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Properties of Low-Density Neutron-Star Matter*

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A nuclear Thomas-Fermi model is used to determine the ground state of matter at subnuclear densities allowing for inhomogeneities on a nuclear scale ("clusters"). The "neutron drip" phase transition occurs at 3.7×10^{11} g/cm^3. As a function of the average density the proton number Z of the clusters first increases from $Z\simeq29$ at 1.4×10^8 g/cm^3 to a maximum of $Z\simeq35$ at $\sim3.0\times10^{12}$ g/cm^3 and then decreases until the clusters gradually evanesce just above 10^{14} g/cm^3; this transition is smooth.

The present Letter reports on a study of the properties of cold catalyzed matter (CCM) for densities ranging from 10^8 to 10^{14} g/cm^3. By catalyzed we mean that matter is considered to be in equilibrium with respect to strong, electromagnetic, and weak interactions. An extensive study is dictated by the existence of neutron stars, a large part of whose interior falls into this density region.

At the very low densities CCM is composed of ^{62}Ni nuclei (which have the highest binding energy per nucleon) imbedded in a sea of electrons. Above a density of $\sim1.0\times10^7$ g/cm^3 (which we refer to as region I), the chemical potential of the electrons is $\gtrsim0.5$ MeV and it is energetically favorable to have neutron-enriched nuclei. Above a density of $\sim3.7\times10^{11}$ g/cm (region II), the chemical potential of the neutrons becomes positive so that no longer are all the neutrons clustered; in addition to the electron fluid there now is an imbedding neutron fluid. Above 10^{14} g/cm^3 (region III), the inhomogeneities on a nuclear scale disappear and one is left with a mixture of neutrons, protons, and electrons, with other particles appearing at still higher densities.[1-4]

Previous authors[1-4] in their study of the composition of matter in regions I and II have relied heavily on the use of a semiempirical (four-parameter) mass formula $B(Z,A)$. Their approach requires $B(Z,A)$ to be applied and, more seriously, to be differentiated far away from the region of stability where it was fitted. The results one obtains with a mass formula are therefore not only quantitatively but also *qualitatively* very sensitive to the particular functional form of $B(Z,A)$, as has previously been pointed out.[5] It is thus important to develop a more fundamental approach which, while remaining tractable, allows the nuclei to change shape and to adjust to their environment, in particular to the imbedding neutron fluid. The natural basis for such an approach is provided by the nuclear Thomas-Fermi model.[6-10] The latter is based on nuclear-matter results derived from a realistic nucleon-nucleon force, and has been quite successful in reproducing the bulk properties of ordinary nuclei,[9,10] which gives confidence in its application to the present problem. We should point out that in region II we do not have the difficulties associated with the neutron tail, characteristic of the Thomas-Fermi model for a self-bound system.

Preliminary results were reported in a previous Letter.[11] The present calculation improves on those results in several respects: First, recent neutron-gas results[12] are used rather than extrapolated values; second, proton-electron and electron-electron Coulomb interactions are now included and are treated in a Wigner-Seitz-sphere approximation; third, the electron density is no longer held constant so that the electrons can screen the nuclear charge; fourth, the minimization problem has been solved exactly, as opposed to variationally, and the numerical accuracy has been greatly improved; finally, the calculations have been extended to region I.

In the density range of interest here, protons cluster together tightly so as to obtain maximum value of the nuclear symmetry energy. In the spirit of the Wigner-Seitz-sphere approximation the system is therefore thought of as consisting of noninteracting identical cells, endowed with spherical symmetry, of volume $\Omega = (4\pi/3)R_c^{\,3}$, centered on the points of highest density (Fig. 1).

The basic physical problem consists in expressing the average energy per unit volume $\bar{\epsilon}$ as a functional of particle densities, $\rho_i(r)$, where $i = (n, p, e)$; the ground-state density distributions which provide the solution to the exact many-body problem are the ones which minimize $\bar{\epsilon}[\rho_i]$ for a given baryon density $\mathfrak{N}$ under the constraint of charge neutrality. In the following we adopt a statistical model (Thomas-Fermi) to approximate the energy-density functional.

