tendency. A careful study of the magnetic moments of nuclei with $j = l - \frac{1}{2}$ could determine which effect is dominant in the f-p shell region.

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Investigation of 2^+ Core Excitation in ^{141}Ce by Inelastic Scattering of Polarized Protons

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Inelastic scattering of polarized protons from ^{140}Ce to the 2_1^+ ($E_x = 1.596$ MeV) state near isobaric analog resonances shows the importance of direct, off-resonant inelastic scattering, and provides enough information to determine parentage coefficients for six p and f states in ^{141}Ce up to an excitation energy of 1.88 MeV.

This Letter is concerned with inelastic scattering of polarized protons near isobaric analog resonances (IAR's). The observed resonance structure of the analyzing power for the reaction $^{140}\mathrm{Ce}(p,p_1)^{140}\mathrm{Ce}^*(2_1^+,\ E_x = 1.596\ \mathrm{MeV})$ shows the importance of direct background scattering, a feature which has been neglected in earlier work.[1,2] The present analysis gives detailed information on the contribution of the 2^+ core excitation to states in ^{141}Ce.

To include core excitation into the shell-model description of a nucleus with even proton number Z and odd neutron number N, the wave function ψ^λ is assumed to be a linear combination (parentage expansion) of single-particle neutron shell-model states $n(j)$ coupled to the ground state $C_0(0^+)$ and to the various excited states $C_i(I)$ of the $(N-1, Z)$ nucleus[3]:

$$\psi^\lambda(J^\pi) = \sum_{i,j} a_{ij}{}^\lambda [n(j) \otimes C_i(I)]_{J\pi}, \qquad (1)$$

where $a_{0j}{}^\lambda$ is the square root of the spectroscopic factor. Excited core states with spin I different from zero may couple with a number of neutron wave functions of different total angular momentum j, restricted by the conditions $\vec{J} = \vec{j} + \vec{I}$ and $\pi(\psi) = \pi(n)\pi(C)$. This expansion is particularly suitable for nuclear spectroscopy with IAR's because the C_i represent target nucleus states, and resonant proton-scattering channels c are observed with partial-width amplitudes proportional to the parentage coefficients $a_{ij}{}^\lambda$.

From the analysis of inelastic scattering the contributions of excited 0^+ core states have been investigated in the $N=50$ region.[4,5] The 2^+ excitation is more complicated since each IAR may have up to five inelastic exit channels. To this problem Abramson *et al.*[6] applied $(p,p'\gamma)$ correlation techniques. They approximated direct off-resonant scattering by Coulomb scattering and obtained the expansion of the lowest d states in the Cd isotopes. The lowest two states in $N=83$ nuclei have been investigated by Zaidi *et al.*[1] and Hiddleston and Riley.[2] They neglected direct background scattering, and analyzed the large-angle inelastic cross section using only $f_{7/2}$ and $p_{3/2}$ neutron waves.

This Letter is concerned with inelastic scattering of polarized protons—again for the investigation of the 2^+ core excitation in $N=83$ states in ^{141}Ce. The pronounced analyzing power, observed for all IAR's, clearly demonstrates the importance of direct inelastic background scattering, since an isolated resonance does not contribute to the analyzing power, if the direct background vanishes.[7] We used the target nucleus ^{140}Ce since Beregi and Lovas[8] had performed nuclear structure calculations for the $N=83$ system, and since the first excited 2^+ state of ^{140}Ce is collective enough to obtain sufficient direct inelastic scattering and hence pronounced effects in analyzing power. The measurements have been performed with the Erlangen polarized-proton beam.[9] We used isotopically enriched targets of ^{140}Ce$_2$O$_3$ (0.6 mg/cm^2) evaporated on thin carbon foils. The scattered protons were detected at the angles 40°, 60°, 120°, 140°, and 160° with symmetric pairs of counters. Elastic-scattering results are reported elsewhere.[10]

Because of the oxygen contamination, good inelastic spectra have been obtained only at scattering angles 140° and 160°. Figures 1(a) and 1(b) show the polarization-independent $(d\sigma/d\Omega)$ and the polarization-dependent $(Ad\sigma/d\Omega)$ parts of the cross section in the energy region $E_p=10.2$ to 12.4 MeV covering the region of IAR's from the first excited state $(\frac{3}{2}^-$, $E_x=0.66$ MeV) up to $E_x=2.5$ MeV.

