

ductivity case. These conditions agree qualitatively with theory, which states that the high-energy ion component arises from linear damping of ion-acoustic waves,<sup>3</sup> and that the ultimate temperature depends directly upon the heating duration  $\tau_H$ , when the energy decay time  $\tau_L$  is long.<sup>5</sup> We know from previous microwave scattering measurements of the turbulence spectrum that the ion-acoustic wave level is high during the unstable period (between  $a$  and  $c$  in Fig. 1),<sup>2</sup> and from transport measurements that the loss rate during heating is only about the Bohm diffusion rate.<sup>3</sup> It is apparent from Fig. 1 that  $\tau_H$  is much shorter than the current pulse duration. Thus the ratio  $\tau_H/\tau_L$  is small in our experiment.

At time  $a$  in Fig. 1 the plasma-column resistance suddenly increases from 2 to 10  $\Omega$ , causing the voltage to depart from the normal oscillatory behavior indicated by dashed line  $b$ . Values of effective conductivity (reciprocal resistivity) are shown by curve  $C$  of Fig. 2. During the turbulent condition  $\sigma \approx 10$  mho/m. From the equation for low-frequency conductivity, Eq. (2), we calculate an anomalous collision frequency  $\nu_{\text{eff}} \approx 10^{11}$ . This is an order of magnitude higher than predicted by theory for the levels of turbulence present in our experiment.<sup>2,3,5</sup> This indicates (a) that the measurements are in error (which we do not believe); (b) that we are making comparisons to the wrong theoretical model; or (c) that anomalous effects

are present that we have not considered. It is interesting to note that other experiments also report resistivities that are higher than expected.<sup>9</sup>

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## Quantum Interference Effects of a Moving Vortex Lattice in Al Films

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An experimental study is reported of steps induced in the flux-flow  $I$ - $V$  characteristics of superconducting aluminum films by a superimposed rf current, whose frequency is a harmonic or subharmonic of the ratio of the vortex velocity and the lattice parameter.

Steps in the dc current-voltage curves of a type-II superconductor in the flux-flow mixed state have been induced by superimposed rf and dc currents. This observation is interpreted as a quantum interference effect between the applied rf current and the local supercurrent oscillations generated by the moving vortex lattice. Interference occurs whenever experimental conditions are such that

$$f = nv/\lambda, \quad (1)$$

where  $f$  is the frequency of the rf current,  $v$

is the average vortex velocity,  $\lambda$  is the magnitude of a two-dimensional lattice vector of the vortex structure, and  $n$  is an integer. Values of  $\lambda$  which fit the data are appropriate to the triangular lattice of singly quantized vortices.<sup>1</sup>

The underlying physical principle is the driven ac Josephson effect observed between weakly coupled bulk superconductors.<sup>2</sup> The moving vortex lattice can be thought of as a coherent two-dimensional array of superconducting weak links.<sup>3</sup> The experiment reported in this Letter demonstrates the coherence property.

Aluminum films, 100 to 1000 Å thick, were evaporated onto glass slides in an oxygen atmosphere, producing normal resistivities at 4.2 K ranging from 1 to 20  $\mu\Omega$  cm.<sup>4</sup> Films made by evaporation through a mask and by photoresist etching gave similar results. The film is wired in series with a 50- $\Omega$  load resistor, terminating a coaxial transmission line immersed in pumped liquid helium.

A magnetic field  $H$  is applied perpendicular to the plane of the film. Because of the large demagnetization coefficient in this geometry, the magnetic induction  $B$  is equal to  $H$ . The vortex density is  $N=B/\phi_0$ , where  $\phi_0=hc/2e$  is the flux quantum. In a perfect triangular lattice,  $N=2/\sqrt{3}d^2$ , where  $d$  is the nearest-neighbor distance between vortex lines.