The energy per cell is expressed as

$$E_{\rm cell} = \int_\Omega \epsilon[\rho_n(r), \rho_p(r), \rho_e(r)]d^3r,$$

where

$$\epsilon = \epsilon_{\rm nuc}[\rho_n, \rho_p] + \epsilon_{\rm el}(\rho_e) + \epsilon_{\rm C}(\rho_p, \rho_e) + M_n\rho_n + M_p\rho_p.$$

The first term represents the nuclear energy density,

$$\epsilon_{\rm nuc} = \rho\mathfrak{B}(\rho_n, \rho_p) - \xi\rho\nabla^2 p,$$

with

$$\rho = \rho_n + \rho_p,$$

where $\mathfrak{B}(\rho_n, \rho_p)$ is the energy per nucleon of homogeneous nuclear matter of neutron and proton densities ρ_n and ρ_p, respectively; the derivative term takes into account the long-range part of the nuclear forces. The constant ξ, which is the only phenomenological parameter in the theory, has been determined to give a fit to the binding energies of the known nuclei. Its fitted value is in good agreement with a theoretical estimate.[7] For further details we refer the reader to Refs. 9 and 11.

The electronic term has the form

$$\epsilon_{\rm el}(\rho_e) = (m_e^{\,4}c^5/8\hbar^3\pi^2) \times\{(2x^3+x)(x+1)^{1/2} - \ln[x+(x+1)]\}^{1/2}$$

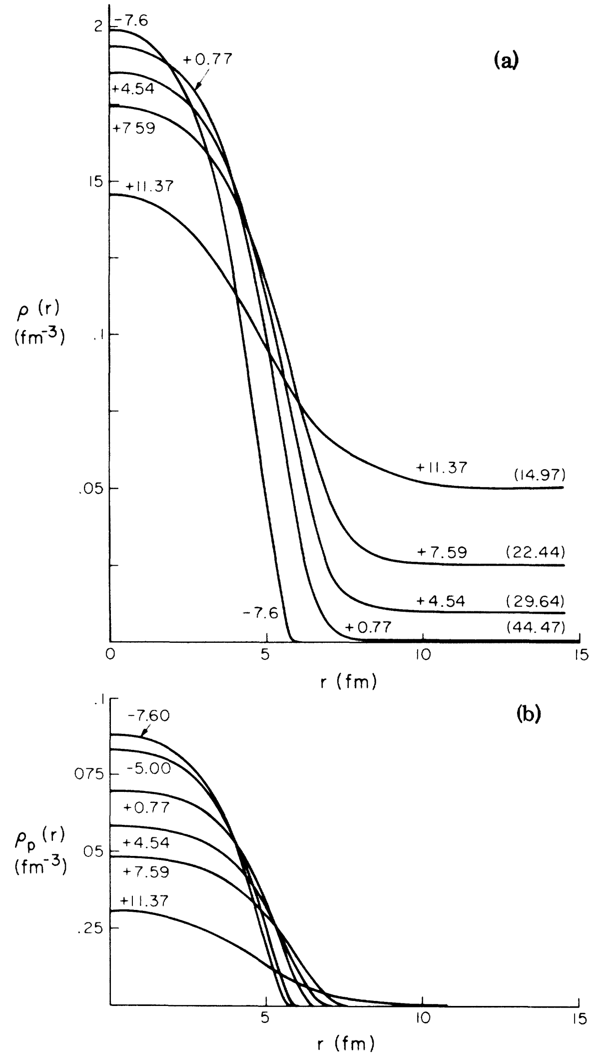

FIG. 1. (a) Total nucleon density distributions $\rho(r)$ as a function of radial distance, labeled according to the corresponding neutron chemical potential λ_n. Shown in parentheses are the cell radii in fm. (b) Proton density distributions $\rho_p(r)$.

