tion of the mass and width was that of the 1.51-GeV isobar since it lies on a relatively slowly varying background. The 1.23-GeV isobar lies on top of the rapidly rising 1π and 2π nonresonant background and the 1.67-GeV isobar lies near the limit of the instrumental acceptance. These factors introduce uncertainties in fitting the background under these peaks and hence also in their masses and widths.

In Table I the results of the present experiment are compared with the most recent preceding pp missing-mass experiment,[3] a recent πp missing-mass experiment,[6] and the results of πp phase-shift analyses.[7] The three missing-mass experiments are in good agreement, demonstrating that widths obtained in missing-mass experiments are narrower than widths obtained from πp phase-shift analyses.

We wish to thank Joe Abate, Dan Bunce, Dave Hartman, and Carl Muehleisen for their assistance and to express our gratitude to the staff of the Princeton-Pennsylvania accelerator. We also thank Dr. Claud Lovelace for useful discussions.

*Work supported by The National Science Foundation.

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Determination of the Photoproduction Phase of ϕ Mesons*

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(Received 28 May 1971)

We have measured wide-angle electron-positron pairs from the reaction $\gamma+\mathrm{C}\to\mathrm{C}+e^+ +e^-$ in the invariant-mass region $920<m<1080$ MeV/c^2 for incident photon energy $6<k<7.4$ GeV. The photoproduction amplitude of the ϕ meson was found to deviate from pure imaginary by $25°\pm15°$ corresponding to a ratio of the real to imaginary part of the ϕ-nucleon amplitude of $\beta=-0.48^{+0.33}_{-0.45}$. The forward photoproduction cross section $[d\sigma(\gamma+C \to C+\phi(\phi\to e^+e^-))dt]_{t=0}$ was found to be 96 ± 14 nb/(GeV/c)^2.

We determine the ratio of the real to imaginary part of the ϕ-nucleon amplitude, β, and the quantity $C_\phi=[d\sigma(\gamma+\mathrm{C}\to\mathrm{C}+\phi(\phi\to ee))/dt]_{t=0}$ by studying the e^+e^- yields from the reaction

$$\gamma+\mathrm{C}\to\mathrm{C}+e^++e^- \tag{1}$$

in the energy region 6.0–7.4 GeV and e^+e^- invariant-mass range $920<m<1080$ MeV/c^2. The motivation for measuring β and C_ϕ is as follows:

(1) The phase of the ϕN scattering amplitude, or the ratio of the real to the imaginary part of the amplitude β, has been of considerable theoretical interest. On the one hand, since the ϕN system does not couple to any of the known high-lying trajectories other than the Pomeranchukon,[1] one will expect the ϕN amplitude to be purely imaginary. On the other hand, according to the quark model[2] the ϕN amplitude is related to the $K^\pm N$, $\pi^\pm N$ amplitudes. Using the existing data on the $K^\pm N$, $\pi^\pm N$ amplitudes, various quark models[2]

have predicted β from −0.3 to 1.0.

(2) Knowledge of β is necessary in the derivation of ϕN total cross section $\sigma_{\phi N}$ and ϕ-photon coupling constant $\gamma_\phi^2/4\pi$ from measurements of photoproduction of ϕ mesons on complex nuclei.[3]

(3) It follows from μ-e universality that $\Gamma(\phi\to ee)$ should equal $\Gamma(\phi\to\mu\mu)$ apart from a small phase-space correction. Since the experiments on photoproduction of lepton pairs measure C_ϕ directly, a comparison between e^+e^- pair-photoproduction data and $\mu^+\mu^-$ pair-photoproduction data[4] around the mass of the ϕ checks μ-e universality at a distance of $\sim 10^{-14}$ cm.

