nication) converges very fast without requiring vast computer storage. This allows one to set  $N$  as large as  $10<sup>3</sup>$  without much effort, and allowed us to obtain quite satisfactory accuracy in the final results, as discussed in the text.

4The methods used in Ref. 1 might be criticized on account of the several approximations (such as the use

of the coherent-potential approximation, and approximating  $F_{e1}$  by the electronic internal energy with neglect of the electronic entropy). The free energy we calculate, on the other hand (denoted  $F^{LRO}$ ), is exact within the postulates of the model and free of any approximations except those which arise in numerical computation.

## Symmetric Fission Observed in Thermal-Neutron-Induced and Spontaneous Fission of  $^{257}$  Fm<sup>†</sup>

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The observed mass distribution for thermal-neutron-induced fission of  $^{257}$ Fm is strongly symmetric, whereas that for spontaneous fission of  $^{257}$ Fm is found to be predominantly asymmetric with, however, some symmetric fission present. The mass distributions were derived from energy measurements on coincident fission fragments. The onset of symmetric fission at  $257$ Fm supports theories relating asymmetric fission to the second hump of the fission barrier.

The understanding of the mass distribution in nuclear fission is an outstanding problem. The simple liquid-drop model predicts symmetric fission, yet mass distributions observed for lowenergy fission have always been asymmetric. Our measurements on the thermal-neutron-induced fission of <sup>257</sup>Fm show a strong mode of symmetric fission. We also observe the symmetric-fission mode occurring weakly in the spontaneous fission of  $257$  Fm, in agreement with the recent data of Balagna  $et al.$ <sup>1</sup> These unusual results afford an additional test of theories of the mass distribution in fission and, indeed, support a recent suggestion by Möller and Nilsson<sup>2</sup> relating the mass distribution to the doublehumped fission barrier.

Measurements were made on the kinetic energies of the coincident fission fragments from a thin sample placed between two Si detectors. The sample, containing  $4 \times 10^8$  atoms of <sup>257</sup>Fm, was obtained from the Hutch underground nuclear explosion.<sup>3</sup>  $\alpha$  emission is the predominant decay mode of this 100-day isotope although a small  $(0.2\%)$  spontaneous-fission branching also occurs. After final purification, a few microliters of solution containing  $257$  Fm was evaporated on a 200- $\mu$ g/cm<sup>2</sup> Pt foil. The foil was then mounted on a four-position sample wheel located between the two Si detectors with collimators to limit the maximum fragment entry angle to 50' from the normal to the detectors. Other positions of

the wheel contained a  $252Cf$  spontaneous-fission source for energy calibration, a <sup>233</sup>U source (2 ng) for neutron-flux determination and a check on the energy calibration, and a blank Pt foil for background measurement. All samples were covered by 200- $\mu$ g/cm<sup>2</sup> Pt foils to prevent detector contamination. The assembly was placed in the thermal column of the Livermore reactor in a flux of  $2 \times 10^{11}$  neutrons/cm<sup>2</sup> sec (cadmium ratio 600). The techniques for counting in the high neutron flux included cooling the detectors and using fast linear electronics as previously described.<sup>4</sup> Data were accumulated alternately with the reactor on and then off. Two separate runs of about three weeks each were made. The fragment masses for each event were calculated from the kinetic energies, assuming conservation of momentum and mass. A correction was made for the pulse-height defect but not for neutron emission. Thus the masses correspond to provisional (approximate pre-neutron) masses of Schmitt, Neiler, and Walter.<sup>5</sup>

In Fig. 1 the results for spontaneous fission of  $^{257}$ Fm are displayed as a contour diagram of counts versus fragment mass and total kinetic energy. The symmetric part of the distribution is evident as a ridge along the line of mass symmetry extending up to 250 MeV. The greatest number of events occurred on the asymmetric peaks at masses 115 and 142 with a total postneutron kinetic energy of 188 MeV. A graph of



FIG. 1. Contour diagram of counts versus fragment mass and total kinetic energy for the spontaneous fission of  $^{257}$ Fm. The numbers on the contours refer to the number of events per 5.0 MeV per 3.4 amu. Provisional masses and post-neutron energies are plotted for a total of 27050 fissions.

counts versus mass is shown in Fig. 2. There is a slight indication of a dip in the middle of the peak. Correction for neutron emission would increase the dip by an amount depending on the assumed dependence of the neutron multiplicity  $\nu$  on fragment mass and kinetic energy. For fission of <sup>257</sup>Fm, however,  $\nu(M, E_h)$  is unknown and difficult to predict because of the unusual mass and kinetic-energy distributions. The correction is especially sensitive to  $\nu$  in the mass region 120-130which is poorly known for any fissioning nucleus.

