Beyond One-Particle Exchange*

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Double-particle exchange in hadronic interactions is investigated with an extension of our strong-absorption model. All parameters are taken from single-particle–exchange fits. Spin effects, variations in coupling values, and unavoidable cancelations result in a cross-section range of $\sim 10^5$ for different forward reactions, in good agreement with existing data. Reactions with exotic–quantum-number exchange are small because of spin and symmetry coupling properties of particles, while some nonexotic double-particle exchanges are quite large.

All two-body hadronic reactions may be divided into three types at high energy.[1] In the first type no internal (nongeometric) quantum numbers are exchanged; this type is dominated by the Pomeranchukon, whose nature is still not known. In the second type the exchanged quantum numbers correspond to those of some low-mass particle. This type is dominated by Regge-pole and Regge-Pomeranchukon–cut exchanges. The third type, with which we deal in this Letter, is scattering in which the exchanged quantum numbers correspond to the sum of the quantum numbers of two particles. It is natural to expect this third type to be dominated by the two-Reggeon and two-Reggeon–Pomeranchukon cuts.

These three types of scattering are ordered in size: Pomeranchukon exchange is largest, Regge exchange smaller, and double-Regge exchange still smaller. In any reaction all allowed mechanisms occur. Thus an understanding of double-particle exchange (DPE) is important for all exchange processes.[2]

Because the two-Reggeon exchange is the smallest, the experimental data are not as good as for the other types of reactions. It may appear reasonable only to attempt a rough estimate of the size of the amplitude. There are reasons, however, why such a procedure gives a very inaccurate picture of double-Reggeon exchange. First, since two Reggeons are exchanged, there are four vertices. Thus, the variation in coupling strengths is relatively more important. Second, there are usually several intermediate states to be summed over. The interference between these states can lead to a significant difference in the size of the cross section. Finally, for single-Reggeon exchange some of the principal features of the amplitude are due to spin effects. It is very possible for spin effects to be even more important in double-Reggeon exchange.

The model we propose[3] for calculating double-Reggeon exchange includes (1) a double-scattering convolution of the same type as is used in the absorption model[4], (2) a sum over intermediate states, and (3) inclusion of absorption effects. We carry out a calculation (corresponding to a diagram as in Fig. 1) as follows: We first transform the Regge-pole amplitudes $R_1(t)$ and $R_2(t)$ to impact-parameter space, obtaining $\tilde{R}_1(b)$ and $\tilde{R}_2(b)$, respectively. Then we form the product $\tilde{M}(b) = -i\tilde{R}_1(b)\tilde{R}_2(b)\tilde{S}_{\text{eff}}(b)/8\pi s$, where $\tilde{S}_{\text{eff}}(b)$ is the transform of the effective elastic S matrix. Finally, we transform back to momentum-transfer space, sum over all intermediate states that are calculable, and sum the two possible orderings of different Reggeons.

The output amplitude consists of two Regge-cut exchanges, with the branch points given by the Amati-Fubini-Stanghellini (AFS) rule for two-Reggeon and two-Reggeon–Pomeranchukon cuts.

The output amplitude is insensitive to the *detailed* t dependence of the input Reggeons.[5] Thus

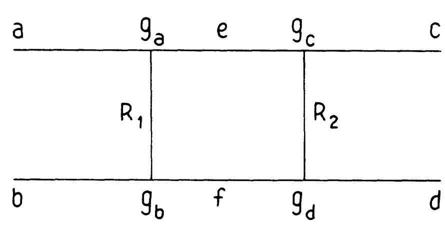

FIG. 1. Diagrammatic representation of double-particle exchange. The reaction $ab \to cd$ has an intermediate state ef. Reggeons R_1 and R_2 are exchanged. The g's are coupling constants.

we make the following approximations for the s-channel helicity amplitudes:

$$R(t)=g_a\left(\frac{\sqrt{-t}}{2M}\right)^{n_a}g_b\left(\frac{\sqrt{-t}}{2M}\right)^{n_b}\frac{s_0}{m_x^{\,2}}\left(\frac{s}{s_0}\right)^{\alpha(0)}e^{At}\exp(i\pi\alpha' m_x^{\,2}/2) \tag{1}$$

