Skin Effect and Anomalous Resistivity Accompanying Turbulent Heating*

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In turbulent heating experiments the electrical resistivity is anomalously large, once turbulence has developed. But before turbulent levels build up in a dense, collisionless plasma, the high conductivity can cause a skin effect that impedes current penetration. We find that this skin effect has an influence on the onset of current-driven instabilities and on the ultimate heating. Low levels of residual turbulence before heating are found to play a role in the rate of current penetration.

In this experiment, intense small-scale current-driven instabilities grow to nonlinear levels, producing a rapid heating of ions and electrons to kilovolt temperatures. The electron drift velocity exceeds the ion sound speed and the ion thermal speed, but is always smaller than the electron thermal speed.

The apparatus has been described in the literature.[1-3] Two conical-pinch guns inject plasma through hollow electrodes into either end of a 2:1 magnetic mirror field of 3000 G, in a vacuum chamber 40 cm in diameter and 200 cm long. The plasma column, 8 cm in diameter and 180 cm long, reaches a maximum density of 8×10^{13} cm^{-3} about 30 μsec after injection. Turbulent heating is accomplished by applying a 50–200-kV pulse between the hollow electrodes, driving currents up to 13 000 A along the magnetic field. Voltage and current wave forms for typical operation are shown in Fig. 1. Because of series inductance, the initial current is zero, then rises sinusoidally until the electron-drift velocity eventually exceeds the threshold for current-driven instabilities, indicated by dotted line a. This circuit connection gives the "constant-current" mode of operation referred to in computer simulation experiments,[3] in which changes in plasma resistivity produce voltage fluctuations, with little influence on the current. The plasma column inductance of 0.6 μH, which remains nearly constant during the rise and fall of turbulence, combined with the external inductance of 3 μH gives an oscillation frequency of 300 kHz.

At the instant the heating current pulse is applied, the plasma column has an electron density n_e between (2 and 8)$\times10^{13}$ cm^{-3}, and a conductivity σ between 10 and 100 mho/m, depending upon settings.[3,4] The conductivity versus time is shown in Fig. 2. Curves A, B and A', B' are

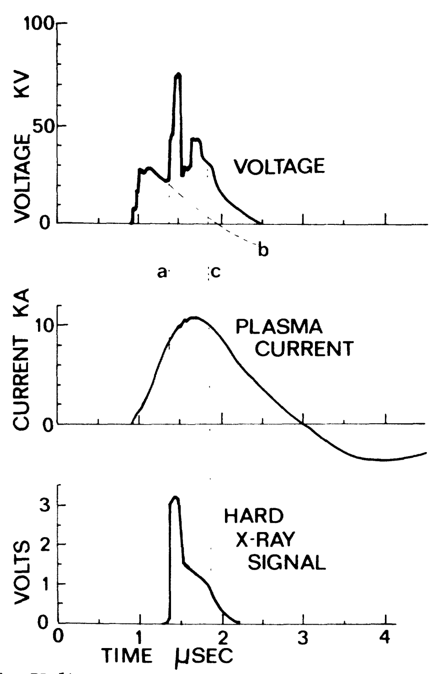

FIG. 1. Voltage, current, and hard x-ray signals. Instability begins at a and ends at c.

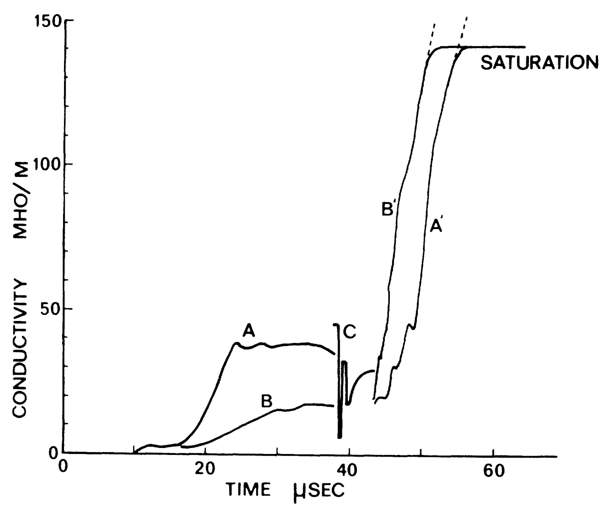

FIG. 2. Conductivity versus time. Curves A, B and A', B' are from data of an rf conductivity probe. Curve C is from analysis of voltage and current traces, removing inductive effects. A: $\sigma_0\approx50$ mho/m. B: $\sigma_0\approx10$ mho/m.