The agreement of our spontaneous-fission data with that of Balagna  $et al.$ <sup>1</sup> is fairly good, particularly for the mass distribution. The energies agree at the high end of the energy distribution but disagree at lower energies. Our energies are about 10 MeV lower at the asymmetric peaks in Fig. 1 near 190 MeV. Judging from the widths of the  $\alpha$ -particle peaks (30 keV full width at halfmaximum), we conclude that the  $257$ Fm-source thickness does not account for the difference with the Los Alamos measurements.

The contour diagram for thermal-neutron-induced fission of  $257$  Fm is shown in Fig. 3. This was obtained by subtracting the spontaneousfission distribution from the reactor-on data (spontaneous plus neutron-induced fission) prorated according to the time, a  $40\%$  subtraction. We estimate the  $257$ Fm cross section to be 2000-3000 b. A small  $(6\%$  of the remaining counts) subtraction was made for fission of  $253Cf$ , the  $\alpha$ -decay daughter of <sup>257</sup>Fm which builds up in



FIG. 2. Mass distribution for the spontaneous fission of  $^{257}$ Fm.

the target. The subtraction was made using the  $252$ Cf spontaneous-fission distribution and 2500 b for the fission cross section. $6$  The highest point on the contour plot is on the line of mass symmetry at  $E_k = 223$  MeV. However, some asymmetric fission is still present in the form of the two ridges extending to lower energies. The data shown in Fig. 3 are entirely from the second run. Data from the first run showed prominent asymmetric peaks near  $E_k = 170$  MeV, which were attributed to uranium contamination. Therefore the Fm sample was chemically repurified before the second run. From trial subtractions of uranium counts from the observed distribution we conclude that less than 10% of the counts of the second run can be attributed to



FIG. 3. Same as Fig. 1 for thermal-neutron-induced fission of  ${}^{257}$ Fm. A total of 15900 net fissions are displayed.



FIG. 4. Mass distribution for the thermal-neutroninduced fission of  $^{257}$ Fm.

uranium. Even if present, uranium counts would not affect the high-energy symmetric component.

Figure 4 shows the counts versus mass for the thermal-neutron-induced fission of  $257$ Fm also taken from the second run. A neutron correction would tend to produce a triple-humped distribution, but again there is considerable uncertainty in estimating the neutron correction. The twoparameter plot (Fig. 3) is less affected, particularly the high-energy portion where fewer neutrons are emitted.

The degree of asymmetry,  $(M_2-M_1)/A$ , for spontaneous fission of  $257$ Fm agrees well with the value 0.10 derived from the systematic plot' of  $(M_2-M_1)^2/A^2$  vs  $Z^2/A$ . Also, the total pre-neutron kinetic energy of 191 MeV agrees with the value 192 MeV from the plot<sup>8</sup> of  $\overline{E}_h$  vs  $Z^2/A^{1/3}$ . We conclude that the mass and energy characteristics of the spontaneous fission of  $257$  Fm can be extrapolated from those of the other actinides except for the addition of a small amount of symmetric fission  $(20\% \text{ of the counts lie above } 205)$ MeV).

The neutron-induced fission exhibits clearly a larger amount of symmetric mass division than in spontaneous fission. The increase of symmetric fission with increasing excitation is similar to the known behavior in the fission of other nuclei. The neutron binding energy introduces 6 MeV of excitation. We estimate from systematics<sup>9</sup> that less than 10% of the  $258$ Fm nuclei fission from the ground state following radiative capture. Recent work of the Argonne group<sup>10</sup> shows that <sup>256</sup>Fm fissions asymmetrically, thus symmetric mass division in low-energy fission of heavy actinides appears abruptly at

 $257$  Fm.

A possible explanation of asymmetric fission has been suggested by Möller and Nilsson<sup>2</sup> who find a lower energy path around the second hump of the fission barrier when asymmetric drop distortions are included in a Strutinsky-type calculation. As heavier elements are approached, the calculated second hump becomes progressively lower in energy. Hence the onset of symmetric fission in  $257$ Fm may be related to the near absence of the second hump. Spontaneous fission of  $257$ Fm may be understood qualitatively by assuming that most of the time the system goes aroung the second hump with asymmetric distortions resulting in asymmetric fission. However, because of the low height of the second hump, sometimes the system tunnels directly through with symmetric distortions. For the neutron-induced fission of <sup>257</sup>Fm the additional excitation places the system above the second hump, facilitating direct passage over, with symmetric distortion. Recent extensive calculasymmetric distortion. Recent extensive calcu<br>tions by Pauli, Ledergerber, and Brack,<sup>11</sup> and by Gustafsson, Möller, and Nilsson, $^{12}$  show that the transition from asymmetric to symmetric mass division for even-even nuclei is expected at  $N \approx 160$ . Our results for <sup>258</sup>Fm ( $N = 158$ ) are consistent with these predictions and imply that nuclei with greater neutron numbers fission symmetrically.

