and A. D. B. Woods, Metal Phys. 2, 5189 (1970).
[3]T. M. Holden, B. M. Powell, M. W. Stringfellow, and A. D. B. Woods, J. Appl. Phys. 41, 1176 (1970).
[4]V. G. Barijaktar and S. V. Maleev, Fiz. Tverd. Tela. 5, 1175 (1963) [Sov. Phys. Solid State 5, 858 (1963)].
[5]B. R. Cooper, in *Solid State Physics*, edited by H. Ehrenreich, F. Seitz, and D. Turnbull (Academic, New York, 1968), Vol. 21, pp. 393-490.
[6]M. S. S. Brooks, Phys. Rev. B 1, 2257 (1970).
[7]D. A. Goodings and B. W. Southern, to be published.
[8]J. W. Cable, E. O. Wollan, W. C. Koehler, and M. K. Wilkinson, Phys. Rev. 140, A1896 (1965).
[9]R. M. Nicklow, N. Wakabayashi, M. K. Wilkinson, and R. E. Reed, Phys. Rev. Lett. 26, 140 (1971).
[10]R. M. Nicklow, to be published.
[11]H. B. Møller, J. C. G. Houmann, and A. R. Mackintosh, J. App. Phys. 39, 807 (1968).
[12]T. Kasuya, in *Magnetism*, edited by G. T. Rado and H. Suhl (Academic, New York, 1966), Vol. II B, pp. 215-294.

Hybrid Model for Pre-Equilibrium Decay in Nuclear Reactions*

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Ideas from Griffin's exciton model are combined with those from the nucleon-nucleon scattering approach to nuclear transition times to provide a simple closed-form expression for predicting pre-equilibrium decay phenomena, including variation of pre-equilibrium emission with target mass, excitation energy, and initial particle and hole numbers. Time estimates for pre-equilibrium emission are given at several excitations.

The question of the attainment of equilibrium in medium-energy nuclear reactions is very old and of great importance to statistical models. Recent approaches to models which may in time provide answers to this question have been the very simple exciton model proposed by Griffin[1] and the more elaborate master-equation approach due to Miller and co-workers.[2,3] We propose a model which in some ways provides a marriage between the simple exciton model and the more elaborate model of Refs. 2 and 3. The result maintains the simple closed-form simplicity of the model of Griffin and may be used to predict such properties as the variation of fraction pre-equilibrium emission (hereafter referred to as fpe) as a function of excitation energy, compound-state mass number, and initial particle and hole numbers. This may be done either on an *a priori* basis, or by a normalization of a "mean free path" (mfp) constant at one excitation. Statements may also be made as to the length of time during which pre-equilibrium emission occurs, as well as to other lifetime averages.

As in high-energy cascade calculations and in Refs. 1–6, it will be assumed that a reaction proceeds through a series of particle-particle or particle-hole interactions, in which the total particle and hole numbers characterizing the nuclear state may either increase by two, decrease by two, or remain unchanged. As in earlier work, we assume that the transitions in which the particle and hole (p-h) or exciton (n) number increases by two dominate in the early stages of the equilibration process. As in Griffin's model, we assume that the intermediate states are characterized by appropriate level density formulas and that all levels may be populated with equal *a priori* probability (within limitations of energy conservation and the Pauli principle) during the equilibration process, at least insofar as they "count" the number of ways different transitions may lead to the various final states. However, as in the model of Ref. 2 we recognize that whether or not one particle in a virtual level is emitted into the continuum or undergoes a transition to a more complicated $(n+2)$-exciton state depends upon the appropriate particle decay rates $\lambda_c(\epsilon)$ and $\lambda_{n+2}(\epsilon)$ for the particle of interest and not for the average over the n-exciton state. The level densities as used therefore represent a bookkeeping procedure for the number of ways excitons from a simple state may populate a given energy range in a more complex state, averaged over time, not at a specific time. As in earlier treatments of the exciton model the total particle-emission probability in a given channel energy range $P_x(\epsilon)d\epsilon$ is given as a sum over the contributions of the intermediate states, although here this has significance as a statistical bookkeeping operation rather than on an absolute time basis. The sum is taken from some initial number of excitons n_0 to the equilibrium number

$\bar{n}$. We write the decay probability as

$$P_x(\epsilon)d\epsilon = \sum_{\substack{n=n_0 \\ \Delta n = n_0}}^{\bar{n}} {}_nf_x \left[\frac{\rho_{n-1}(U)}{\rho_n(E)}\right]\left[\frac{\lambda_c(\epsilon)}{\lambda_c(\epsilon)+\lambda_{n+2}(\epsilon)}\right]\left[\prod_{n'=n_0+2}^{n}(1-P_{n'-2})\right] = \sum_{n=n_0}^{\bar{n}} nP_x(\epsilon)d\epsilon, \quad (1)$$

where E is the compound state and U the residual nucleus excitation. The symbols of (1) which have not been previously defined will be defined further on in the text.