A steady transport current through the film induces motion of the vortex lattice and resistance. The vortex velocity is computed from the usual expression,

$$\vec{E} = -c^{-1}\vec{v} \times \vec{B}, \quad (2)$$

where  $\vec{E}$  is the electric field (voltage per unit length).<sup>5</sup>

Representative data, for a 1000-Å-thick, 1.27-mm-wide aluminum film with a 9- $\mu\Omega$ -cm normal resistivity, are plotted in Fig. 1. In all three curves the abscissa is the dc electric field  $E_{dc}$ . In the dc characteristics, Fig. 1(a), one finds the typical critical current for the onset of voltage, and therefore flux flow. When rf current is superimposed, steps appear at certain values of the electric field. In order to bring out the step structure in clearer detail, the differential dc resistivity,  $(dV/dI)_{dc}$ , is plotted in Fig. 1(b). When interferences or minima in  $(dV/dI)_{dc}$  are observed, they occur at values of the dc electric field,  $E_{nn'}$ , which satisfy the simultaneous solution of Eqs. (1) and (2) with  $\lambda=n'd$  ( $n'$  an integer):

$$E_{nn'} = fBn'd/nc = n'f(2\phi_0B/\sqrt{3})^{1/2}/nc. \quad (3)$$

Interferences with  $n'$  up to 8 and  $n$  up to 4 have been easily distinguished. Figure 1(b) is annotated with the ratios  $n'/n$ . Extensive additional data were taken to verify the linear dependence upon  $f$  and the square-root dependence upon  $B$ . Also,  $E_{nn'}$  does not depend upon the rf or dc current or the temperature.

$E_{nn'}$  has been resolved to within a few percent precision over the magnetic field interval  $1 \leq B \leq 400$  G, for which  $49\,000 \geq d \geq 2500$  Å. Qualitatively, interference is detected in magnetic

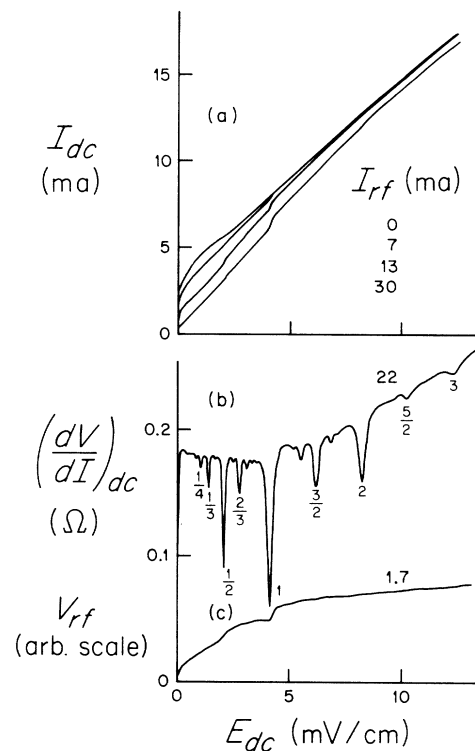


FIG. 1. Data taken at  $T=1.36$  K,  $H=80$  G, and  $f=96$  MHz on a 1000-Å  $\times$  1.27-mm  $\times$  2.54-mm Al film. (a) dc current for several rf current levels, (b) differential dc resistivity, and (c) rf voltage plotted against dc electric field. Values of the rf current and the ratio  $n/n'$  are also shown.

fields up to  $0.9H_{c2}$  in the purest specimens, and up to  $0.5H_{c2}$  in the least pure specimens. As these limits are approached, the negative peaks in  $(dV/dI)_{dc}$  become broad, shallow dips. The effect apparently washes out when the current patterns of the individual vortices greatly overlap.

$V_{rf}$ , the rf voltage in phase with the rf current which is kept constant, also shows structure at  $E_{nn'}$ . In Fig. 1(c) we plot  $V_{rf}$  against  $E_{dc}$ . An abrupt increase by as much as 20% occurs in the rf resistivity at  $E_{11}$ , with smaller changes at other interferences. This structure is visible for an rf current as small as 2% of the dc current. With only a dc current applied, no enhancement in rf noise emitted at the frequency  $v/d$  was detected.

