manner is presently available for any single binary system.

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Possibility of Thermal Corrections in Radiative Level-Shift Determination

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We give an estimate, based on nonrelativistic quantum mechanics, of the thermal contribution to the Lamb shift in the blackbody approximation. We show that observable corrections may exist.

Despite the excellent agreement between the measured and the calculated Lamb shifts[1] in H, there remain small differences between theory and experiment. A thorough review of the status of the agreement as of 1969 is given by Taylor, Parker, and Langenberg[2] who suggest that the difference, although small by normal standards, is large enough to warrant concern. The experimental shifts discussed in Ref. 2 are for the most part slightly larger than theory predicts. Recent calculations of $O(\alpha^2)$ corrections by Appelquist and Brodsky[3] have led to somewhat larger α^2 corrections and theoretical shifts which are in turn slightly larger. In the present note, we discuss the possibility that the thermal background which is necessarily present in any experiment may give a measurable contribution to the level shift. The thermal contribution to the mass of a free particle has been discussed previously by Englert.[4]

Since the effective radiation temperature should be small compared with the energies of the levels in question, we do the calculation in the nonrelativistic approximation.[5] We write the unsubtracted expression for the shift of the mth level using the notation of Ref. 5:

$$W = -\frac{2}{3\pi}\frac{\alpha}{c^2}\int_0^{k_c} dk\, k \sum_n \frac{|v_{mn}|^2}{E_n - E_m + k}. \qquad (1)$$

If the additional shifts which occur in the presence of a thermal background are to be taken into account, the emission probability must include the stimulated contribution. Furthermore, since this is a self-energy correction arising from the scattering of real photons, absorption must now be included. The integrand of Eq (1) will then have two additional terms: the emission term having a coefficient $\bar{n}+1$, and an absorption term having a coefficient $\bar{n}$. We will evaluate these contributions in the blackbody approximation, i.e.,

$$\bar{n} = (e^{k/T} - 1)^{-1}. \qquad (2)$$

The term arising from the spontaneous emission is the usual Lamb shift (in the nonrelativistic approximation) and may be treated in the usual way. The thermal contribution to the shift, W_T, is

$$W_T = -\frac{2}{3\pi}\frac{\alpha}{c^2}\sum_n |v_{mn}|^2 \int_0^\infty dk\, k (e^{k/T} - 1)^{-1} \times \left[\frac{1}{E_n - E_m + k} + \frac{1}{E_n - E_m - k}\right], \qquad (3)$$

where the first term comes from emission and the second from absorption. The upper limit of the integral has been carried to infinity since it is convergent. The terms within the brackets in Eq. (3) may be expanded in a power series in $k/(E_m - E_n)$ and the resulting integrals expressed in terms of ζ functions. Since we expect that the ratio of the temperature to the energy differences will be small, we write only the first two terms in the expansion:

$$W_T \cong -\frac{4\alpha}{3\pi c^2} \times \sum_n |v_{mn}|^2 \left[\frac{\pi^2}{6}\frac{T^2}{(E_n - E_m)} + \frac{\pi^4}{15}\frac{T^4}{(E_n - E_m)^3}\right]. \qquad (4)$$

The first term in Eq. (4) may be evaluated with the dipole sum rule.[6] The shift is

$$W_T^{(1)} = \tfrac{1}{3}\pi\alpha T^2/mc^2. \quad (5)$$

It is the same for all levels and therefore cannot be observed spectroscopically. There is, however, a contribution to the total mass. We should point out that Eq. (4) is larger by a factor of 4π than Englert's free-particle correction. Although relativistic formalism was used in Ref. 4 it does not seem that it should differ by such a large factor. The origin of the discrepancy has not been pinned down. The second term in Eq. (4) may be rewritten as

$$W_T^{(2)} = \frac{4\pi^3}{45}\left(\frac{T}{\alpha mc^2}\right)^3 TB, \quad (6)$$

where

$$B = \frac{-e^2}{a^3}\sum_n \frac{|r_{mn}|^2}{E_m - E_n}, \quad (7)$$

$|r_{mn}|^2$ is the dipole matrix element, and a is the Bohr radius. The contribution to the thermal shift is related to the polarizability of the state. Although it may be possible to obtain a closed solution[7] for the sum in Eq. (7) for certain initial states E_m, it is quite straightforward to evaluate it numerically when the states in question are hydrogenic. The dipole matrix elements for the Balmer series are readily available.[8] All the shifts predicted are of course positive. However, the $2p_{1/2}$ shift is greater than the $2s_{1/2}$ shift which makes the contribution to the Lamb shift negative. This appears to be in a helpful direction for some of the most recent calculations.[3] The thermal shift of the $2s_{1/2}$ level minus that of the $2p_{1/2}$ level is

$$\Delta W_T^{(2)} = \frac{4\pi^3}{45}\left(\frac{T}{\alpha mc^2}\right)^3 T[B(2s_{1/2}) - B(2p_{1/2})]. \quad (8)$$