The curves in these figures result from a fitting procedure described further on. The solid curve is the result of a best fit, including six resonances, obtained from elastic-scattering analysis. The dash-dotted curve is calculated with the same set of resonance parameters, but neglecting direct background scattering. The dashed curve indicates an additional incoherent cross section. Obviously the direct background scattering is important; in the differential cross section it produces an asymmetric behavior of the tails of the resonances, and shifts the peak of the resonance pattern. In the analyzing power the resonance effect for the more isolated resonances is nearly completely due to the interference with the direct background and not due to the tails

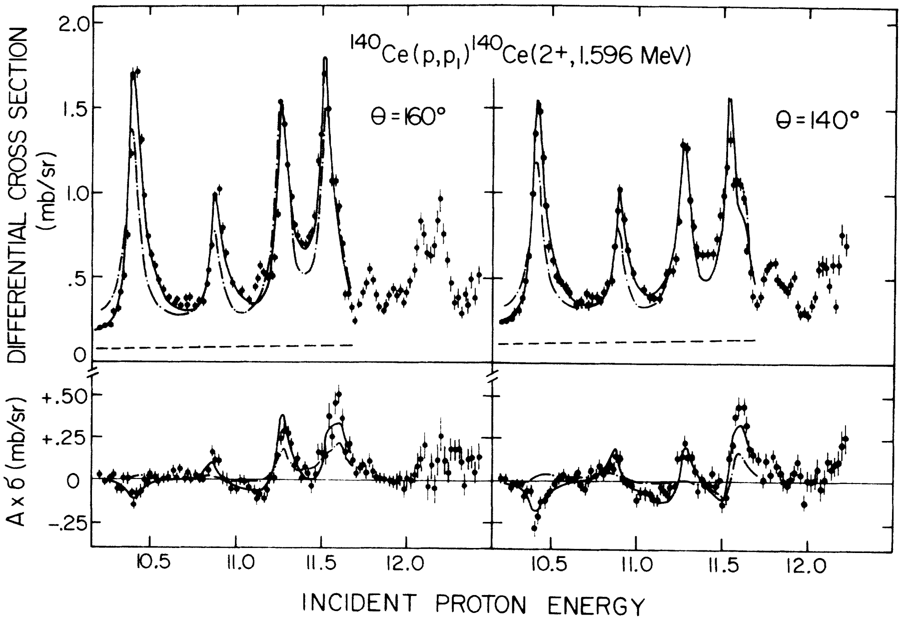

FIG. 1. $d\sigma/d\Omega$ and $Ad\sigma/d\Omega$ for $\theta=140°$ and 160°. The solid curve includes six resonances interfering with direct background, the dashed line indicates an additive incoherent cross section. The dash-dotted curve is calculated without direct background.

of nearby resonances. The latter become important in the case of overlapping resonances ($\frac{7}{2}^-$ at $E_x = 1.75$ MeV, $\frac{3}{2}^-$ at $E_x = 1.86$ MeV, and $\frac{5}{2}^-$ at $E_x = 1.88$ MeV).

The fit has been obtained by a search program[11] using the formulas of Simon and Welton[12] for cross section and analyzing power, and an energy-averaged scattering matrix describing direct background and resonance scattering due to the IAR's,[13]

$$S_{cc'} = \exp[i(\delta_c + \delta_{c'} + \omega_c + \omega_{c'})]\left[u_{cc'} \exp[-(\eta_c + \eta_{c'})] + \frac{i \exp[i(\varphi_c{}^\lambda + \varphi_{c'}{}^\lambda)](\Gamma_c{}^\lambda \Gamma_{c'}{}^\lambda)^{1/2}}{E_\lambda - E - \frac{1}{2} i \Gamma_\lambda}\right]. \tag{2}$$

Coulomb and nuclear phase shifts $\omega_c, \omega_{c'}$ and $(\delta_c + i\eta_c)$, $(\delta_{c'} + i\eta_{c'})$ have been calculated from an optical model[14] at the energy of the elastic (c) or inelastic channel (c'), respectively. The direct transition strength is represented by $u_{cc'}$; it may be calculated from a distorted-wave Born-approximation (DWBA) program, using the collective properties of the excited 2^+ state, or from a phase-shift analysis. In this analysis, background scattering for the two scattering angles $\theta = 140°$ and $160°$ is described by eight fitted, energy-independent, complex numbers $u_{cc'}$ for s and p waves in the entrance channel. This does not mean that the actual values of the direct scattering matrix elements have been determined; however, it produces effective background scattering amplitudes for these two angles. This is quite similar to the case of elastic scattering, where meaningful fits have been obtained for both cross section and analyzing power by the use of only one complex number for each angle to describe the influence of all off-resonant direct scattering elements.[15]

The resonance terms contain the following real magnitudes: resonance energy E_λ, total width Γ_λ, mixing phases $\varphi_{c'}{}^\lambda$, and partial-width amplitudes $(\Gamma_{c'}{}^\lambda)^{1/2}$, the latter carrying the spectroscopic information:

$$(\Gamma_{c'}{}^\lambda)^{1/2} = a_{ij}{}^\lambda [\Gamma_{c'}{}^{\text{s.p.}}/(2T_0 + 1)]^{1/2}.$$

Most of these parameters are determined by the elastic-scattering analysis; the inelastic-channel resonance-mixing phase $\varphi_{c'}$ and single-particle widths $\Gamma_{c'}{}^{\text{s.p.}}$ have been obtained from the tables of Thompson at the energy of channel c'.[16] The program fitted both $u_{cc'}$ and $a_{ij}{}^\lambda$.

