vious estimations of the nuclear isospin impurity, without considering the proton-neutron correlation, have already shown that it is quite small, of the order of a fraction of a percent.[6,8]

Finally it can be shown that, for the special nondegenerate case (8), the superallowed Fermi-β^+-decay matrix element does not deviate from the expected value between states of good isospin, i.e.,

$$\langle\tilde{0}|T_+B_{\mathrm{IAS}}^\dagger|\tilde{0}\rangle = c_\alpha x_\alpha + c_\alpha C_{\alpha\bar{\alpha}} d_{\beta\bar{\alpha}} x_\beta - c_\alpha d_{\alpha\bar{\alpha}} y_{\bar{\alpha}} + x_\alpha d_{\alpha\bar{\alpha}} c_{\bar{\alpha}} - c_{\bar{\alpha}} d_{\alpha\bar{\alpha}} C_{\alpha\bar{\beta}} y_{\bar{\beta}}$$
$$\simeq (2T_0)^{1/2} = \langle T_0, T_0|T_+|T_0, T_0-1\rangle, \tag{14}$$

to second order in Δ, in the approximation discussed here.

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[1]C. A. Engelbrecht and R. H. Lemmer, Phys. Rev. Lett. 24, 607 (1970).

[2]We restrict ourselves to particle-hole pairs that can couple to $J^\pi = 0^+$ only.

[3]See, e.g., J. Jänecke, in *Isospin in Nuclear Physics*, edited by D. H. Wilkinson (North-Holland, Amsterdam, 1969).

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[6]H. C. Lee and R. Y. Cusson, to be published.

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Excitation of Abnormal Parity States by α Particles Acting with Velocity-Dependent Central Forces*

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The violation of the normal parity rule in (α,α') scattering is related to a velocity dependence in a purely central, spin-independent α-N force. The distorted-wave Born-approximation formalism of the process is presented, and the specific case of the 3^+ state in Ni^{58} is calculated in the plane-wave Born approximation. The role of the velocity dependence is discussed in physical terms, and a formal similarity with Weber's old theory of electromagnetism is found.

It is well known that in single-stage inelastic scattering the transfer of orbital angular momentum l from the projectile to the target and the parity change $\Delta\pi$ of the target are related by the so-called "normal" parity rule $(-)^l = \Delta\pi$, if the force between the projectile and the target nucleons is static and local, and exchange is ignored. Essentially, this is because the only operator in the multipole expansion of such a force that has the required multipolarity l is Y_l. One particularly striking consequence of this rule is the absolute prohibition of the direct, single-stage excitation of states of abnormal parity from a 0^+ ground state by α particles since in this case there cannot, of course, be any question of spin flip.

Nevertheless, such processes do occur, and as one of several possible explanations a spin-orbit α-N force has been proposed by Eidson and Cramer.[1] Although no calculation has been published to our knowledge, a more complete discussion was given by Satchler[2] who showed that the essential role of the spin-orbit force was to permit the formation of composite tensors through the coupling of the $\vec{L}$ operator with spherical harmonics. Without at all developing the idea he then indicated that a $\vec{p}$ operator (momentum) would do just as well as the $\vec{L}$ operator coming from the spin-orbit force.

In the light of this suggestion, we wish to point out that excitation of abnormal parity states can take place with a central, spin-independent α-N force, provided it is allowed to be velocity dependent. We have, in fact, found that we can get a rough fit to the α-N s-wave phase shifts[3] between 0 and 40 MeV with an interaction of the form[4]

$$V_{\alpha i}=\hbar^{-2}[p^2 g(r)+g(r)p^2], \tag{1}$$

where $g(r)=B\exp(-\beta^2 r^2)$, with $B=20.7$ MeV fm^2 and $\beta=0.2$ fm^{-1}. This potential is, of course, far from unique, but we would remark that there must be *some* velocity dependence: The basic nucleon-nucleon force is known from meson theory to have a nonlocal character, and the first term in the expansion of this will involve a p^2 factor. Actually, most earlier attempts to find an α-N potential have involved energy-dependent parameters[5] (note that such potentials are non-Hermitean), and in fact we believe that it would be rather difficult to fit the data over the above range with a purely static, local potential. This is especially the case if care is taken to avoid the spurious 1s bound state.