To second order, the amplitude for Reaction (1) is

$$A_T = A_\rho(\gamma) + A_\phi(\gamma) + A_{\rm BH}(2\gamma) + A_{\rm BH}(3\gamma) + A_x(\gamma), \tag{1a}$$

where $A_\rho(\gamma)$ and $A_\phi(\gamma)$ are the diffractive photoproduction amplitudes of ρ and ϕ mesons decaying into e^+e^- via one photon. $A_{\rm BH}(2\gamma)$ is the ordinary Bethe-Heitler (BH) amplitude (which is real), where the final e^+e^- states are connected to two γ rays. $A_{\rm BH}(3\gamma)$ is the second-order BH pair amplitude in which the e^+e^- are connected to three γ rays. $A_x(\gamma)$ is the incoherent ρ,ϕ meson production amplitude. It follows from charge-conjugation invariance that $2|A_{\rm asy}|^2 = |A_T(e^+,e^-)|^2 - |A_T(e^-,e^+)|^2$ can come only from interference terms involving an odd number of photons[5]:

$$|A_{\rm asy}|^2 = 2\,{\rm Re}\{[A_\rho(\gamma) + A_\phi(\gamma)]A_{\rm BH}(2\gamma) + A_{\rm BH}(2\gamma)A_{\rm BH}(3\gamma)\}.$$

At high energy on complex nuclei in the region of the ϕ mass, one has

$$|A_{\rm asy}|^2 \simeq 2\,{\rm Re}\{A_\phi(\gamma)A_{\rm BH}(2\gamma)\}.$$

Since $A_{\rm BH}(2\gamma)$ is real, the measurement of asymmetric e^+e^- pairs yields information on the phase $ie^{i\varphi}$ of $A_\phi(\gamma)$.

The contribution to e^-e^+ yield from coherent ρ and ϕ production[6] is

$$A_{\rm V} = g_{\gamma \rm V}\frac{1}{m_{\rm V}^2}A(VA\to VA)D_{\rm V}\frac{g_{\gamma \rm V}}{-m^2}A(\gamma\to e^+e^-);\quad D_{\rm V} = (m_{\rm V}^2 - m^2 - im_{\rm V}\Gamma_{\rm V})^{-1},$$

where $g_{\gamma\rm V} = em_{\rm V}^2/2\gamma_{\rm V}$ are the vector-meson-photon coupling constants, Γ_V is the width of the resonance, and $A(\gamma\to e^-e^+)$ is the amplitude for $\gamma\to e^-e^+$ pairs. The contribution of the Compton process then is[7]

$$\sigma_{\rm C} = \frac{d\sigma}{dE_+dE_-d\Omega_+d\Omega_-} = \frac{4\alpha}{\pi^2}\,e^{at}|\vec{\rm p}_+ + \vec{\rm p}_-|\frac{E_+E_-}{m^4}(E_+E_- - p_{+z}p_{-z})S(k)|\Lambda_1|^2. \tag{2}$$

The interference between the BH and Compton processes is then described by the cross section[7]

$$\sigma_i = \frac{d\sigma}{dE_+dE_-d\Omega_+d\Omega_-} = \frac{Z\alpha^2}{\pi^2}G_E(t)E_+E_-\frac{e^{at/2}}{t}\frac{1}{m^2}\,S(k)\,[{\rm Re}(\Lambda_1)]\Lambda_2 \tag{3}$$

with

$$\Lambda_1 = ie^{i\varphi}\Lambda_0;\quad \Lambda_0 = g_{\gamma\phi}\Sigma_\phi D_\phi + g_{\gamma\rho}\Sigma_\rho D_\rho\exp[i(\varphi_\rho - \varphi)];$$

$$\Lambda_2 = 2m^2\left(\frac{E_-}{k\cdot p_+} - \frac{E_+}{k\cdot p_-}\right) + 2\left(\frac{1}{k\cdot p_+} + \frac{1}{k\cdot p_-}\right)\left[\frac{m^2}{2}(E_+ - E_-) + E_-k\cdot p_+ - E_+k\cdot p_-\right]$$
$$- \frac{2}{M}(p_{+x}p_{-x} + p_{+y}p_{-y})\left(\frac{Q\cdot p_+}{k\cdot p_-} - \frac{Q\cdot p_-}{k\cdot p_+}\right),$$

where

$$\Sigma_V = [d\sigma(\gamma A\to VA)/dt|_{t=0}]^{1/2},$$

φ_ρ is the production phase of the ρ meson, Z is the charge of the target, k is the photon four-momentum, $p_\pm$ is the four-momentum of the $e^\pm$, $E_\pm$ is the energy of the $e^\pm$, Q is the recoil four-momentum of nucleus, $t = (k - p_+ - p_-)^2$, $S(k)$ is the bremsstrahlung energy spectrum, and $G_E(t)$ is the elastic form factor of the target. The metric is $g_{00} = 1$, $g_{ii} = -1$ $(i = 1, 2, 3)$, with the z axis defined to be the beam direction. The effect of the ω contribution is $\leq 1\%$ and has been omitted.