from data obtained with an rf conductivity probe,[4] and curve C is from voltage and current data, with inductive effects properly accounted for.

Using the equation for skin depth δ,[5]

$$\delta = \frac{c}{\omega_{pe}}\left(\frac{2\nu_{\rm eff}}{\omega}\right)^{1/2} = \left(\frac{2}{\sigma\mu_0\omega}\right)^{1/2}, \qquad (1)$$

the two initial conditions should lead to current-penetration depths of 10 and 3 cm, respectively, where $\omega = 2\pi(3\times10^5) = 1.9\times10^6$.

To measure the actual current penetration, we probed the azimuthal magnetic field B_θ with a small, shielded loop, mounted inside an insulating glass tube. We observed that when the initial conductivity was $\lesssim 20$ mho/m, the current penetrated to the axis, and the turbulence levels rose rapidly, as soon as the current reached the critical value. But when the initial conductivity was $\gtrsim 50$ mho/m, the current channel remained hollow for ~0.5 μsec, and the onset of turbulence was delayed until a higher current had been reached.

Plots of $B_\theta(r)$ at several instants of time are shown in Fig. 3. The dashed curve shows the spatial dependence to be expected for a current channel having uniform density out to 3 cm, and zero density beyond. Our data points outside $r = 4$ cm do not depart much from this $1/r$ curve, indicating that the current channel is mostly within $r = 4$ cm. X-ray data, obtained with a collimated detector and movable tantalum target, indicate that the high-energy electrons also are confined to this region. (See Fig. 1 for the time history of x-rays.) No evidence of self-pinching was observed.

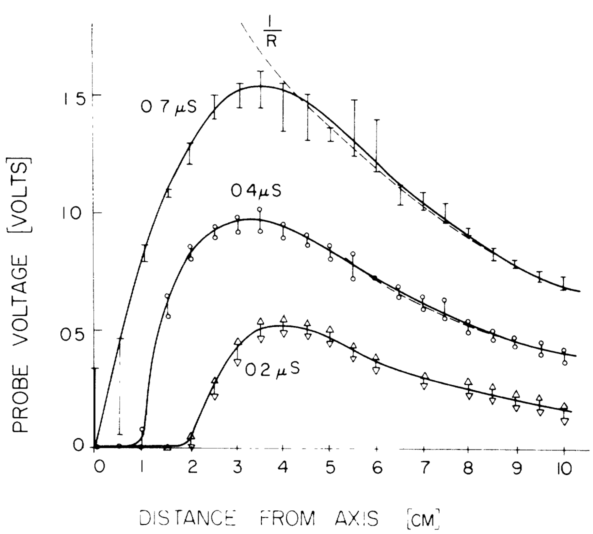

FIG. 3. $B_\theta(r,t)$ obtained from magnetic-probe data taken at different radial positions to demonstrate the skin effect.

The fact that our electrodes are hollow probably does not influence the skin effect, since the plasma is in good electrical contact with both electrodes. We note that in an experiment using plane electrodes,[6] the results obtained are very similar to those reported here.

The initial conductivity is much lower than classical, presumably due to a low level of turbulence in the counterstreaming plasmas. From the equation for conductivity,

$$\sigma = \epsilon_0\omega_p^2/\nu_{\rm eff}, \qquad (2)$$

we obtain an anomalous collision frequency $\nu_{\rm eff} \approx 10^9$ in the injected plasma before heating. The classical collision rate is $\nu_{ei} \approx 10^8$ for our plasma conditions. One concludes then that residual levels of turbulence in the initial plasma may play a significant role in the problem of current penetration during turbulent heating experiments. At our large current levels there may also be an anomalous skin effect,[7] although we have no direct evidence of it.