Several new tensor products appear in the multipole expansion of the force when it contains a p^2 dependence, but the only ones that contribute to transitions that violate the normal parity rule are $[X_l^{(1)}(\alpha)X_l^{(1)}(i)]_{I=0}$ and $[X_l^{(2)}(\alpha)X_l^{(2)}(i)]_{I=0}$, where

$$X_l^{(1)}=[[Y_1L_1]_1Y_l]_l, \tag{2a}$$

$$X_l^{(2)}=[Y_l[Y_1L_1]_1]_l, \tag{2b}$$

where the brackets denote tensor coupling and L_1 is the orbital angular momentum operator.

The generalization of Satchler's distorted-wave Born-approximation description of inelastic scattering[6] is then relatively straightforward. We get for the "reduced amplitude"

$$\beta_l^m=\sum_{l_\alpha,l_\alpha'}\sum_{\omega=1,2}\Gamma_{l,l_\alpha l_\alpha'}{}^{\omega,m}I_{l,l_\alpha l_\alpha'}{}^{\omega}P_{l_\alpha'}{}^{|m|}(\cos\theta), \tag{3}$$

where

$$\Gamma_{l,l_\alpha l_\alpha'}{}^{\omega,m}=i^{l_\alpha'-l_\alpha+l+|m|+m}\left[\frac{(l_\alpha'-|m|)!}{(l_\alpha'+|m|)!}\,\frac{(2l_\alpha+1)(2l_\alpha'+1)}{(2l+1)}\right]^{1/2}\langle Y_{l_\alpha'}{}^m|X_l^{(\omega)m}|Y_{l_\alpha}{}^0\rangle \tag{4}$$

and

$$I_{l,l_\alpha l_\alpha'}{}^{\omega}=\frac{16\pi^2}{3k_\alpha k_\alpha'}\int_0^\infty u_{l_\alpha'}(r)\,\mathcal{G}_l{}^\omega(r)u_{l_\alpha}(r)\,dr. \tag{5}$$

In this latter equation we have introduced the radial form factor

$$\mathcal{G}_l{}^\omega(r_\alpha)=-4\pi i^l\sum_{i(A)}\left\langle j_A'\,\Big\|\frac{1}{r_\alpha r_i}h_l(r_\alpha,r_i)X_l^{(\omega)}\Big\|\,j_A=0\right\rangle, \tag{6}$$

where the multipole h_l is given in terms of the function $g(r)$ appearing in the force (1) by

$$g(r)=4\pi\sum_{l=0}^{\infty}\sum_{\mu=-l}^{+l}h_l(r_\alpha,r_i)Y_l{}^\mu(\hat r_0)Y_l{}^{*\mu}(\hat r_i). \tag{7}$$

The differential cross section is then given by

$$\frac{d\sigma}{d\Omega}=\frac{M_\alpha{}^2}{4\pi^2\hbar^4}\,\frac{k_\alpha'}{k_\alpha}(2j_A'+1)\sum_m|\beta_l{}^m|^2. \tag{8}$$

By way of example, we have chosen to consider the excitation of the 3^+ state at 3.414 MeV in Ni^{58} since this is a case which is particularly simple from the point of view of the nuclear matrix elements,[7] although unfortunately there is very little in the way of data.[8] Performing the calculation in plane-wave approximation, the above interaction gives 79 mb for the integrated cross section at 18 MeV, while at 40 MeV this has risen to 213 mb. No doubt the plane-wave Born approximation has considerably overestimated this cross section; but we believe, nevertheless, that we have here a viable mechanism for violating the normal parity rule, and that it is worthy of further study along, of course, with the other possible explanations.[1]

Despite the appearance of new tensors it may be found rather surprising that with a purely central force it is possible to change the selection rules simply by introducing a momentum dependence. We shall conclude, therefore, by attempting to give some physical insight into the way this new reaction mechanism works. With $\vec{l}_\alpha$ and $\vec{l}_\alpha'$ being the orbital-angular-momentum vectors of the α particle before and after impact, respectively, we shall have $\vec{l}_\alpha=\vec{j}_A'+\vec{l}_\alpha'$ since $j_A=0$. Then, if there is no change in the *direction* of the α-particle's angular momentum, it follows that

$j_A' = |l_\alpha \pm l_\alpha'|$. Since the parity change of the target is given by $\Delta\pi = (-)^{l_\alpha + l_\alpha'}$, we see at once that abnormal parity transitions require that the α-particle's angular momentum change direction during the scattering,[9] i.e., there must be "orbit tilt" (for nucleons, "spin flip" plays the same role). In this respect it is significant that of the several tensors associated with a p^2 force, the only ones that contribute to abnormal parity transitions are those that contain L_1: It is $L_1^{\pm 1}$ that are responsible for the tilting.