Using the more detailed notation of Ref. 8, p. 254, the terms within the brackets become

$$B(2s_{1/2})\frac{8}{a^2}\sum_n \frac{n^2|R_{20}{}^{n1}|^2}{n^2-4}, \quad (9)$$

and

$$B(2p_{1/2}) = \frac{8}{a^2}\sum_n \frac{n^2}{n^2-4}\left(\tfrac{1}{3}|R_{21}{}^{n0}|^2 + \tfrac{2}{3}|R_{21}{}^{n2}|^2\right), \quad (10)$$

where the term $n=2$ is to be deleted from the sum. The matrix elements as a function of n (Ref. 8, p. 262) can now be used to give an expression for the terms within the square brackets in Eq. (8):

$$B(2s_{1/2}) - B(2p_{1/2}) = \frac{2^{18}}{3}\sum_n\left[\left(\frac{n-2}{n+2}\right)^{2n-8} \times \frac{1}{(1+2/n)^{16}}\left(\frac{1}{n^3} - \frac{48}{n^5} + \frac{48}{n^7}\right)\right]. \quad (11)$$

Evaluating Eq. (11) numerically, multiplying by the coefficients in Eq. (8), and dividing by Planck's constant gives the thermal correction to the Lamb shift,

$$\Delta f = -1.13[T(\mathrm{eV})]^4 \ \mathrm{MHz}. \quad (12)$$

Rather large temperatures are necessary in order to produce measurable shifts. It would be presumptous, therefore, to claim that Eq. (12) explains any particular experimental discrepancy before a systematic study of the thermal background is undertaken. However, it is interesting that in recent measurements[9] of the shift for C^{5+}, deviations of approximately 5% are observed and a large background is also reported. While this indicates that background radiation may be important, a temperature of $\sim 10^2$ eV is necessary in order to produce such a deviation since the thermal correction also goes as Z^{-4} for hydrogenic ions. This may be unreasonably large even for a very nonequilibrium effective temperature, although such a temperature may be a good deal larger than the ambient temperature.

The actual radiation temperature in any given experiment will be a complicated function of beam and receiver geometries. Strictly speaking the corrections to the line profile should be calculated. This, however, will depend upon the details of a particular apparatus. A systematic dependence of the line center on the effective radiation temperature is predicted by the above calculation which does indicate that an experimental study of the effects of artificially imposed background radiation may be of interest.

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Solar Oblateness and the Abundance of Lithium in the Sun*

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The Goldreich-Schubert thermally driven turbulence extending to the deep interiors of solar-type stars which exhibit rotational slowing is precluded by spectroscopic observations showing the presence of lithium and beryllium. These observations, similar observations of the sun, the solar-wind torque, and the solar oblateness are consistent with a model for which such a turbulence is terminated at the surface of a rapidly rotating core containing 95% of the star's mass.

It has been suggested that an excess solar oblateness might be due to the gravitational quadrupole moment of a rapidly rotating core in the sun and that the resulting quadrupole moment might induce ~10% of the part of the motion of Mercury's perihelion considered to be relativistic.[1,2] The required rotation rate of the core is about the same as that of the surfaces of very young solar-type stars slightly more massive than the sun. This surface rotation decreases with age.[3] The central question concerns the interior. As surface rotation decreases, is a rapidly rotating core left in the stellar interior?

Goldreich and Schubert[4,5] have suggested that, because of a thermally excited instability in the solar interior, the angular velocity of the sun is substantially constant down to the hydrogen burning core, which could be rapidly spinning. Then, a slowing of the surface rotation would imply a slowing of most of the sun's interior, a mild thermally driven turbulence transporting angular momentum upward from the deep solar interior by turbulent diffusion.

It has been difficult to assess the existence or efficacy of the Goldreich-Schubert instability. It is very easily quenched by slight molecular-weight gradients or by nonrotational fluid motion.[4] A molecular-weight variation of $\sim 2\times10^{-3}$ is enough to terminate it, and a nonrotational fluid motion with velocities as small as a few centimeters per second can quench or modify the instability.

The discussion below pivots on the following observations: (1) For stars of mass 1.2 the values at the surface of angular velocity and lithium abundance decrease with age without a decrease in beryllium. (2) The sun is losing angular momentum at a rate of roughly 5×10^{30} dyn cm and has lost 99.5% of its surface lithium without loss of beryllium.

The following points will be discussed in order:

(1) Assuming angular-momentum transport by fluid motion ("spin-down" currents or turbulent diffusion), a slowing of the rotation of deep interiors of stars of mass 1.2 along with the surface is precluded by the slight loss of lithium and the constancy of beryllium at the surfaces of these stars.

(2) The assumption that the angular momentum of the outer 5% of a star (by mass) is transported to the surface by means of a mild turbulent diffusion is compatible with the depletion of lithium and the constancy of beryllium.

(3) Under the same assumptions, the "observed" solar-wind torque implies a present loss rate for lithium compatible with the total observed loss in the sun.

(4) The solution of equations governing slowing of the sun's surface rotation and lithium depletion yields values for the solar-wind torque close to that "observed."

(5) For a solar core rotating fast enough to account for the excess solar oblateness the maximum rate of loss of angular momentum by vis-