We have assumed that $\nu_{\rm eff}$ is uniform in radius. This is justified for this particular experiment, since the plasma has not become unstable during the skin time. The conditions may be fortuitous. If turbulence had developed while the current was soaking in, the increased local resistivity would be expected to increase the rate of current penetration. The measured radial profile of conductivity before the heating current begins[4] substantiates the above assumption.

The initial conductivity has a large influence on the final energy density of heated plasma. The low-conductivity condition (curve $B-B'$) led to a final $nkT_\perp$ of 7.5×10^{16} eV/cm^3 when ± 100 kV (1 kJ) was impressed. By contrast, the high-conductivity case (curve $A-A'$) gave only 3.3×10^{16} eV/cm^3, for the same conditions. Most of the difference is involved with the length of time that the heating persists before the instability quenches itself.[3] In the low-conductivity case the instability begins as much as 0.1 μsec earlier. The power-input level of $\sim 10^9$ W persists for about 0.5 μsec, giving an electrical input of 500 J. Of this energy 6.5% is deposited in the form of heated plasma. The high-conductivity case leads to an efficiency of only 3–4%.

This distribution of ion energies following heating is nearly isotropic, but has two temperature components.[8] The high-temperature component ($T_i \approx 1$–2 keV) is about 30% higher for the low-conductivity case. These conditions agree qualitatively with theory, which states that the high-energy ion component arises from linear damping of ion-acoustic waves,[3] and that the ultimate temperature depends directly upon the heating duration τ_H, when the energy decay time τ_L is long.[5] We know from previous microwave scattering measurements of the turbulence spectrum that the ion-acoustic wave level is high during the unstable period (between *a* and *c* in Fig. 1),[2] and from transport measurements that the loss rate during heating is only about the Bohm diffusion rate.[3] It is apparent from Fig. 1 that τ_H is much shorter than the current pulse duration. Thus the ratio τ_H/τ_L is small in our experiment.

At time *a* in Fig. 1 the plasma-column resistance suddenly increases from 2 to 10 Ω, causing the voltage to depart from the normal oscillatory behavior indicated by dashed line *b*. Values of effective conductivity (reciprocal resistivity) are shown by curve *C* of Fig. 2. During the turbulent condition $\sigma \approx 10$ mho/m. From the equation for low-frequency conductivity, Eq. (2), we calculate an anomalous collision frequency $\nu_{\text{eff}} \approx 10^{11}$. This is an order of magnitude higher than predicted by theory for the levels of turbulence present in our experiment.[2,3,5] This indicates (a) that the measurements are in error (which we do not believe); (b) that we are making comparisons to the wrong theoretical model; or (c) that anomalous effects are present that we have not considered. It is interesting to note that other experiments also report resistivities that are higher than expected.[9]

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Quantum Interference Effects of a Moving Vortex Lattice in Al Films

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An experimental study is reported of steps induced in the flux-flow *I-V* characteristics of superconducting aluminum films by a superimposed rf current, whose frequency is a harmonic or subharmonic of the ratio of the vortex velocity and the lattice parameter.

Steps in the dc current-voltage curves of a type-II superconductor in the flux-flow mixed state have been induced by superimposed rf and dc currents. This observation is interpreted as a quantum interference effect between the applied rf current and the local supercurrent oscillations generated by the moving vortex lattice. Interference occurs whenever experimental conditions are such that

$$f = nv/\lambda, \tag{1}$$

where f is the frequency of the rf current, v is the average vortex velocity, λ is the magnitude of a two-dimensional lattice vector of the vortex structure, and n is an integer. Values of λ which fit the data are appropriate to the triangular lattice of singly quantized vortices.[1]

The underlying physical principle is the driven ac Josephson effect observed between weakly coupled bulk superconductors.[2] The moving vortex lattice can be thought of as a coherent two-dimensional array of superconducting weak links.[3] The experiment reported in this Letter demonstrates the coherence property.