(except for pion exchange because of the small mass of the pion) and $\tilde{S}_{\rm eff}(b)=1-\exp(-b^2/4A')$, with A' the slope of an elastic amplitude; g_a and g_b are dimensionless coupling constants, and n_a and n_b are the helicity flips at the vertices. The factors $(s_0/m_x^{\,2})(s/s_0)^{\alpha(0)}\exp(i\pi\alpha' m_x^{\,2}/2)$ come from the extrapolation from the pole at $t=m_x^{\,2}$ to $t=0$. Finally, all of the t dependence is approximated by e^{At} [e.g., $A=\alpha'\ln(s/s_0)-i\pi\alpha'/2$]; note that A increases with s. The form for $\tilde{S}_{\rm eff}$ is similar to that used in the strong absorptive Regge-cut model, with a λ factor of about 1.5. With these approximations, all the transforms can be done analytically.

For Regge-pole exchange with the helicity flip *at each vertex* equal to 0 or 1, the spin properties can be classified into four classes: $n_a=n_b=0$, nonflip, symbol N; $n_a=1$ and $n_b=0$, or $n_a=0$ and $n_b=1$, flip, symbol F; $n_a=n_b=1$, no net flip, evasive, symbol E; and $n_a=n_b=1$, helicity flip equals 2, double flip, symbol D. These can be combined into various kinds of cut amplitudes which can be labeled by the spin properties of each exchange.

A nonflip amplitude can be formed as NN, FF, NE, etc. A flip amplitude can be formed as NF, EF, etc. Let us define $A''=(1/A+1/2A')^{-1}$, and

$$X=\frac{-ig_ag_bg_cg_ds_0(s/s_0)^{\alpha_1(0)+\alpha_2(0)-1}\exp[i\pi\alpha'(m_{1x}^{\,2}+m_{2x}^{\,2})/2]}{32\pi m_{1x}^{\,2}m_{2x}^{\,2}A}. \tag{2}$$

Some s-channel helicity amplitudes are the following: nonflip NN,

$$M=X[e^{At/2}-(A''/A)e^{A''t/2}]; \tag{3}$$

nonflip FF,

$$M=(X/8M^2A)[e^{At/2}(1+At/2)-(A''/A)^2e^{A''t/2}(1+A''t/2)]. \tag{4}$$

In our normalization $d\sigma/dt=0.39\sum|M|^2/64\pi q^2s$ mb/GeV^2, where the sum is over the independent helicity amplitudes included. Similar equations can be obtained when a pion is involved, and for all other amplitudes.

Some of the parameters in our model have well-established values; all others are taken from strong-absorptive Regge-cut model fits.[6] Thus there are no new parameters in this model. A very important fact is that, *for equal values of the couplings, the FF amplitude is smaller than the NN amplitude at t=0 by a factor* $(1+A''/A)/8M^2A=0.10$. Thus, in the cross section, the FF contribution is *two orders of magnitude* smaller than would be estimated by neglecting spin effects. *The absorption reduces the NN amplitude by about a factor of 4 at t=0,* and the FF amplitude by about a factor of 2.5. The important factor of 10 between NN and FF amplitudes is the product of a factor of about 18 due to spin effects, and a factor of about 1/1.75 due to absorption.

Example (1), calculations at 5 GeV/*c*.—The largest DPE we have found is the double ω exchange in $\bar{p}p\to\bar{p}p$, with e and f being nucleon lines in Fig. 1, an NN reaction. Using standard couplings in Eqs. (2) and (3) gives a cross section $d\sigma/dt$ at $t=0$ of about 15 mb/GeV^2 (to be compared with about 50 mb/GeV^2 from the absorbed single ω exchange and about 400 mb/GeV^2 for the actual cross section). Such large DPE contributions will give rise in many reactions to large differences in particle-antiparticle cross sections[7] such as $K^{\pm}p\to K^*\Delta$ or $pp\to\Delta p$ vs $\bar{p}p\to\bar{\Delta}p$, as is observed. Similarly, such large DPE will lead to significant effects[8,7] on polarizations and cross sections for line-reversed reactions. On the other hand, in cases such as $pp\to pp$ or $K^+p\to K^+p$, where it is known that the single-ω exchange is largely canceled by P' exchange, one should think of the exchanges R_1 and R_2 in Fig. 1 as $\omega\pm P'$. Thus there is a large cancelation and one does not expect DPE to cause any rapid variation in the s-channel exotic total cross sections. Similar cancelations occur elsewhere.[7]

