electrons.

We are indebted to D. N. E. Buchanan for assistance with the measurements.

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Onsager Phase Factor in Cadmium

P. T. Coleridge and I. M. Templeton
National Research Council of Canada, Ottawa, Canada
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Precise de Haas–van Alphen phase measurements for the lens orbit in Cd with the magnetic field in the basal plane show that the Onsager phase factor γ for this orbit is $\frac{1}{2}$ and not $\frac{3}{4}$ as predicted recently by Hosack and Taylor. A spin-splitting zero is observed in the $(10\bar{1}0)$ plane, giving a value for $g \approx 1.6$.

Hosack and Taylor (HT) have recently published[1] calculations on energy levels of Bloch electrons in a magnetic field. A major conclusion of this paper is that for the "lens" orbit in hexagonal metals, with the magnetic field in the basal plane, the undetermined phase factor γ in the Onsager[2] relationship has the value $\frac{3}{4}$ rather than the free-electron value of $\frac{1}{2}$.

We have recently developed a technique[3,4] whereby the phase/field relationship of de Haas–van Alphen (dHvA) oscillations may be defined with very high precision. Under optimum conditions the phase can be measured to better than 0.01 cycle, and the field determined with a relative precision approaching 1 in 10^6. We have used this technique to test HT's prediction for the lens orbit in Cd. For this particular case we expect to be able to define the dHvA phase factor to a few hundredths of a cycle.

According to the dHvA data of Grassie[5] and the cyclotron resonance data of Galt *et al.*,[6] the lens orbit in Cd has a minimum frequency $\sim 6.25 \times 10^7$ G with the field in the basal plane, and the cyclotron mass at this point is $\sim 0.52 m_0$. Grassie explains his relatively strong second-harmonic signals on the basis of the product gm^*/m_0 being close to unity (the spin-splitting zero condition for the fundamental) but makes no estimate of g. It is important for the present study to know whether gm^*/m_0 is in fact greater or less than 1 since there is an ambiguity of $\frac{1}{2}$ in the phase according to the sign of the spin-splitting factor $\cos(\pi p g m^*/2m_0)$, where p is the harmonic number.

In our experiments we measure the second derivative of magnetization. If we consider the oscillatory part of the signal to be

$$d^2m/dH^2 \propto \cos[2\pi(F/H + \varphi)],$$

then $\varphi = \frac{1}{2} - p\gamma \mp \frac{1}{8} \pm \frac{1}{4}$; see, for example, Shoenberg.[7] We see that φ includes γ, the phase factor; p, the harmonic number; $\mp \frac{1}{8}$, according as the extremal area is a maximum or minimum; and $\pm \frac{1}{4}$, according as the spin-splitting factor is positive or negative. For the case of the lens in Cd, where the cross section is a maximum, where γ may be $\frac{1}{2}$ (free electron) or $\frac{3}{4}$ (HT), and where gm^*/m_0 is close to 1, we may tabulate the possible values of φ for the fundamental and second harmonic as shown in Table I.

Using the technique referred to above, we studied dHvA oscillations from the lens orbit in a sample of Cd with the magnetic field aligned to within $\sim 1°$ of the $\langle 10\bar{1}0 \rangle$ axis. The value of φ for the fundamental was obtained experimentally by extrapolating to $1/H = 0$ dHvA phases measured over a field range of 17–48 kG. The phase of second-harmonic signals were extracted by Fourier analysis over the range 36–48 kG, and the corresponding value of φ was derived by reference to the precise dHvA frequency obtained from the slope of the fit to the fundamental phases. We

TABLE I. Predicted and observed dHvA phases for the lens in cadmium.

	$\frac{1}{2}<gm^*/m_0<1$		$1<gm^*/m_0<\frac{3}{2}$	
	$p=1$	$p=2$	$p=1$	$p=2$
Hosack and Taylor ($\gamma=\frac{3}{4}$)	$\frac{7}{8}$	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{5}{8}$
Free electron ($\gamma=\frac{1}{2}$)	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{5}{8}$	$\frac{1}{8}$
Experimental result	$0.13_7\pm0.02_5$	$0.15_4\pm0.05$	...	...

took care to minimize interaction effects by appropriate choice of temperature. The result for $p=1$ was $\varphi=1.3_7\pm0.02_5$; for $p=2$, $\varphi=0.15_4\pm0.05$. These results are consistent *only* with the choice $\gamma=\frac{1}{2}$ and $\frac{1}{2}<gm^*/m_0<1$, and hence disagree with HT's conclusion.

The requirement $gm^*/m_0<1$ was somewhat surprising in that it implies a value of g appreciably less than 2. To check this point we performed a separate experiment in which we observed carefully the amplitude of dHvA signals from the lens with the field rotated in the $(10\bar{1}0)$ plane. Although complicated by the presence of other frequencies, the fundamental appeared to vanish over a fairly narrow angular range about 55° from the hexad axis although the second harmonic was still visible. This behavior, which is entirely consistent with the results of Grassie, is typical of a spin-splitting zero; and, taken in conjunction with the data of Galt *et al.*[5] ($m^*\sim0.6m_0$ at 55° from $\langle0001\rangle$), implies $g\sim1.6$, in support of our conclusion above.

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Optical Detection of Site Selectivity for Rare-Earth Ions in Flux-Grown Yttrium Aluminum Garnet

J. P. van der Ziel, M. D. Sturge, and L. G. Van Uitert
Bell Telephone Laboratories, Murray Hill, New Jersey 07974
(Received 15 June 1971)

Linear dichroism of the optical transitions of rare-earth ions in the garnets can be used to determine growth-related site preferences. Site selectivity is detected for Pr^{3+} and Nd^{3+} in yttrium aluminum garnet but not for Eu^{3+}, Tb^{3+}, or Ho^{3+}.

Although the garnet structure is cubic, it has been found that mixed rare-earth iron garnets grown from solution show noncubic magnetocrystalline anisotropy.[1,2] The magnetic axes are related to the orientation of growth facets, and the anisotropy is believed to result from the selective incorporation of rare-earth ions into sites which, while crystallographically equivalent, are inequivalent with respect to growth direction.[3,4] That such site selectivity does indeed occur has recently been demonstrated by spin resonance measurements on Nd^{3+} and Yb^{3+} ions dilutely incorporated into yttrium aluminum garnet (YAlG).[5]

We report here a powerful new technique for measuring the relative site populations through the linear dichroism of the optical transitions of the rare-earth ion. We discuss in particular the linear dichroism of Pr^{3+}-doped YAlG. Dichroism has also been observed in the YAlG:Nd^{3+} spectrum, whereas the spectra of Eu^{3+}, Tb^{3+}, and Ho^{3+} appear to be quite isotropic for the crystals studied.