As can be seen from Eq. (3), the interference cross section σ_i is antisymmetric under exchange of the four-momenta of the electron p_- and positron p_+. Thus the effect of the interference term is to produce an asymmetric distribution of experimental events as a function of variables antisymmetric in p_+ and p_-. Such an asymmetry is a measure of σ_i and therefore of φ.

Asymmetries introduced by the spectrometer are removed by taking equal amounts of data for each polarity of the spectrometer.

The experiment was done using the DESY–Massachusetts Institute of Technology spectrometer. The apparatus and experimental procedure were the same as described earlier.[7] Symmetric spectrometer settings for the two arms were found to be the optimum condition for running because of the large acceptance of the spectrometer in $p_\pm$ and $\theta_\pm$. The target was chosen to be 1.5 cm carbon. The data were collected with $k_{max} = 7.4$ GeV, central electron momentum 3.350 GeV/c, and central electron angle $\theta_0 = 8.6°$ and 8.8°. The pions and the muons from the decay of pions were rejected by four large-aperture Cherenkov counters with a rejection efficiency of better than 10^{-7}. To check the absolute normalization of the detecting system, we measured the e^+e^- pair yield at $\theta_0 = 4°$ to be within 3% of the quantum electrodynamics prediction. The data are corrected for target out (4%), bremsstrahlung loss, dead time, accidentals (8%), etc.

In order to describe the results of the measurements, we adopt the following notation: The subscripts "+" and "−" denote the sign of the charge of the lepton passing through the right arm of the spectrometer. The experimental results for $N_+(m)+N_-(m)$ are presented in Fig. 1 and the experimental results for $N_+(m)-N_-(m)$ are given in Fig. 2. The total number of e^+e^- pair events is 390 among which 242 are attributed to the BH process and 46 events are attributed to target-out and accidental events. The analysis was done in two steps.

To compare the observed spectrum of Fig. 1 with Eq. (1a), we fit the symmetric data with the contribution from ρ, ϕ production and the BH process. The BH contributions are calculated using the measured elastic form factor on C and the inelastic form factor from the Drell-Schwartz sum rule.[8]

Taking[6,7] $\Gamma_\phi = 4.0$ MeV, $m_\rho = 765$ MeV, $\Gamma_\rho = 130$ MeV, $\gamma_\rho^2/4\pi = 0.5$, $[d\sigma(\gamma C \to C\rho)/dt]_{t=0} = 11.0$ mb/(GeV/c)^2, $\varphi_\rho = 12°$, and the slope of the diffraction peak of the ϕ meson $a = 58$ (GeV/c)$^{-2}$, we fit for $C_\phi(k)$, the resolution of the pair mass Δm, and m_ϕ. The best values are (Fig. 1)

$$m_\phi = 1021.0 \pm 1.5 \text{ MeV},$$

$$\Delta m = \pm 7.0^{+2.9}_{-1.9} \text{ MeV},$$

$$C_\phi = 96 \pm 14 \text{ nb/(GeV/}c)^2.$$

Using these fitted values, we compare the data in Fig. 2 with Eq. (3). The fitted parameter is φ. We find

$$\varphi = 25° \pm 15°.$$

The error is statistical only. The sensitivities of the fit results to various input parameters are shown in Table I. As seen, the result for the