Example (2).—The exotic-exchange process $\bar{p}p\to\bar{\Sigma}^+\Sigma^-$ has several significant contributions, R_1 and R_2 being K^* and ρ and e, f being various baryon lines. All are of order 0.1–0.2 μb/GeV^2 for $d\sigma/dt$ at $t=0$ so we expect a net contribution of ~1 μb/GeV^2, a little smaller than the observed value. NN and NE contributions are similar in size.

Example (3).—For the backward exotic reaction $K^-p \to pK^-$ there is a sizable NN contribution from the diagram, e and f being Σ^+ and π^-, and R_1 and R_2 being Δ^{++} and K^{*0}. Ignoring possible cancelations we get from Eqs. (2) and (3) $(d\sigma/dt)_{t=0} \approx 0.7$ μb/GeV^2 to be compared with the experimental result of 0.2 μb/GeV^2.[9]

Example (4).—Next we consider $K^-p \to \pi^+\Sigma^-$, including vector and tensor exchanges, and baryon and decuplet lines for f. Here the NN contribution is considerably suppressed due to the values of the couplings. Thus the biggest contribution is FF, with its associated suppression of about 10^2 in $d\sigma/dt$. In addition, there is a cancelation between the contributions with octet baryons and decuplet as line f, giving $(d\sigma/dt)_{t=0} \approx 0.45\ (1 - g^2/500)^2$ μb/GeV^2, where g^2 is the $\rho N\Delta$ coupling. For g^2 given its expected value of about 22, this contribution is extremely small. Thus we expect $d\sigma(K^-p \to \pi^+\Sigma^-)/dt$ to be less than $\frac{1}{2}$ μb/GeV^2 but it is not easy to say how small; other contributions have to be calculated also. This is in agreement with the experimental data.[10]

Example (5).—A recent experiment[11] at 8 GeV/c finds the ratio $[d\sigma(\pi^+n \to \rho^0\Delta^+)/dt][d\sigma(\pi^+n \to \rho^+\Delta^0)/dt]^{-1} \equiv R$ to be 2.4 ± 0.5 times the expected value for $I=1$ exchange near $t = t_{min}$, decreasing to the expected ratio for $-t \gtrsim 0.3$. This reaction can be qualitatively described (magnitude, shape, and density matrices) by π exchange in a single helicity amplitude (no flip at either vertex). Indeed, we find that a pure $I=2$ π-p DPE with the phase of Eq. (2) interferes with the absorbed π exchange in a way that gives the observed effect, both in magnitude and in t dependence.

We are in the process of examining other reactions, including those in which several helicity amplitudes are important, to examine the size, phase, and t dependence of exotic exchanges.[7]

The large DPE such as the $\omega\omega$ cut have the quantum numbers of the vacuum, while the smaller ones have exotic quantum numbers. We can examine exotic cuts in more generality to see if they are systematically suppressed. All high-lying $I=1$ trajectories, the ρ, the A_2, and the π, are mainly helicity flip at a baryon vertex. Thus $I=2$ and $I=\frac{3}{2}$ (exotic) nonflip amplitudes are FF type and are suppressed by the factor $1/8M^2A$.

So we see that there is a systematic suppression of exotic cuts. This important suppression is largely a consequence of the spin structure of the ρ and A_2 couplings.