The expression in the first set of brackets is the fraction of the population of the n-exciton state which has one particle in a virtual level which would have energy between ϵ and $\epsilon+d\epsilon$ in the continuum. The expression in the second set of brackets is the ratio of transition rates into the continuum to total transition rate. It is similar in form to that given in Ref. 3 except that the transition rate is taken here to depend on a particular particle, and this will be simplified to the transition rate $\lambda_{n+2}(\epsilon)$ of the particle of interest by a nucleon-nucleon interaction with a particle below the Fermi energy to give a state with an additional excited particle plus hole. To evaluate $\lambda_{n+2}(\epsilon)$ we will use the average mfp for nucleons scattering in nuclear matter as calculated in Kikuchi and Kawai,[7] dividing these into the classical particle velocities to generate transition rates. In the range of energies up to 100 MeV above the Fermi energy this result is very well approximated by

$$\lambda_{n+2}(\epsilon) = [1.4\times10^{21}(\epsilon+B_x) - 6.0\times10^{18}(\epsilon+B_x)^2]/k \text{ sec}^{-1}. \quad (2)$$

where ϵ and B_x are given in MeV. In (2) a value of $k=1$ corresponds to mfp values as calculated in Ref. 7; in the subsequent discussion k will be treated as a parameter. The rates for emission into the continuum, $\lambda_c(\epsilon)$, of (1) are calculated as in Ref. 3,

$$\lambda_c(\epsilon) = \sigma(\epsilon)(2\epsilon/m)^{1/2}\rho_c(\epsilon)/g\Omega, \quad (3)$$

where $\sigma(\epsilon)$ is the inverse cross section, m the nucleon mass, $\rho_c(\epsilon)$ the free-particle translational state density, g the single-particle density in the nucleus, and Ω the laboratory volume.

The term in the third set of brackets of (1) is the depletion factor, which reduces the population of each state according to the amount of particle emission from simpler states. With this definition P_{n-2} is given by

$$P_{n-2} = \sum_{x=n,p}\int_{\epsilon=0}^{\epsilon_{max}} n - 2P_x(\epsilon)\,d\epsilon, \quad (4)$$

where P_{n-2} is zero for the first term in the summation of (1).

The level densities used in (7) were those given by Williams,[8]

$$\rho_n(E) = \rho_{p,h}(E) = g(gE-\theta)^{n-1}/[p!h!(n-1)!], \quad (5)$$

where $\theta = f(p,h)$ is a correction term for the Pauli principle.

The density of n-exciton states having one particle in the energy range $\epsilon+B_x$ to $\epsilon+B_x+d\epsilon$ (where B_x is the nucleon binding energy) above the Fermi energy, leaving $p-1$ particles and h holes to share the residual energy U, is

$$\rho_{p-1,h}(U) = g(gU-\theta)^{n-2}g\,d\epsilon/[p!h!(n-2)!]. \quad (6)$$

The denominator of (6) differs from earlier results in that it is recognized that the particle at $\epsilon+B_x$ as well as the other $p-1$ excited particles are all indistinguishable.

Using Eq. (1) we have calculated (α,p) particle spectra for reactions on ^{93}Nb corresponding to bombarding energies of 30.5 and 42 MeV. The published spectra[9,10] were integrated over angles and are compared with these calculations in Fig. 1. The experimental cross sections are each divided by the calculated total reaction cross sections.[11] The error bars given on the experimental results are our own estimates, based on the number of angles for which results were available, accuracy of reading graphical data, etc. A value of 6 MeV^{-1} was used for g and optical-model results were used for neutron and proton inverse cross sections.[12] A value of $n_0=4$ was required to give the proper spectral distribution, and in this sense it was not a free parameter. It was assumed that the initial distribution was a 2-proton, 2-neutron state, increasing on the average by one hole, half proton and half neutron in each subsequent transition [in order to evaluate ${}_nf_x$, the fraction of particles of type x in an n-exciton state of Eq. (1)].

