TABLE III. Comparison of calculated[a] and experimental[b] amplitudes.

Decay	$A\times 10^5$ sec$^{1/2}$ Experiment	$A\times 10^5$ sec$^{1/2}$ Calculated	$B\times 10^5$ sec$^{1/2}$ Experiment	$B\times 10^5$ sec$^{1/2}$ Calculated
Λ_-^0	1.53 ± 0.02	1.56	10.50 ± 0.33	10.4
Σ_+^+	0.06 ± 0.02	-0.35	19.07 ± 0.34	19.9
Σ_-^-	1.89 ± 0.03	1.81	-0.72 ± 0.01	-0.5
Ξ_-^-	-2.07 ± 0.02	-1.96 (-2.04)	6.68 ± 0.70	6.3 (3.9)

[a]We have used the π-N phase shifts of Ref. 9 to evaluate U, V, and Z and have chosen $\eta=0.69$, $D=-2.69\times10^{-5}$ MeV, and $F=3.90\times10^{-5}$ MeV.

[b]The experimental amplitudes are from Ref. 10.

components in the $G_{\beta\alpha}$; however, it takes an unacceptably large violation of the $\Delta I=\frac{1}{2}$ rule to remove it.

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Explanation of Excess High-Momentum Spectators in High-Energy Deuteron-Breakup Reactions*

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The multiple-scattering correction to the spectator momentum distribution is calculated using Glauber theory. It is shown to dominate when the transverse component of the spectator momentum is greater than about 400 MeV/c, yielding an overall enhancement of high-momentum events.

To obtain information about inelastic scattering on neutrons, it is necessary to study reactions in which a deuteron breaks up. For such processes one assumes that the proton is a spectator which plays no part in the scattering; the neutron reaction is then given by the impulse approximation. To test the validity of this assumption, it is usually important to measure the momentum distribution of the spectator proton. In the impulse approximation, this distribution should correspond precisely with the momentum-space deuteron wave function. Experimentally, it is generally true[1] that the spectator distribution shows a significant excess of high-momentum events as compared with the predictions of the Hulthên[2] wave function.

It is well-known that multiple-scattering effects become dominant for large momentum transfer. In this paper we wish to point out that an analogous result holds for the spectator momentum. The multiple-scattering contributions to this distribution can be calculated straightforwardly using the Glauber[3] multiple-scattering formalism, and their effects fall off less rapidly at large momentum than the impulse approximation. In order to present this result as simply as possible, we shall neglect spin and isospin; it is also helpful to assume that the spectator can be identified unambiguously. (For example, in $\pi^+d\to K^+\Lambda^0 p$ the proton is certainly the spectator to the inelastic reaction $\pi^+n\to K^+\Lambda^0$; whereas in $\pi^+d\to\pi^0pp$, one must decide *which* proton was the spectator. In the latter case the effects of symmetrization will probably be more important than double-scattering corrections.[4]) Neither of these assumptions is crucial to the calculation.

In the Glauber theory the scattering amplitude is written using an impact-parameter representation,

$$F(\vec{\Delta})=\frac{ik}{2\pi}\int d^2b\, e^{i\vec{\Delta}\cdot\vec{b}}\langle\Psi_f|\Gamma(\vec{b},\vec{r}_1,\vec{r}_2)|\Psi_i\rangle, \qquad (1)$$

where $\vec{\Delta}$ is the transverse momentum transfer, Γ is the profile function, and ψ_f and ψ_i are the final and initial spatial wave functions for the deuteron. The positions of the two nucleons are $\vec{r}_1$ and $\vec{r}_2$; and we shall choose the nucleon at $\vec{r}_2$ to be spectator, assuming that the scattering process of interest takes place only on the nucleon at $\vec{r}_1$. The appropriate expression for the profile function is then

$$\Gamma(\vec{b},\vec{r}_1,r_2)=\Gamma_1(\vec{b}-\vec{r}_1)-\Gamma_1(\vec{b}-\vec{r}_1)\Gamma_e(\vec{b}-\vec{r}_2). \qquad (2)$$

