Experimental Verification of Negative Faraday Rotation

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Magnetoionic theory predicts that positive Faraday rotation, i.e., the direction without collisions, decreases to zero near $\nu/\omega = 1$ and becomes negative for $\nu/\omega > 1$. Using a 35-GHz microwave beam propagating parallel to the magnetic field and the precursor plasma from an exploding wire, rotation of an initially plane wave is measured throughout the region $\nu/\omega \lesssim 1$. The negative Faraday effect, as well as the crossover from positive to negative with increasing collisions, is qualitively verified.

In a collisional plasma, magnetoionic theory predicts that the Faraday rotation of a plane polarized electromagnetic wave can be positive, zero, or negative depending on the ratio of the collision frequency ν to the radian wave frequency.[1] Here "negative" means only that the sense of rotation is opposite that of the zero-collision case. (Changing the direction of the magnetic field will also alter the sense of the Faraday rotation. In these studies the field is generated by an electromagnet with a constant polarity.) Figure 1 illustrates the theory for a particular value of magnetic field and electron density. When the absorption indices of the two circular polarizations differ, an initially plane wave becomes elliptical. Normally the major axis of the ellipse dominates measurements of rotation. The effect of the minor axis can be accounted for in the measurements (see Ref. 1, Appendix A and Fig. 2).

The curious behavior of the rotation when $\nu/\omega \simeq 1$ is of interest both on physical grounds and as a measurement tool. Physically, it is easy to understand the collisional interruption of the electron Larmor orbits, causing the refractive indices for the ordinary and extraordinary waves to approach each other with increasing collision frequency.

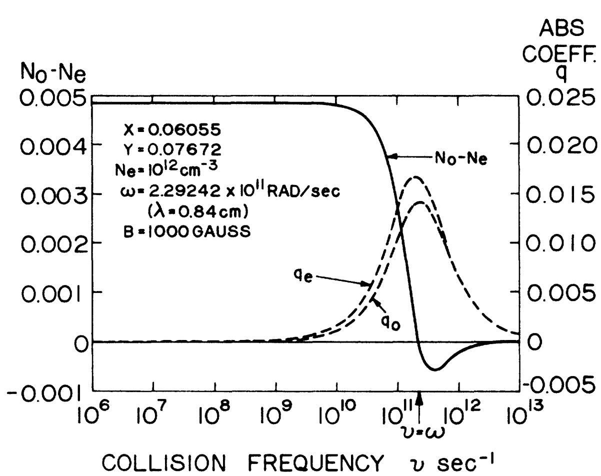

FIG. 1. The solid curve is the difference between the refractive indices $(n_o - n_e)$ for propagation along the magnetic field. The dashed lines labeled q_o, q_e are the absorption coefficients. All calculations are made with the standard magnetoionic equation. This graph is explained in detail in Ref. 1.

Explaining the negative overshoot beginning at $\nu \cong \omega$ is more difficult. Normally the idealized picture of collisions interrupting electron gyromagnetic orbits would lead to a monotonic decrease in rotation, with the angle approaching zero as the collision frequency becomes very large. Thus, the surprising activity predicted in the refractive index difference function $(n_o - n_e)$ argues for experimental verification.

From the measurement standpoint, it is always important to have reference points, such as the rotation crossover at $\nu \cong \omega$ and the slope change at $\nu \cong 1.7\omega$. Since the total rotation is small in these areas, with a resulting loss of precision, sign changes in the rotation or its slope can serve to locate particular points uniquely.

The arrangement used in these experiments is the same as shown in Ref. 1, Fig. 3. An exploding wire is aligned along a magnetic field, with a 35-GHz microwave beam parallel to the wire, but displaced outward several centimeters. As the wire explodes, it releases an ultraviolet pulse, weakly ionizing the gas in the chamber. This precursor ionization has been studied in detail and differs from gas to gas.[2] It was decided to utilize CO_2 as the test gas because of the high collision frequency.[3] The level of ionization attainable is adequate to insure sufficient rotation and the collision frequency is high enough so that the electron temperature is the same as the gas temperature during the measurements.