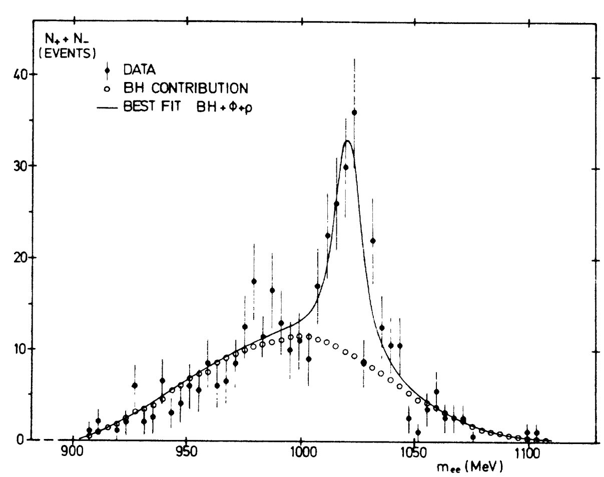

FIG. 1. Observed e^+e^- spectrum from Reaction (1). The curve is the best fit of the data with Eq. (1a) with $m_\phi = 1021.0 \pm 1.5$ MeV, $\Delta m = \pm 7.0^{+2.9}_{-1.9}$ MeV, and $C_\phi = 96 \pm 14$ nb/(GeV/c)^2.

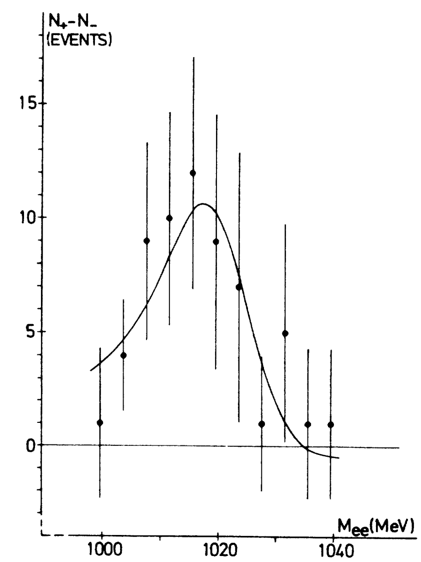

FIG. 2. Spectrum of interference events. The curve is the best fit of the data with Eq. (3), from which one obtains $\varphi = 25° \pm 15°$.

TABLE I. Sensitivities of the fitted results to input parameters.

Fit	δm_ϕ (MeV)	$\delta\Delta m$ (MeV)	δC_ϕ [nb/(GeV/c)^2]	$\delta\varphi$ (deg)
Without ρ contribution	+1.7	+1.8	+10	+7°
Binning shifted by +2 MeV	+0.2	+1.9	± 0	+3°
Include incoherent part (5%)	±0	+0.0	− 5	±0
Slope $a=47$ ($\Delta a=-11$) (GeV/c)$^{-2}$	±0	+1.0	−11	+1°
Slope $a=70$ ($\Delta a=12$) (GeV/c)$^{-2}$	+0.3	+0.4	+10	+1°
Fixed mass resolution to ±4.5 MeV	−0.2	⋯	−11	−5°

production phase is very insensitive to variation of the input parameters used, while the normalization constant C_ϕ is sensitive (to 10% level) to reasonable variation to input parameters.

The measured mass of the ϕ meson is compatible with the world average value $m_\phi = 1019.5 \pm 0.5$ MeV.[9] Since this average value also has some systematic uncertainty greater than the indicated error, no correction to the mass measurement is made. The measured mass resolution $\Delta m = \pm 7.0^{+2.9}_{-1.9}$ MeV is also consistent with the calculated value, $\Delta m = \pm 4.5$ MeV for this spectrometer, using a Monte Carlo technique. The value $C_\phi(e^+e^-) = 96 \pm 14$ nb/(GeV/c)^2 compares very well with a value obtained previously by this group[6,10] $C_\phi(e^+e^-) = 99 \pm 27$ nb/(GeV/c)^2.

With the colliding-beam value[11] of $\gamma_\phi^2/4\pi = 3.2$ we obtain the photoproduction cross sections, in μb/(GeV/c)^2, for the 6.7-GeV reaction $\gamma + \mathrm{C} \to \mathrm{C} + \phi$: $(d\sigma/dt)_{t=0} = 270 \pm 40$ or $(d\sigma/dt)_{\theta=0°} = 190 \pm 30$, in good agreement with the earlier DESY values from the 5.2-GeV $\gamma + \mathrm{C} \to \mathrm{C} + K^+ + K^-$ experiment.[6] The $\theta = 0°$ value is also in agreement with the Cornell 6.4-GeV photoproduction experiment.[6]

Conversely, using our production cross section on carbon, we obtain a branching ratio $R = \Gamma(\phi \to ee)/\Gamma(\phi \to \mathrm{all}) = (2.8 \pm 0.4) \times 10^{-4}$ and $\gamma_\phi^2/4\pi = 4.0 \pm 0.7$, comparable with our earlier value[10] of $R = (2.9 \pm 0.8) \times 10^{-4}$ and the Orsay colliding-beam value of $\gamma_\phi^2/4\pi = 3.2 \pm 0.3$.