If the exotic cut were large, then the threshold cuts associated with the Regge cut passing through integers would also be large. This large scattering would very likely lead to resonances, in this case exotic resonances. This argument also works in reverse, so that small cuts are associated with the lack of resonances and large cuts are associated with the presence of resonances. Thus suppression in size of exotic cuts below nonexotic cuts is associated on the one hand with spin properties of Reggeons and on the other hand with the absence of exotic mesons. We have, therefore, a connection between spin effects and the nonexistence of exotic particles.

Although they are of considerable practical value, the calculations described in this paper cannot be quantitatively accurate, because in some reactions similar calculations lead to unacceptable conclusions. It is then necessary that more than two nonexotic-particle intermediate states be included; i.e., diagrams more complicated than those in Fig. 1 are needed.

For example, the signature of a cut is expected to be the product of the signatures of the input poles. Thus the $\rho\rho$ cut is absent in $\pi^-p \to \pi^0n$. There is no reason for individual diagrams to be zero. Instead one expects[12] a cancelation between intermediate states (line f in Fig. 1) with isospin-$\frac{1}{2}$ baryons and states with isospin-$\frac{3}{2}$ baryons. In fact, resonances of the two isospins enter with opposite signs and can easily cancel.[12] The same considerations apply to $\pi^-K^+ \to \pi^0K^0$,[13] although here the contributions canceling the K^* states must either be exotic (isospin $\frac{3}{2}$) or nonresonant (isospin $\frac{3}{2}$ or $\frac{1}{2}$).

As another example[15] in which exotic states appear to be required, consider $K^-p \to \bar{K}^0n$ and $K^+n \to K^0p$. Since these reactions share the same t channel, they must have the same Regge singularities, e.g., the double K^* cut. In the former, we expect the cut to come from $K^-p \to \pi\Lambda$ (or $\pi\Sigma$) $\to \bar{K}^0n$. In the latter, however, it must come from diagrams with an exotic intermediate line (not necessarily resonant) at one side.

Because of the above contributions, a quantitative cut calculation cannot be trusted to better than perhaps a factor of 2. We hope that in cases of interest all important diagrams are included with our methods. In any case, the variation in magnitude due to the effects (spin, absorption, coupling) we have included is very large compared to any uncertainty introduced by the above difficulty.

In conclusion, we would like to emphasize that we have presented a model, for any double-particle–exchange contribution in hadronic reactions, that predicts contributions varying in size in cross section by about 10^5 and that appears to be *consistent with experimental data over this range.* The model is a natural extension of the strong absorption model for describing single Reggeon exchanges, so if our considerations are correct, an internally consistent approach is available for discussing all nondiffractive two-body hadronic reactions. Finally, we have briefly discussed the interesting possibility that one can relate the spin dependence of couplings to the size of exotic amplitudes—if the ρ-coupling did not flip s-channel baryon helicities, exotic amplitudes would be an order of magnitude larger.

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Comparison of an Inclusive Multiperipheral Model to Secondary Spectra in p-p Collisions*

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The quantitative predictions of an inclusive multiperipheral model are compared with the measured secondary spectra in pp collisions from 12 to 30 GeV/c.

For proton-proton collisions at high energies the dominant contributions to the total cross section come from inelastic processes. The simplest probe of these processes are inclusive experiments of the type $p+p\to x+\text{anything}$, where x is one of the several types of final-state particles (e.g., $\pi^\pm$, $K^\pm$, or p). The usual procedure is to measure the momentum spectra of x for various angles of its production. Recently, three such experiments have been performed at 12.4,[1] 19.2,[2] and 30 GeV/c.[3] In this report we show that the prominent characteristics of these spectra can be understood with a simple multiperipheral model that contains the known properties of two-body scattering processes. We compare the $\pi^\pm$, $K^\pm$, and proton spectra measured by these experiments to the predictions of an inclusive multiperipheral model that is a modified version of that first proposed by Caneschi and Pignotti (CP).[4]

Our modified CP inclusive multiperipheral model is illustrated by the diagrams of Fig. 1. Figure 1(a) represents the case of the proton remaining at the end of the multiperipheral chain; Fig. 1(b) represents the baryon-exchange case of the proton traveling down one link, emitting a