With only the precursor ionization to cause rotation, coupled with the small rotation magnitude near $\nu = \omega$, a number of separate measurements at various pressures would be required

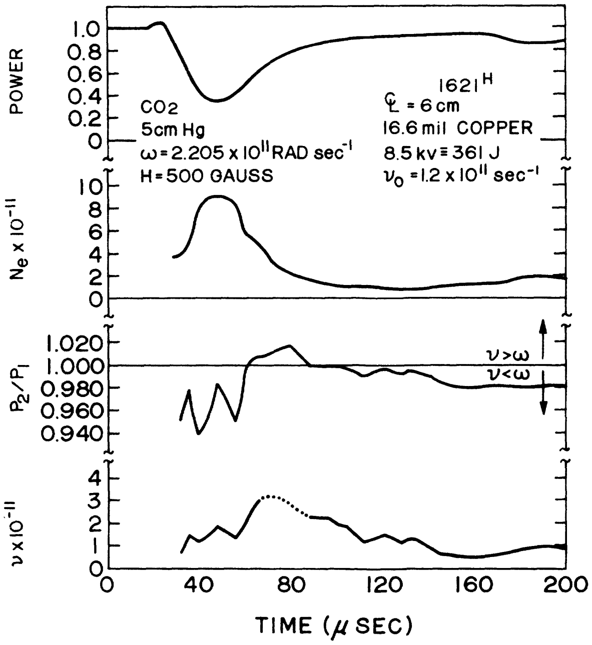

FIG. 2. This set of measurements were taken with the experimental conditions shown near the top curve. The experimental setup is the same as reported in Ref. 1.

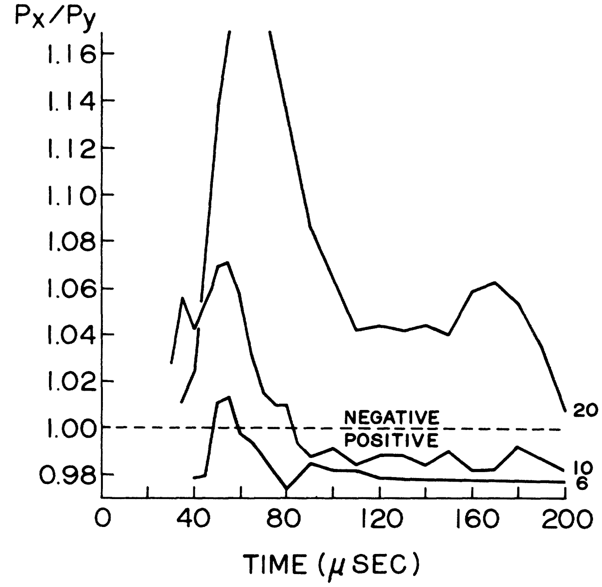

FIG. 3. Faraday rotation is shown for three pressures, 200, 100, and 60 Torr. Otherwise, experimental conditions are the same as for Fig. 2.

to verify the crossover at $\nu \cong \omega$. It was decided to monitor the rotation caused by the precursor plasma during the passage of a weak shock wave. The shock compresses the gas, raising the collision frequency, then relaxes back. By setting the initial gas pressure so that ν_0 is somewhat less than ω, the rotation from the precursor ionization is initially positive, then goes negative as the shock comes through, finally relaxing back to positive.

Figure 2 illustrates the precursor ionization with superposed shock situation, outlined above. Experimental conditions are shown in the upper curve: The microwave beam ($\lambda = 0.84$ cm, half-power beam width ~ 1.1 cm) is displaced 6 cm from the wire axis; the gas is CO_2 at a pressure of 50 Torr and a collision frequency of 1.2×10^{11} $\sec^{-1}$ (with a relaxation time for the precursor electron temperature of much less than 1 μsec[4]); the energy of 361 J is quite low, insuring a weak shock.[5] The upper curve is the net microwave power at the receiver. The small bump that rises above 100% power is believed to be caused by microwave reflections from the approaching shock front. The second curve shows the time evolution of the electron density. In the third curve, the ratio of the power on the two arms of the polarimeter is rough near the start, again probably from reflections from the shock. However, there is clearly a crossover from positive (below the curve) to negative (above the curve) and then back to positive. The bottom curve gives the time history of the collision frequency.

A set of three separate experiments is shown in Fig. 3. Again, the polarimeter is set up so that rotation into arm P_y ($P_x/P_y < 1$) is the situation experienced when there are no collisions. The experimental parameters are also the same as for the previous figure. Labels on the curves (20, 10, and 6) refer to the initial gas pressure in centimeters of mercury. For 200 Torr, the electron-neutral collision frequency is 4.8×10^{11} $\sec^{-1}$, about twice the radian wave frequency. Under these conditions one would expect all rotation to be negative unless the expansion phase of the shock reduced the density sufficiently to cause ν to drop below ω. The overpressure at the front is

$$\frac{\Delta P}{P_0} = \frac{2\gamma}{\gamma+1}(M^2-1),$$

where γ is the adiabatic constant and M is the Mach number. For a weak blast wave, as in this experiment, the N-wave analysis is approximately correct.[6] That is, the compression phase is immediately followed by an expansion phase that is equal in magnitude. For instance, when $M = 1.1$, $\Delta P = 0.24P_0$ indicating that the collision frequency at the front has increased by

24% over the initial preshock value ν_0 with a like 24% decrease from ν_0 at the tail. This effect is illustrated by the middle curve, 100 Torr, with a $\nu_0/\omega = 1.1$. The bottom curve for 6 cm Hg again illustrates the case of mostly positive rotation, since initially $\nu_0/\omega = 0.65$.