The unusually high kinetic energy associated with the symmetric fission of  $^{257}$ Fm and  $^{258}$ Fm is probably related to the proximity of the fragment nucleon numbers  $Z = 50$  and  $N = 82$ . These nearly doubly magic fragments will be more resistant to deformation; hence relatively less energy will go into excitation, fewer neutrons will be emitted, and more energy will go into kinetic energy. The effect is particularly pronounced for symmetric fission of <sup>257</sup>Fm and  $^{258}$ Fm, since *both* fragments are magic or nearly magic.

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## Properties of Low-Density Neutron-Star Matter\*

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A nuclear Thomas-Fermi model is used to determine the ground state of matter at subnuclear densities allowing for inhomogeneities on a nuclear scale ("clusters"). The "neutron drip" phase transition occurs at  $3.7 \times 10^{11}$  g/cm<sup>3</sup>. As a function of the average density the proton number Z of the clusters first increases from  $Z \approx 29$  at  $1.4 \times 10^8$  g/cm<sup>3</sup> to a maximum of  $Z \approx 35$  at  $\sim 3.0 \times 10^{12}$  g/cm<sup>3</sup> and then decreases until the clusters gradually evanesce just above  $10^{14}$  g/cm<sup>3</sup>; this transition is smooth.

The present Letter reports on a study of the properties of cold catalyzed matter (CCM) for densities ranging from  $10^8$  to  $10^{14}$  g/cm<sup>3</sup>. By catalyzed we mean that matter is considered to be in equilibrium with respect to strong, electromagnetic, and weak interactions. An extensive study is dictated by the existence of neutron stars, a large part of whose interior falls into this density region.

At the very low densities CCM is composed of  $^{62}$ Ni nuclei (which have the highest binding energy per nucleon) imbedded in a sea of electrons. Above a density of  $\sim 1.0 \times 10^7$  g/cm<sup>3</sup> (which we refer to as region I), the chemical potential of the electrons is  $\geq 0.5$  MeV and it is energetically favorable to have neutron-enriched nuclei. Above a density of  $\sim 3.7 \times 10^{11}$  g/cm (region II), the chemical potential of the neutrons becomes positive so that no longer are all the neutrons clustered; in addition to the electron fluid there now is an imbedding neutron fluid. Above  $10^{14}$  g/cm<sup>3</sup> (region Ill), the inhomogeneities on a nuclear scale disappear and one is left with a mixture of neutrons, protons, and electrons, with other particles appearing at still higher densities.<sup>1-4</sup>

res appearing at stiff inglier defisities.<br>Previous authors<sup>1-4</sup> in their study of the composition of matter in regions I and II have relied heavily on the use of a semiempirical (four-parameter) mass formula  $B(Z,A)$ . Their approach

requires  $B(Z, A)$  to be applied and, more seriously, to be differentiated far away from the region of stability where it was fitted. The results one obtains with a mass formula are therefore not only quantitatively but also qualitatively very sensitive to the particular functional form of  $B(Z,A)$ , as has previously been pointed out.<sup>5</sup> It is thus important to develop a more fundamental approach which, while remaining tractable, allows the nuclei to change shape and to adjust to their environment, in particular to the imbedding neutron fluid. The natural basis for such an approach is provided by the nuclear Thomas-Fermi model. $6-10$ The latter is based on nuclear-matter results derived from a realistic nucleon-nucleon force, and has been quite successful in reproducing the bulk properties of ordinary nuclei,<sup>9,10</sup> which gives leo:<br>pro:<br>9,10 confidence in its application to the present problem. We should point out that in region II we do not have the difficulties associated with the neutron tail, characteristic of the Thomas-Fermi model for a self-bound system.

Preliminary results were reported in a previ-Preliminary results were reported in a prev<br>ous Letter.<sup>11</sup> The present calculation improve on those results in several respects: First, recent neutron-gas results<sup>12</sup> are used rather than extrapolated values; second, proton-electron and electron-electron Coulomb interactions are now included and are treated in a Wigner-Seitz-sphere