Considerably better agreement is shown in Fig. 1 for the result with $k=5$, which implies a mfp for nucleon-nucleon scattering which is 5 times the value given in Ref. 7. There are several reasons as to why mfp values larger than those calculated in (7) may be reasonable. First, it is thought that pre-equilibrium emission results mainly from high-impact-parameter, peripheral

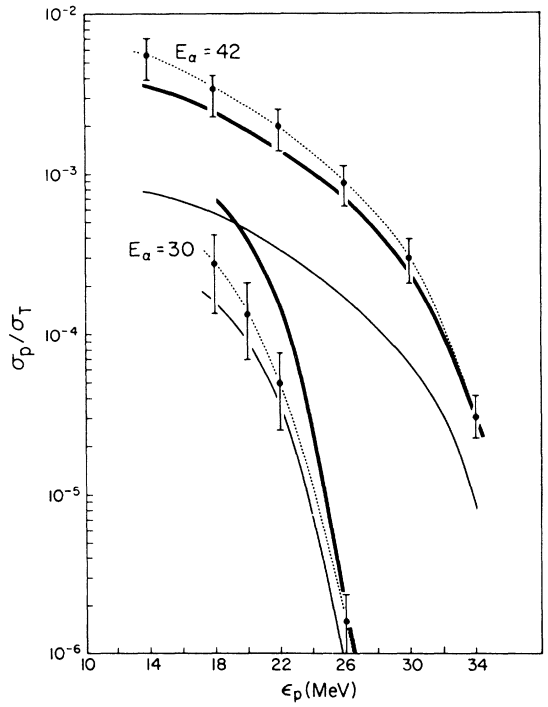

FIG. 1. Experimental (Refs. 9 and 10) and calculated spectra for the ^{93}Nb(α,p) reaction at $E_\alpha = 30.5$ and 42 MeV. Calculated results for $k = 1$ are given by the thin solid lines and for $k = 5$ by the heavy solid lines.

target-projectile interactions. The lower nuclear density in the nuclear surface should result in a longer mfp for scattering. Secondly, the calculations of Ref. 7 were based on free-scattering cross sections, with only the requirement for scattering within the nucleus that energy be conserved and that no scattering be allowed into levels below the Fermi energy. However, if the nucleons involved in the scattering process have well-defined quantum states before and after the transition in question, then the final states must be accessible from the initial states through coupling with the angular momentum associated with the scattering process. Clearly only a fraction of the total states at a given energy are accessible within this restriction of angular momentum conservation, and this additional restriction should result in longer mfp for nucleons in nuclear matter.

In Fig. 2 the predicted variation of fpe is shown as a function of excitation energy for the first particle emitted in the α-induced reaction of ^{93}Nb. For all curves shown it may be seen that the fpe initially increases very rapidly with an increase in excitation energy and then at a much lower rate. This implies a difficulty in extract-

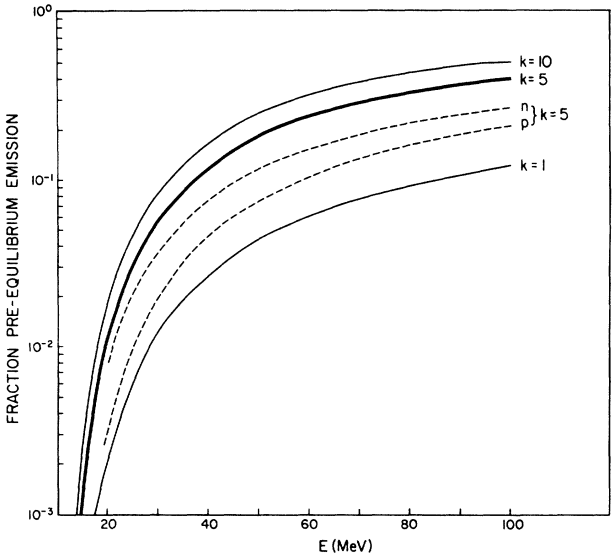

FIG. 2. Predicted variation of the fpe with excitation energy for the α-induced reaction of ^{93}Nb. Calculations are for k values [Eq. (2)] of 1, 5, and 10. For $k = 5$ the separate neutron and proton contributions are shown as dashed lines.

ing the fpe from experimental data at lower excitations since calculated values for comparison will have huge uncertainties due to uncertainties in the intermediate state level densities on account of, for example, pairing and shell effects. What are very much needed to test the predictions set forth by this model are data at a range of excitations and on a selection of several targets. Nontheless, the values given in Fig. 2 are in qualitative to semiquantitative agreement with values determined from normalizations to the spectral shapes predicted by the exciton model.[6,13]

A mass dependence for pre-equilibrium emission is also predicted by substitution into (1). One can get a rough estimate of the variation by considering high-energy particle emission as resulting from pre-equilibrium emission. The mass dependence is then given by the quantity in the second set of brackets in (1); it cancels in the quantity in the first set of brackets. Taking $\sigma(\epsilon) \propto A^{2/3}$, $g \propto A$, and assuming $\lambda_{n+2} \gg \lambda_c$ (which is only a fair approximation), a mass dependence of $A^{-1/3}$ is indicated for the fpe. This is in very good agreement with the values of fpe of 0.35 for $A = 201$ and 0.50 for $A = 60$ in Ref. 6. The analyses of Ref. 6 were fairly crude, however, and once again firm conclusions await the acquisition of more and better data. The mass dependence discussed here is, with suitable approximations, present in the exciton-model formulation of Ref.