Although the results in Fig. 2 and 3 show crossover from positive to negative Faraday rotation, it still remains to find the change in slope in $n_0 - n_e$ at $\nu/\omega = 1.7$ experimentally. This change in slope may exist in the data of the 200-Torr curve of Fig. 3. However, the qualitative nature of the experiments and the changing nature of the collision frequency and electron density as they were shocked precluded locating that point.

[1]D. L. Jones, in *Exploding Wires*, edited by W. Chace and H. Moore (Plenum, New York, 1968), Vol. 4, p. 96, Fig. 1.

[2]D. L. Jones, Phys. Fluids 11, 247 (1968).

[3]F. C. Fehsenfeld, J. Chem. Phys. 39, 1653 (1963).

[4]B. W. McDaniel, *Collision Phenomena in Ionized Gases* (Wiley, New York, 1964), p. 19.

[5]D. L. Jones and R. M. Gallet, in *Exploding Wires*, edited by W. Chace and H. Moore (Plenum, New York, 1962), Vol. 2, p. 127 ff.

[6]L. D. Landau, in *Collected Works of Landau*, edited by D. ter Haar (Gordon and Breach, New York, 1965), p. 437.

Nuclear-Spin-Ordering Effects in Magnetized Solid ^{3}He†

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The thermodynamic pressure $P_V(T,H)$ has been measured in solid ^{3}He in the presence of high magnetic fields. Modification of nuclear-spin ordering by the field is observed, indicating antiferromagnetic behavior. The results indicate that the behavior is not quantitatively as expected from the Heisenberg model. A depression of the melting curve in a field has also been observed.

We have measured the thermodynamic pressure $P_V(T,H)$ in solid ^{3}He for molar volumes $V \simeq 24$ cm^3/mole in applied magnetic fields of 60, 40, and ~0 kG. In addition to the specific interest in magnetized solid ^{3}He,[1-4] this study provides information of use in extending our understanding of fundamental problems in magnetism.

The major results of this study are as follows: (1) An applied field has a pronounced effect on the thermodynamic properties of solid ^{3}He. This is the first time these effects have been seen. In a recent study no effect of the field was observed.[5] (2) Time constants are short enough to permit solid ^{3}He to be magnetized in high fields. (3) The exchange energy J is negative, corresponding to antiferromagnetism. (4) The behavior at high magnetic fields is not adequately described by the series expansions[6] for the Heisenberg Hamiltonian.

In solid ^{3}He the magnetic properties of the nuclear spin system are determined by the exchange interaction which arises from the large zero-point motion and the resulting overlap of wave functions of neighboring atoms. An interesting consequence of this is that there should be a transition to a magnetically ordered state at $T_c \simeq 3|J|/k$, where J is the exchange energy. Measurements of $P_V(T)$ in zero magnetic field have been used to determine $|J|$.[7] For the solid near melting with $V \simeq 24$ cm^3/mole, it was found that $|J|/k \simeq 0.7$ mK or $T_c \simeq 2$ mK.[8] Recently, susceptibility measurements have shown that J is negative, corresponding to antiferromagnetic ordering.[9,10]

Solid ^{3}He has been considered to be probably the best example of a Heisenberg antiferromagnet because of the absence of interactions other than exchange and the symmetry provided by the bcc lattice. In order to make a detailed comparison between theory[1,6] and experiment, the measurements in zero magnetic field need to extend well into the critical region near $T_c = 2$ mK. However, as Goldstein[1] has pointed out, a detailed comparison should be possible in the paramagnetic region at temperature well above T_c if one applies a large magnetic field.

Taking the Heisenberg Hamiltonian to represent the exchange interaction, the Hamiltonian for the system is[11]

$$\mathcal{H} = -2J \sum_{i<j}^{N} \vec{\mathrm{I}}_i \cdot \vec{\mathrm{I}}_j - H \sum_{i}^{N} \mu_{zi}, \tag{1}$$

where $\vec{\mathrm{I}}$ is the nuclear spin operator, H the mag-