The first term is the impulse approximation, with Γ_1 related to the scattering amplitude for a free (although off-mass-shell) nucleon by

$$F_1(\vec{\Delta})=(ik/2\pi)\int d^2b\, e^{i\vec{\Delta}\cdot\vec{b}}\Gamma_1(\vec{b}). \qquad (3)$$

(A corresponding term involving $\vec{r}_2$ is absent since we are assuming that the proton at $\vec{r}_2$ *must* be the spectator.) The second term describes the double-scattering process in which the inelastic reaction at $\vec{r}_1$ is preceded or followed by an elastic scattering on the spectator, with the corresponding free elastic amplitude

$$F_e(\vec{\Delta})=(ik/2\pi)\int d^2b\, e^{i\vec{\Delta}\cdot\vec{b}}\Gamma_e(\vec{b}). \qquad (4)$$

The initial wave function ψ_i is simply the deuteron wave function,

$$\psi_i(\vec{r}_1,\vec{r}_2)=\int d^3p\, \varphi(\vec{p})\exp[i\vec{p}\cdot(\vec{r}_1-\vec{r}_2)], \qquad (5)$$

in the deuteron rest frame. To consider coherent processes, we would take ψ_f of the same form as (5) with an appropriate center-of-mass momentum. For breakup reactions, it is appropriate instead to take ψ_f as the product of two free-particle states,

$$\psi_f=(2\pi)^{-6}\exp(i\vec{p}_1\cdot\vec{r}_1)\exp(-i\vec{p}_2\cdot\vec{r}_2). \qquad (6)$$

The single-scattering amplitude then becomes

$$F_s(\vec{\Delta},\vec{p}_2)=\delta(\vec{p}_2-\vec{p}_1+\vec{\Delta})\varphi(\vec{p}_2)F_1(\vec{\Delta}), \qquad (7)$$

guaranteeing conservation of momentum and showing the impulse-approximation result that the distribution of spectator momenta is determined directly by $\varphi(\vec{p}_2)$. The double-scattering term is straightforward to evaluate, and yields the correction term

$$F_d(\vec{\Delta},\vec{p}_2)=\int d^2b\, e^{i\vec{\Delta}\cdot\vec{b}}\langle\psi_f|\Gamma_1(\vec{b}-\vec{r}_1)\Gamma_e(\vec{b}-\vec{r}_2)|\psi_i\rangle=\delta(\vec{p}_2-\vec{p}_1+\vec{\Delta})(i/2\pi k)\int d^3q\, F_1(\vec{\Delta}-\vec{q})F_e(\vec{q})\varphi(\vec{p}_2+\vec{q}). \qquad (8)$$

The differential cross section for scattering with momentum transfer $\vec{\Delta}$ and spectator momentum $\vec{p}_2$ following from (7) and (8) is

$$d^5\sigma/d\Omega d^3p_2=|f_1(\vec{\Delta},\vec{p}_2)+f_2(\vec{\Delta},\vec{p}_2)|^2, \qquad (9)$$

with

$$f_1(\vec{\Delta},\vec{p}_2)=\varphi(\vec{p}_2)F_1(\vec{\Delta}),$$
$$f_2(\vec{\Delta},\vec{p}_2)=\frac{i}{2\pi k}\int d^3q\, F_1(\vec{\Delta}-\vec{q})F_e(\vec{q})\varphi(\vec{p}_2+\vec{q}). \qquad (10)$$

It is clear in (10) that the correction term contains $\varphi(\vec{p})$ in a more complicated form than does the impulse approximation, and that a correction to the spectator momentum distribution will be introduced. Knowing approximate forms for F_1 and F_e, one may calculate $f(\vec{\Delta},\vec{p}_2)$ using either the Hulthèn wave function for $\varphi(\vec{p})$ or, if desired, a more accurate form such as Moravcsik's.[5] The modifications to (10) which must be introduced when either of two final-state protons could be the spectator consist essentially of adding terms with $\vec{p}_2$ replaced by $\vec{p}_1$. Other corrections, such as those which arise from spin and isospin effects,[6] are also straightforwardly included.