3 although the authors did not choose to discuss it.

Averages over time may be calculated in various ways for Eq. (1). We present only a few numbers for the lifetimes during which pre-equilibrium emission occurs (in excess of 0.1% of the total cross section), based on (2) with $k = 5$. For the example cited in Figs. 1 and 2, typical values are $\tau \simeq 1.2\times10^{-21}$ sec at an excitation energy $E = 32$ MeV; $\tau \simeq 1.4\times10^{-21}$ sec at $E = 42$ MeV; and $\tau \simeq 2\times10^{-21}$ sec at $E = 80$ MeV.

By the marriage prescribed in this Letter between the exciton model and higher-energy cascade models we are able to calculate the variation of pre-equilibrium emission with excitation energy and mass number of the compound state. Statements can also be made as to the time during which pre-equilibrium emission occurs and the variation of time with excitation energy. The first two quantities cited (E and A dependence) are in reasonable agreement with analyses of existing data on a qualitative to semiquantiative basis.[6,13] Equation (1) is also ideal for use in calculations involving multiple particle emission, as for excitation functions, since the gross approximations previously required need no longer be made.[6] The data may be used to test the model implied by Eq. (1) rather than the inadequacy of the approximations as was largely the case in the past. The need for more and better data is clearly indicated before the validity or inadequacy of (1) is clearly defined, but we feel that (1) offers very distinct advantages over earlier treatments of the exciton model, while preserving the simplicity of formulation. Surely additional questions are raised by this formulation and the approximations implied by it. It is a first approach, which must be confirmed in first order before it is worthwhile to worry about higher-order effects.

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[1]F. C. Williams, Jr., Phys. Lett. 31B, 184 (1970).
[2]G. D. Harp, J. M. Miller, and B. J. Berne, Phys. Rev. 165, 1166 (1968).
[3]G. D. Harp and J. M. Miller, Phys. Rev. C 3, 1847 (1971).
[4]J. J. Griffin, Phys. Rev. Lett. 17, 478 (1966).
[5]M. Blann, Phys. Rev. Lett. 21, 1357 (1968).
[6]M. Blann and F. M. Lanzafame, Nucl. Phys. A142, 559 (1970).
[7]K. Kikuchi and M. Kawai, *Nuclear Matter and Nuclear Reactions* (North-Holland, Amsterdam, 1968), p. 40.
[8]F. C. Williams, Jr., Nucl. Phys. A166, 231 (1971).
[9]W. Swenson and N. Cindro, Phys. Rev. 123, 910 (1961).
[10]R. W. West, Phys. Rev. 141, 1033 (1966).
[11]J. R. Huizenga and G. J. Igo, Nucl. Phys. 29, 462 (1962).
[12]G. S. Mani, M A. Melkanoff, and I. Iori, Centre d'Études Nucléaires de Saclay Reports No. CEA 2380 and No. CEA 2379, 1963 (unpublished).
[13]C. K. Cline and M. Blann, University of Rochester Report No. UR-3591-17, 1970 (to be published).

Concentric Spherical Cavities and Limits on the Photon Rest Mass*

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The concentric spherical cavity is used as a vehicle for study of the complete breakdown of the "relativistic particle in a box" formula for the photon rest-mass effect. The results are applied to a determination of the limit on the photon's rest mass which can be inferred from the Schumann resonances.

We consider a cavity consisting of the space between two concentric conducting spheres and study the dependence of its resonant frequencies upon an assumed photon rest mass for various values of the ratio of inner to outer radius. Our interest arises, on the one hand, from its relevance to a suggestion of Kendall[1] that the Schumann resonances in the cavity formed by Earth and its ionosphere[2] be used to set a limit in the photon's rest mass, and on the other, from its utility as a vehicle for exploring in detail the breakdown of the "relativistic particle in a box" formula for the photon rest-mass effect.

For the case of a Klein-Gordon particle whose wave function is required to vanish at the boundary of a closed cavity, it is easy to show that