with

$$x = [(3\pi^2)^{1/3}\hbar/m_e]\rho_e^{\,1/3};$$

and, finally, the Coulomb term

$$\epsilon_{\rm C} = \tfrac{1}{2}e(\rho_p - \rho_e)\Phi - \tfrac{3}{4}(3/\pi)^{1/3}e^2(\rho^{4/3}{}_p + \rho^{4/3}{}_e)$$

contains both the direct and exchange energies with Φ satisfying the Poisson equation

$$\nabla^2\Phi = -4\pi e(\rho_p - \rho_e).$$

The average baryon density $\mathfrak{N}$ is given by

$$\mathfrak{N} = \Omega^{-1}\int_\Omega(\rho_p + \rho_n)d^3r.$$

Rather than minimizing $\bar{\epsilon} = E_{\rm cell}/\Omega$ at constant

$\mathfrak{N}$, one can equivalently minimize the quantity

$$\mathfrak{F}=\Omega^{-1}\int_{\Omega}[\epsilon-\mu_n(\rho_n+\rho_p)-\mu_e(\rho_e-\rho_p)]d^3r$$

at constant μ_n with the other Lagrange parameter μ_e chosen to satisfy charge neutrality.

This minimization problem has been approached by solving the associated Euler-Lagrange equation with the required boundary and matching conditions. Details of their derivation and of the computational procedure are rather lengthy and are being published elsewhere, together with more detailed results and with a tabulation of the equation of state. The results are reported in Figs. 1-3. Two interesting features stand out:

First, the proton number Z of the clusters goes through a maximum of ~35 at ~3×10^{12} g/cm³. This is in sharp contrast to the results that have been found by other authors[1-3] who have relied on the use and extrapolation of a mass formula. It has been previously pointed out,[5] however, that such a behavior of Z can be obtained through changing the functional form of $B(Z,A)$ by including a surface-symmetry term. This term appears in the most recent mass formulas[13,14] and improves the fit to the known nuclei. The fact, however, that the qualitative behavior of the results undergoes a drastic change shows that extreme care has to be used in extrapolating a mass formula. Terms which are unimportant in the fitting region can become very important, especially when derivatives have to be taken in order to provide the chemical potentials

$$\lambda_p=-(\partial B/\partial Z)_A-(\partial B/\partial A)_Z \text{ and } \lambda_n=-(\partial B/\partial A)_Z.$$

The second feature concerns the transition from a clustered system to a homogeneous gas. The nuclei are seen to evanesce gradually as the average density increases, and the proton "radius" a_p [point where $\rho_p(r)=0$] tends towards the cell radius. The most striking result is exhibited in Fig. 3 where, as a function of λ_n (the neutron chemical potential excluding rest mass), the proton chemical potential λ_p is shown for both the clustered and the homogeneous phase (the latter results have been obtained as described by Buchler and Ingber[12]). The two curves are numerically found to be tangent, thus making it both difficult and irrelevant to pinpoint the exact transition density. The transition from the clustered to the homogeneous phase is therefore smooth. This result is again in contrast to the findings of other authors[1-4], namely, that the proton number Z reaches[4] ~160 when the nuclei start to touch and

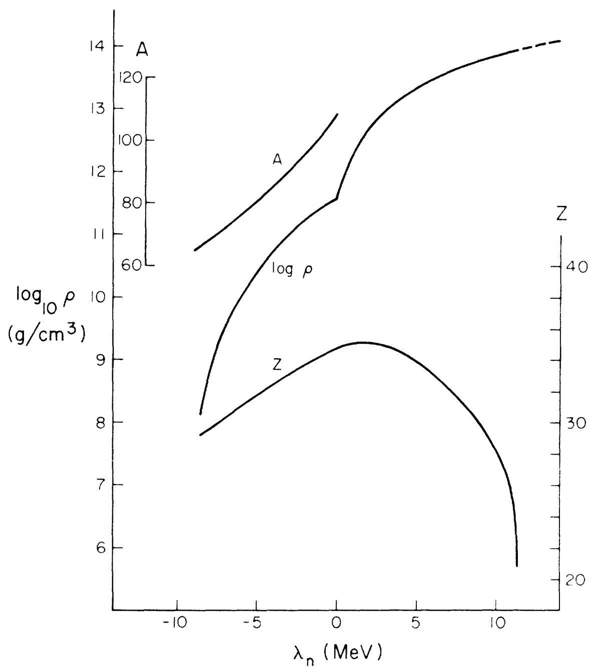

FIG. 2. Proton number Z, atomic number A of the clusters, and average $\log_{10}$(mass density in g/cm³) as a function of the neutron chemical potential λ_n. The dashed line for $\log\rho$ above $\lambda_n=10$ MeV corresponds to the homogeneous system, showing that the transition is smooth, which contrasts with the "neutron drip" phase transition ($\lambda_n=0$) at 3.7×10^{11} g/cm³ where ρ vs λ_n shows a cusp, i.e., $d\rho/d\lambda_n$ is discontinuous. The mass number A does not have a well-defined meaning for $\lambda_n>0$ as the neutrons extend throughout the medium.