In order to see clearly the consequences of (10), it is appropriate to choose particularly simple forms of the functions involved, for which the integration of $f_2(\vec{\Delta},\vec{p}_2)$ can be carried out analytically. We therefore assume exponential diffraction peaks[7] for F_1 and F_e,

$$F_1(\vec{\Delta})=A\exp(-a\Delta^2),$$

$$F_e(\vec{\Delta})=(ik\sigma/4\pi)\exp(-b\Delta^2),$$

and choose a Gaussian form also for φ,

$$\varphi(\vec{p}) = N \exp(-cp^2),$$

with a normalization factor $N = (2c/\pi)^{3/4}/\sqrt{2}$. Then it follows that

$$f_2(\vec{\Delta}, \vec{p}_2) = -\frac{AN\sigma}{8\pi(a+b+c)} \exp\left[-\frac{a(b+c)\Delta^2 + c(a+b)p_t^2 + 2ac\vec{\Delta}\cdot\vec{p}_t}{a+b+c} - cp_l^2\right], \tag{11}$$

where $\vec{p}_l$ and $\vec{p}_t$ are the longitudinal and transverse components of $\vec{p}_2$. The sum of f_1 and f_2 is

$$f_1(\vec{\Delta}, \vec{p}_2) + f_2(\vec{\Delta}, \vec{p}_2) = AN \exp(-a\Delta^2 - cp_2^2)\left\{1 - \frac{\sigma}{8\pi(a+b+c)} \exp[a\vec{\Delta} - c\vec{p}_t)^2/(a+b+c)]\right\}. \tag{12}$$

It should be noted that in (11) the exponential fall off in p_t^2 has a coefficient $c(a+b)/(a+b+c) < c$, so that for large enough p_t it will dominate over $\varphi(\vec{p}_2)$. It is also intriguing to find a dependence on $\vec{\Delta}\cdot\vec{p}_t$, implying angular correlations between the momentum transfer and the spectator momentum. This effect should be present even if more complicated amplitudes and wave functions are used, and should be checked experimentally. The prediction is that, particularly for high-momentum spectators, the differential cross section should be somewhat more sharply peaked, as a function of Δ^2, when $\vec{\Delta}$ and $\vec{p}_2$ are parallel than when they are antiparallel.

A net spectator momentum distribution may be calculated by integrating (9) over all transverse momentum transfers Δ. For diffractive processes the solid-angle element $d\Omega$ can be written using the approximation $d\cos\theta = d\Delta^2/2k^2$, the azimuthal angle being defined as the angle between $\vec{\Delta}$ and $\vec{p}_t$. The result is appropriately expressed as a distribution with respect to p_l and p_t,

$$\frac{d^2\sigma}{dp_l dp_t} = 2\pi p_t \int d\Omega |f_1(\vec{\Delta}, \vec{p}_2) + f_2(\vec{\Delta}, \vec{p}_2)|^2, \tag{13}$$

yielding terms corresponding to single scattering, double scattering, and the interference between them. The first is the impulse approximation:

$$\left(\frac{d^2\sigma}{dp_l dp_t}\right)_1 = 2\pi p_t |\varphi(\vec{p}_2)|^2 \int d\Omega |F_1(\vec{\Delta})|^2,$$

which for these parametrizations becomes

$$\left(\frac{d^2\sigma}{dp_l dp_t}\right)_1 = \frac{\pi^2 p_t |A|^2 N^2 \exp(-2cp_2^2)}{ak^2}. \tag{14}$$