A comparison of our data with that of Hayes *et al.*[4] shows a difference of 2 standard deviations if one assumes μ-e universality.

For light nuclei the effect of nuclear physics is small, and using the Margolis multiple-scattering theory[3] we relate the production phase angle φ on C to that on a nucleon, thus to β, the ratio of the imaginary to the real part of the ϕ production amplitude on a nucleon. We find

$$\beta = -0.48^{+0.33}_{-0.45}.$$

In conclusion, our measurement indicates that the ratio of the real to the imaginary part of the ϕ-N scattering amplitude is negative, 1.5 standard deviations from zero.

We are grateful for the support of W. Jentschke, V. F. Weisskopf, P. Demos, A. G. Hill, H. Joos, and G. Weber, who made this collaboration possible. We also thank E. Lohrmann, M. Krammer, and P. Biggs for interesting comments. We are grateful to D. Lublow, H. Kumpfert, Mrs. H. Feind, Miss I. Schulz, and Mr. P. Berges for technical assistance.

*Work supported in part through funds provided by the U. S. Atomic Energy Commission under Contract No. AT (30-1) 2098.

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Multiperipheral-Model Predictions Concerning Pion Production in High-Energy Collisions*

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The pion-exchange multiperipheral model is used to predict the amount and spectrum of pion production at small c.m. momenta in high-energy collisions. The transverse momentum distribution, which agrees with the present experimental data, is predicted to remain unchanged at all higher energies, but the number of pions produced per unit volume of phase space, which is related to the average multiplicity, is predicted to decrease.

There has recently been a revival of interest in the theoretical development[1] and phenomenological application[2] of the pion-exchange multiperipheral model originally proposed by Bertocchi, Fubini, and Tonin, and Amati, Stanghellini, and Fubini in 1962 (the ABFST model).[3] The results of these studies have been generally encouraging, and there appears to be some possibility that the model will actually be able to account for the majority of inelastic events at the very high energies that will soon be experimentally accessible.

The purpose of this article is to present and discuss the predictions of the ABFST model concerning inclusive pion production in a kinematic region that will become more and more important at higher energies, namely, the region in which the pion energy is only a small fraction of the total available energy in the overall c.m. system. We shall refer to this loosely as the region of "small c.m. momenta," although the highest pion momenta included may actually be quite large [a fixed small fraction of $s^{1/2}$, the c.m. energy] at very high energies. This is an important region for multiperipheral models because its growth with increasing energy is supposed to give rise to a logarithmically increasing multiplicity of secondaries, a prediction most characteristic of these models and at present in agreement with experiment.[4]

If pion exchange is to be the dominant multiperipheral mechanism, it should give a detailed account of the production of pions in this limited kinematic region at small transverse momenta, which are associated with small momentum transfers, as it does in the case of low-energy processes that are dominated by single pion exchange.[5] The predictions provide a good test of the essential features of the model, and any shortcomings will lead to a disagreement with experiment that becomes more serious with increasing energy.

A striking consequence of the hypothesis of multiperipheral pion exchange is that secondary pions must be produced in pairs of predominantly low invariant mass. Since most of the cross section for low-energy $\pi\pi$ scattering is associated with the ρ resonance, this means that most secondary pions result from the decay of ρ mesons. Accordingly, the principal features of the model should be represented by the diagram shown in Fig. 1, and we shall adopt this diagram as our basis of discussion.

A problem that must be faced at the outset, and which always arises in the ABFST model, is that the vertices in Fig. 1 refer to scattering processes in which one or two particles are not on the mass shell. For several reasons this is not a serious difficulty here. Consider first

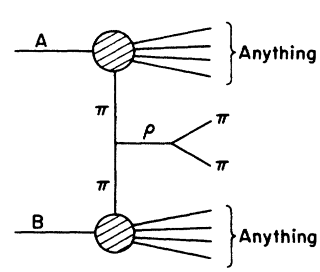

FIG. 1. The multiperipheral diagram to be considered.