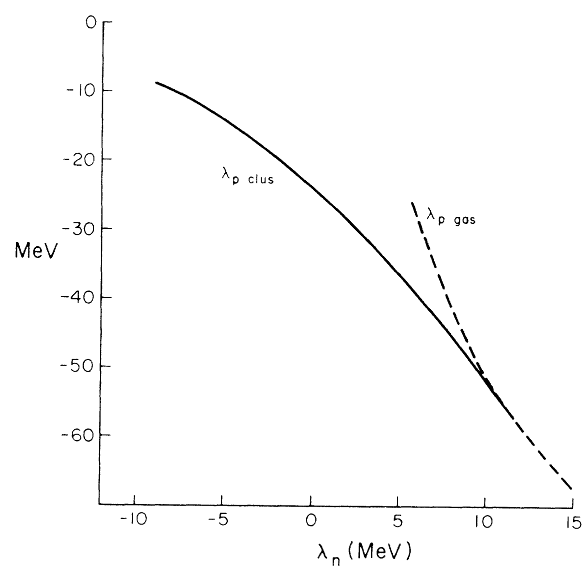

FIG. 3. Chemical potential of the protons in the cluster ($\lambda_{p\,\mathrm{clus}}$) and in the homogeneous phase ($\lambda_{p\,\mathrm{gas}}$) as a function of the neutron chemical potential λ_n, showing that the transition is smooth.

when the spherical symmetry and the Wigner-Seitz approximation breaks down.

It is interesting to note that at a density of 1.4 $\times 10^8$ g/cm^3 we find a nucleus of $Z \simeq 29$ and $A \simeq 66$, which is in the region of known nuclei. This is in good agreement with the results obtained with a mass formula. We find a total energy per nucleon of −8.14 MeV. Subtracting the electronic energy (0.69 MeV) leaves a nuclear binding energy of 8.83 MeV. Considering that the statistical model neglects shell effects, the agreement with the experimental binding energy of ^{66}Cu (8.73 MeV) is excellent.

The nuclear Thomas-Fermi model thus allows a continuous and consistent study of matter between 10^8 and 10^{14} g/cm^3. We feel that the qualitative behavior obtained is correct, but that the uncertainty in the parameter ζ may provoke a small quantitative shift. A sensitivity analysis is being completed and will be published together with the rather lengthy details associated with the present report.

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Cloud-Chamber Quark Quest—Negative Results*

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More than 100 000 cosmic-ray tracks from extensive air showers were scanned in pictures of cloud chambers after the manner of McCusker. In addition, positive- and negative-ion columns were separated by an electric field, and individual droplets were counted; histograms of numbers of tracks versus droplet density are shown. Artificially produced low-droplet-count tracks have been used to measure the scanning efficiency for quarklike tracks. To date the upper limit for the $\frac{2}{3}e$ quark flux is 3×10^{-11} cm^{-2} sec^{-1} sr^{-1}, and the limit for $\frac{1}{3}e$ particles is 3×10^{-10} cm^{-2} sec^{-1} sr^{-1}.

Following McCusker and co-workers,[1] cloud-chamber experimenters[2,3] assume that quark tracks differ from normal cosmic-ray tracks only in the amount of ionization: A $\frac{2}{3}e$ quark would make a track $\frac{4}{9}$ as dense as a normal track, and a $\frac{1}{3}e$ quark, $\frac{1}{9}$ as dense. Further, McCusker suggested that quarks should be produced by the interaction of high-energy cosmic rays with the atmosphere. Thus, cloud chambers are triggered on extensive air showers produced by primary particles with energies mostly greater than 10^{15} eV.