The interference term and the double scattering term yield, respectively,

$$\left(\frac{d^2\sigma}{dp_l dp_t}\right)_{\rm int} = -\frac{\pi p_t |A|^2 N^2 \sigma}{2k^2 a(a+2b+2c)} \exp\left(\frac{-2cp_2^2 + 2c^2 p_t^2}{a+2b+2c}\right), \tag{15}$$

$$\left(\frac{d^2\sigma}{dp_l dp_t}\right)_2 = \frac{p_t |A|^2 N^2 \sigma^2}{64k^2 a(b+c)(a+b+c)} \exp\left(\frac{-2cp_2^2 + 2c^2 p_t^2}{b+c}\right). \tag{16}$$

Combining these terms and writing $\int d\Omega |F_1(\Delta)|^2 = \pi|A|^2/2k^2 a = \sigma_1$, we may define an overall correction factor,

$$d^2\sigma/dp_l dp_t = 2\pi p_t \sigma_1 |\varphi(p_2)|^2 C(p_t^2),$$

with

$$C(p_t^2) = 1 - \frac{\sigma}{2\pi(a+2b+2c)} \exp\left(\frac{2c^2 p_t^2}{a+2b+2c}\right) + \frac{\sigma^2}{(8\pi)^2(b+c)(a+b+c)} \exp\left(\frac{2c^2 p_t^2}{b+c}\right). \tag{17}$$

Since $C(p_t^2)$ depends only on the transverse component of the spectator momentum, the distribution with respect to p_l at fixed p_t should depend only on $\varphi(p_2) = \varphi((p_l^2 + p_t^2)^{1/2})$.

An estimate of the behavior of $C(p_t^2)$ can be obtained by using $c = 1\ \text{fm}^2$ and taking $a = b = 4.5\ \text{GeV}^{-2}$, $\sigma = 25$ mb as appropriate values for typical meson-nucleon reactions above 5 GeV/c. It follows then

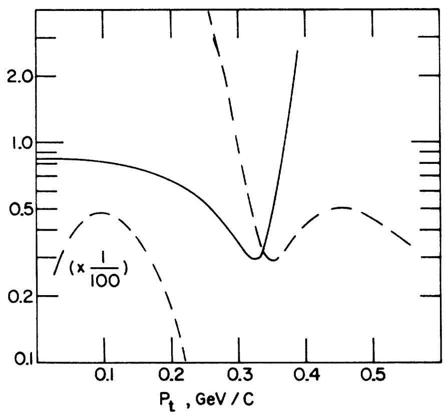

FIG. 1. The correction factor $C(p_t^2)$ (solid curve) and the net distribution in p_t (dashed curve) resulting from Eq. (18).

that

$$C(p_t^2) = 1 - 0.158 \exp(20.4 p_t^2) + 0.006\,49 \exp(43.7 p_t^2), \quad (18)$$

with p_t in GeV/c. As shown in Fig. 1, $C(p_t^2)$ decreases to a minimum value of 0.29 at $p_t = 337$ MeV/c, then rises sharply and is completely dominated by the double-scattering term for $p_t \gtrsim 400$ MeV/c. It is in precisely this region that excess spectators have been observed. Integrating over p_l yields a net distribution in p_t which is proportional to $\exp(-cp_t^2)C(p_t^2)$; this curve is also shown in Fig. 1.

The quantitative result (18) depends, of course, on the approximations involved in evaluating (10) as well as those inherent to the Glauber theory. Its general shape, however, and the dominance of double scattering for p_t larger than about 400 MeV/c, reflect the relations between the magnitudes and slopes of the diffraction peaks and the rms radius of the deuteron. For this reason the qualitative aspects of this result are relatively independent of the parametrizations used. We therefore urge experimental checks, in particular, of the presence of the $\vec{\Delta}\cdot\vec{p}$ dependence in (11) and the structure in p_t resulting from (18).

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