Table I shows the result: a matrix of parentage coefficients in the basis of p and f neutron waves, and the ground and first excited states of ^{140}Ce. In brackets are the results of nuclear structure calculations of Beregi and Lovas[8] using the same basis. There is good overall agreement. For the higher states we get smaller contributions of the configuration $(f_{7/2} \otimes 2^+)$. The expansion of the $\frac{3}{2}^-$, $E_x = 0.66$ MeV state is similar to that obtained in previous work[1,2] neglecting $p_{1/2}$ and $f_{5/2}$ neutron waves in the exit channel. These waves

TABLE I. Spins, excitation energies, and parentage coefficients for six states in ^{141}Ce in the basis of p and f neutron waves and the 0^+ g.s. and 2^+ excited state of ^{141}Ce. In brackets, theoretical values of Beregi and Lovas (Ref. 8).

E_x^λ (MeV)	$J^{\pi\lambda}$	$a_0^\lambda(0_1^+$ g.s.$)$ $=(S^\lambda)^{1/2}$	$a_{i,j}^\lambda(2_1^+$ $E_x = 1.596$ MeV$)$				$\sum_{ij} a_{ij}^2$
			$3p_{1/2}$	$3p_{3/2}$	$2f_{5/2}$	$2f_{7/2}$	
.66	3/2$^-$	.83 (.8131)	-.19 (-.1556)	-.20 (-.1849)	-.05 (-.0802)	-.84 (-.5236)	1.48
1.14	1/2$^-$	.74 (.8745)		+.53 (+.3872)	-.58 (-2922)		1.17
1.53	5/2$^-$	.67 (.7233)	-.52 (-.1659	-.07 (+.1323)	-.55 (-.1623)	+.65 (+.6388)	1.44
1.79	7/2$^-$	.33 (.2296)		+.31 (-.0392)	+.04 (-.0193)	+1.07 (+.9723)	1.27
1.86	3/2$^-$	.40 (.4527)	+.10 (-.1734)	-.18 (-.2211)	-.26 (-.0702)	+.29 (+.8433)	.35
1.88	5/2$^-$	.59 (.5860)	-.11 (-.1651)	+.33 (+.0831)	+.02 (-.1725)	-.40 (-.7699)	.67

contribute considerably to the next $\frac{1}{2}^-$ ($E_x = 1.4$ MeV) and $\frac{5}{2}^-$ ($E_x = 1.53$ MeV) states. In the last column, the sum of the absolute squares of the coefficients is given; this number should be unity if the basis is sufficient. The values obtained are systematically larger than 1 for the lower $\frac{3}{2}^-$, $\frac{1}{2}^-$, $\frac{5}{2}^-$, and $\frac{7}{2}^-$ states and smaller than 1 for the higher $\frac{3}{2}^-$ and $\frac{5}{2}^-$ states. The deviations of the lower states are assumed to be due to an underestimation of the single-particle widths.

This is the first time that an extensive analysis of inelastic scattering of polarized protons to excited collective states has been performed in the region of IAR's. The description of the direct background scattering should be refined by DWBA calculations and by extending the experiments to more scattering angles. However, the results reported here indicate that important nuclear structure information will be obtained from inelastic scattering of polarized protons.

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Electromagnetic Test Fields Around a Kerr-Metric Black Hole*

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Weak electromagnetic perturbations (test fields) in the exterior of a Kerr-metric black hole are studied. It is shown that (i) the only physically acceptable, time-independent perturbation is an axisymmetric field which corresponds to adding charge to the source and (ii) all axisymmetric normal modes are stable which, assuming completeness, guarantees stability for arbitrary axisymmetric perturbations. The proof of the nonexistence of time-independent, nonaxisymmetric test fields actually holds for perturbations associated with any physical (e.g., gravitational) field.

Present theoretical evidence within general relativity suggests strongly that when a rotating star collapses completely it leaves behind a black hole whose exterior geometry is the Kerr metric.[1-4] If so, the study of electromagnetic (em) perturbations of the Kerr metric relates directly to the history of the em field of a star which undergoes gravitational collapse and to the em information about the collapse which would be received by a distant observer. Also, the study of em perturbations might point out a path to be followed in attacking the more difficult problem of gravitational perturbations.

In this Letter we prove two theorems concerning physically acceptable (specific definition given below) weak em perturbations[5] of the exterior of a Kerr black hole:

First, the only time-independent em perturbation is axisymmetric and corresponds to the addition of charge to the black hole. An immediate corollary is that when a star collapses the higher multipoles of its (weak) em field must be attenuated.[6] The proof of the nonexistence of time-independent, nonaxisymmetric perturbations is very general and is valid for perturbations associated with any physical field. The proof thus extends Carter's theorem[4] that a Kerr black hole has no time-independent, axi-