dictions.

In conclusion, we have shown that, depending whether the quantity ($|\omega_c| - v_0^2/4D$) is <0 or >0, the instability of a semiconductor biased in the NDM region will take either the form of a traveling space-charge layer starting from a cathode-type fluctuation, or of a stationary, high-field layer initiated by an anode-type fluctuation. In the first case, a Gunn diode would exhibit the usual Gunn-effect behavior, whereas in the second situation, it would switch from a high- to a low-current stable state. The author is very much indebted to Dr. P. Wolf and Dr. H. Thomas for very stimulating discussions on the subject of this paper.

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Deduction of Band Structure from Elastic Tunneling Studies

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It is pointed out that the incorporation of band structure in the theory of elastic tunneling in metal-insulator-metal structures does not make an appreciable enough change in the tunnel current to warrant its deduction from measured voltage-current data. It is emphasized that if it is to be at all meaningful, the essential barrier parameters (effective work function, thickness, and effective mass) and the barrier shape must be consistently and precisely specified. This has so far not been achieved.

Recently there have been a few attempts at deducing the electron energy-momentum dispersion relation[1,2] in the forbidden energy region of insulators and semiconductors from elastic tunneling studies[3] in metal-insulator-metal[4,5] (MIM) and metal-semiconductor[6,8] contacts. From the experimental plot of tunnel voltage current data, the $E(k)$ relation for an amorphous AlN thin film[1,4] was deduced to be approximately Franz's empirical relation (by fitting it with the theoretical curve). Recently Kurtin, McGill, and Mead[5] have deduced the $E(k)$ relation for a single-crystal GaSe insulating film by taking the inverse transform the equation of tunnel current which involves the $E(k)$ relation term explicitly in the transmission probability, assuming some characteristic (trapezoidal) model of electron tunneling.

We wish to point out in this regard that such deductions from the voltage-current experimental data of MIM structures are not very meaningful unless a high order of precision in barrier parameters is ascertained *uniquely* and *consistently* from independent estimates. The latter point is very doubtful at this time, and perhaps it will be extremely complex to accomplish. This is because the change brought about in voltage-current characteristics by invoking the nonparabolicity term is not very appreciable compared to the known uncertainties in the essential

barrier parameters[9] (viz the effective work function, thickness, and effective mass in the insulator region), the effects of which are very high. Hence it is more likely that the discrepancy between theory and experiment lies in the uncertainties in the barrier parameters rather than in neglecting the actual band structure of the insulator (i.e., assuming it to be a simple parabolic relation).

Rather than resorting to the exact numerical integration[10] of the tunnel integral, we estimated these changes from the following approximate tractable expression, valid for the nonparabolic $E(k)$ relation[11] of the type $\hbar^2k^2/2m_i = E + aE^2$, obtained recently by the authors[12]:

$$J = \frac{16\pi e m_i}{h^3 A^2}\left\{\frac{A_1}{A_0}\exp(-A\sqrt{A_1})\left[\frac{\pi B_1 k_0 T}{\sin(\pi B_1 k_0 T)}\right][1-\exp(-B_1 eV)]\right.$$
$$\left.-\left[\frac{A_1+\eta m_c/m_i}{A_0-m_c/m_i}\right]\exp[-A(A_1+\eta m_c/m_i)^{1/2}]\left[\frac{\pi B_2 k_0 T}{\sin(\pi B_2 k_0 T)}\right][1-\exp(-B_2 eV)]\right\}. \quad (1)$$

In obtaining this expression we have used the average-barrier-height method[13] for making it analytically tractable. The various parameters appearing in (1) are

$$A = 2(2m_i)^{1/2}(x_{2F}-x_{1F})\beta/\hbar, \quad A_0 = (1-2a\bar{\varphi}),$$
$$A_1 = (\bar{\varphi}-a\bar{\varphi}^2), \quad B_1 = AA_0/2\sqrt{A_1}, \quad (2)$$
$$B_2 = \tfrac{1}{2}A(A_0-m_c/m_i)(A_1+\eta m_c/m_i)^{-1/2}.$$

Here φ is the potential-barrier height above the Fermi level of the negatively biased electrode as a function of distance in the insulator region, and the bar represents the average of the quantity involved between x_{2F} and x_{1F} (x_{1F} and x_{2F} are the classical turning points corresponding to the Fermi level); η is the Fermi energy of the negatively biased electrode; and β is a numerical factor depending upon the barrier shape and generally lies in between[13] 0.95 and 1.0. The rest of the symbols have their usual meaning.

To appreciate the influence of a nonparabolic relation for $E(k)$, we computed the tunnel current [Eq. (1)] for Al-AlN-Al junctions assuming that the empirical Franz relation[1,4] represents a good approximation of the true band structure [i.e., a in Eq. (2) has been replaced by the inverse of the forbidden energy gap]. To have some idea about the difference between parabolic and nonparabolic cases, we assumed, typically, an idealized trapezoidal barrier.[14] Assuming the parameters for the Al-AlN-Al junction[15] to be $E_g = 4.0$ eV, $m_i/m_c = 0.47$, $\eta = 11.7$ eV, $\varphi_1 = 1.68$ eV, $\varphi_2 = 2.01$ eV, thickness = 20 Å, we find that the tunnel currents in nonparabolic and parabolic cases differ by nearly 2 orders of magnitude at an applied bias of 1 eV. Since this difference is very small and can be obtained even for parabolic cases with small fluctuations in the insulator thickness, work function, effective mass, etc. (while actually the fluctuations are much larger), we conclude that the effect of a nonparabolic relation for $E(k)$ is not really very significant, particularly in view of the other uncertainties.

It is worthwhile here to point out the reason, already known, for uncertainty in the barrier parameters. Leaving aside a few cases of single-crystal and muscovite mica films, the tunnel junctions in general are prepared by either thermal oxidation or plasma anodization.[16] Since these processes involve random deposition of insulator atoms onto the parent metal, the thickness of the film is nonuniform at least on an atomic scale.[17] This causes a wide difference and uncertainty in the net thickness of the insulator region. Further, because of different extents of chemical contamination and characteristic growth kinetics,[18] the effective work function, too, differs from sample to sample and laboratory to laboratory.[9] Finally, there is no definitive technique[19] available to deduce the electron effective mass in thin films of insulators, especially for amorphous (e.g., Al_2O_3) films. The values which are used in calculations to date are arbitrarily chosen (in the light of the value of band gap) so as to fit the theory to experiments.[20] In addition, if the image force is also included in the tunnel model, there is still a dispute[21] over the use of a static or optical value of the dielectric constant of the insulator film.

Fortunately, these comments are not directly applicable to single-crystal films (like that of GaSe) where the major uncertainties in the work function and thickness values are removed to a great extent. However, in general, since the agreement between the theory and experiments is not unique[22] to the band-structure effect (in the sense that it may be ascribable to other effects

as well), such a deduction is approximate for single-crystal films and unacceptable for amorphous cases.

Finally, a general drawback of all band-structure deductions (for insulators and semiconductors) is that the charge-transfer mechanism has not been convincingly shown to be due to tunneling only. Also the validity of the independent-particle approach has not been fully established as far as MIM structures are concerned. These points together with the justification in using WKB approximation have been discussed in detail by Duke.[23]

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