My conclusion is that the vortex structure is readily induced to flow in such a way that a primitive lattice vector, whose length is determined by  $B$ , is aligned parallel to  $\vec{v}$ . Several specimens have shown additional resonances for which  $\lambda=n'\sqrt{3}d$ , so that another high-sym-

metry direction, obtained by rotating the lattice through  $30^\circ$ , is parallel to  $\vec{v}$ .

Voltages in the flux-flow state are a manifestation of the free-running ac Josephson effect. The phase of the order parameter between two points changes by  $2\pi$  for each vortex line that crosses between them. Hence, the average rate of change in phase per unit length along the direction of  $\vec{E}$  equals  $2\pi vN = 2evB/\hbar c$ . By invoking the Josephson equation relating voltage to the time derivative of the phase,

$$2eV = \hbar \partial \phi / \partial t, \quad (4)$$

one obtains Eq. (2).<sup>6</sup>

The theory of vortex motion based upon a time-dependent Ginsburg-Landau equation has been worked out by Kulik,<sup>6</sup> Schmid,<sup>7</sup> Caroli and Maki,<sup>8</sup> and Thompson.<sup>9</sup> These authors have calculated the supercurrent generated by the moving vortex lattice. Although the local supercurrent is spatially inhomogeneous and time dependent, the spatially averaged ac current turns out to be zero. In particular, superimposed ac and dc fields give ac and dc currents without any interference structure when Eq. (1) is satisfied.<sup>10</sup> We therefore do not have a perfect analog of the Josephson junction or Dayem bridge,<sup>11</sup> or even a physical array of Josephson junctions,<sup>3</sup> where in a variation in the phase difference corresponds to a net ac current flow which can be synchronized to an external rf current.

A possible explanation for the experimental observation is that a collective fluctuation in the vortex lattice is induced by the rf current. For example, we can get an interference term in the theoretical equations for the supercurrent by allowing the velocity to vary spatially with periodicity commensurate to that of the lattice.<sup>8,10</sup> An alternative possibility is that the interference arises from the finite width of the film, i.e., it

is an edge effect. The latter is considered less likely in view of the following: Al and Pb-20% Tl films showing stronger vortex pinning did not display observable interference. Dirty aluminum films were chosen because they tend to have weak pinning.<sup>12</sup> Also, varying the number of vortices across the specimen from 500 to 10 000 does not appear to change appreciably the character of the interference effect. Apparently, a sufficiently coherent motion of the vortex lattice is needed in order to observe the interference effect.

<sup>1</sup>For a review of type-II superconductivity, see A. L. Fetter and P. C. Hohenberg, in *Superconductivity*, edited by R. D. Parks (Marcel Dekker, New York, 1969), Vol. 2, Chap. 14.

<sup>2</sup>For a review of the Josephson effect, see P. W. Anderson, in *Progress in Low Temperature Physics*, edited by C. J. Gorter (North-Holland, Amsterdam, 1967), Vol. 5, Chap. 1.

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<sup>4</sup>Fire-polished Corning 7059 slides were used. Also, see R. W. Cohen and B. Abeles, *Phys. Rev.* **168**, 444 (1968).

<sup>5</sup>For a review of flux flow, see Y. B. Kim and M. J. Stephen, in *Superconductivity*, edited by R. D. Parks (Marcel Dekker, New York, 1969), Vol. 2, Chap. 19.

<sup>6</sup>The analogy to the Josephson effect was first discussed by I. O. Kulik, *Zh. Eksp. Teor. Fiz.* **50**, 1617 (1966) [*Sov. Phys. JETP* **23**, 1077 (1966)], who suggested looking for interference at the frequency  $v/\xi$ , where  $\xi$  is the coherence length.

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<sup>10</sup>Current in rf fields is calculated by G. Fischer, R. D. McConnell, P. Monceau, and K. Maki, *Phys. Rev. B* **1**, 2134 (1970).

<sup>11</sup>P. W. Anderson and A. H. Dayem, *Phys. Rev. Lett.* **13**, 195 (1964).

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