=\frac{1}{r^2}\frac{\partial}{\partial r}\left[\frac{r^2-2Mr+a^2}{(r-ia\cos\theta)^2}\frac{\partial\Phi_1}{\partial r}\right]+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\frac{\sin\theta}{(r-ia\cos\theta)^2}\frac{\partial\Phi_1}{\partial\theta}\right]$$
$$+\omega^2\frac{1}{r^2(r-ia\cos\theta)^2}\left[\frac{(r^2+a^2)^2}{r^2-2Mr+a^2}-a^2\sin^2\theta\right]\Phi_1. \tag{4}$$

If we multiply this equation by the complex conjugate of Φ_1, integrate over the volume exterior to the black hole, and use the boundary conditions in performing an integration by parts, we thereby obtain for the real part of the result, when $\omega^2\leqslant 0$,

$$0=\int\sin\theta\,d\theta\,dr\frac{(r^2-a^2\cos^2\theta)}{(r^2+a^2\cos^2\theta)^2}\left\{(r^2-2Mr+a^2)\left|\frac{\partial\Phi_1}{\partial r}\right|^2+\left|\frac{\partial\Phi_1}{\partial\theta}\right|^2-\omega^2\left[\frac{(r^2+a^2)^2}{r^2-2Mr+a^2}-a^2\sin^2\theta\right]|\Phi_1|^2\right\}. \tag{5}$$

Each term is non-negative. Thus there is no acceptable solution with $\omega^2\leqslant 0$ besides the solution $\Phi_1=\text{const}$ (if $\omega=0$). This proves that no axisymmetric normal modes grow exponentially in time and that the only acceptable, time-independent axisymmetric perturbation is that which corresponds to charging the source.

The above method fails for nonaxisymmetric perturbations. Still, there is a simple way to see why there is no acceptable, time-independent, nonaxisymmetric perturbation. The key point is that within the region between the event horizon and the so-called "ergosphere"[1] [defined by $r=M+(M^2-a^2\cos^2\theta)^{1/2}$] an observer must have $d\varphi/dt>0$ (positive angular velocity as measured from infinity), or else his world line would be spacelike.[10] Hence a perturbation independent of the time coordinate t (static in time as observed from outside the ergosphere), and with azimuthal-angle dependence $e^{im\varphi}$, must actually be changing in time according to any observer within the ergosphere! Near the event horizon such a perturbation has the behavior

$$\Omega_1 \propto \exp[\pm i(am/r_+^2)r^*] \text{ near } r=r_+.$$

The plus and minus signs refer to ingoing and outgoing em radiation, respectively, at the horizon. Since the black hole is assumed to have formed in the remote past, the radiation must be ingoing at the horizon. Consequently, as measured locally, em radiation, and hence energy, is continually pouring into the black hole. Its source is the energy gained by photons falling in the gravitational field in the dynamical region. This continual influx of energy must eventually cause the black hole to change by an arbitrarily large amount—a contradiction of our basic assumption that the perturbations are weak. Thus we can rule out the existence of "time-independent," nonaxisymmetric em perturbations.

Notice that the above proof is of sufficient generality that with only minor modifications it can be used to prove the nonexistence of time-independent, nonaxisymmetric perturbations associated with any physical field, including gravity itself. The idea is simply that a nonaxisymmetric perturbation independent of the time coordinate t would actually be dynamical inside the ergosphere. Associated with this would be a continual influx of radiation into the horizon, which would make the black hole change in time by a non-negligible amount.

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