Physically, it is very east to see how a spin-orbit component in the two-body force can give rise to this tilting. That a purely central force can have the same effect provided it is velocity dependent is less obvious but can be understood on noticing the similarity with the electromagnetic interaction between a long wire and a small circuit: The two tend to swing together into the same plane. Although his theory has other defects, and was eventually replaced by the field theory of Maxwell, an adequate description of this phenomenon was given by Weber[10] in terms of a central force

$$\vec{F} = (e^2/r^2)[1 - (\dot{r}^2 - 2r\ddot{r})/c^2]\hat{r} \tag{9}$$

acting between charges. Now the potential (1) corresponds classically to the velocity-dependent force[11]

$$\vec{F} = [f(r)\ddot{r} + \tfrac{1}{2}f'(r)\dot{r}^2]\hat{r}, \tag{10}$$

where

$$f(r) = (4M^2/\hbar^2)g(r)[1 + (4M/\hbar^2)g(r)]^{-1}. \tag{11}$$

But the Weber force (9) is simply a special case of (10), with $f(r) = 2e^2/c^2r$ and the static Coulomb term added. Thus velocity-dependent potentials of the form (1) may be effectively regarded as corresponding to generalized Weber forces, whence the tilting action can be understood at once since the radial form $f(r)$ is immaterial.

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[2]G. R. Satchler, in *Lectures in Theoretical Physics*, edited by W. E. Brittin *et al.* (U. of Colorado Press, Boulder, Colorado, 1966), Vol. VIII-C, p. 91.

[3]See, for example, J. L. Gammel and R. M. Thaler, Phys. Rev. 109, 2041 (1958); also, R. A. Arndt and L. D. Roper, Phys. Rev. C 1, 903 (1970).

[4]The fit might be improved by the addition of a static term; but, since this would not contribute to the abnormal parity transition, the only effect would lie in a slight modification of the parameters B and β.

[5]See, for example, G. R. Satchler, L. W. Owen, A. J. Elwyn, G. L. Morgan, and R. L. Walter, Nucl. Phys. A112, 1 (1968).

[6]G. R. Satchler, Nucl. Phys. 55, 1 (1964), and 77, 481 (1966). We are using essentially the same notation, except that we denote the target spin by j_A (=0) and j_A'; the reduced matrix elements are defined in the same way. The generalization of the distorted-wave Born-approximation theory of inelastic scattering to take account of p^2 forces is considerably more complicated for normal parity transitions and for nucleons: A complete account is being presented elsewhere. See also E. Boridy and J. M. Pearson, Bull. Amer. Phys. Soc. 16, 580 (1971).

[7]We use the wave functions of M. M. Stautberg, Phys. Rev. C 3, 1541 (1971); note that these do not include any excitation of the Ni^{56} core.

[8]O. N. Jarvis, B. G. Harvey, D. L. Hendrie, and J. Mahoney, Nucl. Phys. A102, 625 (1967).

[9]This can also be seen in Eq. (4), which, applying the Wigner-Eckart theorem, vanishes for $m=0$.

[10]W. Weber, Ann. Phys. (Leipzig) 64, 337 (1845). See also E. Whittaker, *A History of Theories of Aether and Electricity* (Thomas Nelson and Sons Ltd., London, England, 1951).

[11]See Chap. 1 of H. Goldstein, *Classical Mechanics* (Addison-Wesley, Reading, Mass., 1950). Note that velocity-dependent potentials V, such as (1), are to be interpreted as defining the Hamiltonian according to $H = p^2/2m + V$. Expressing the canonical momentum $\vec{p}$ in terms of $\dot{\vec{r}}$ then enables Goldstein's generalized potential U to be obtained from V, whence the force follows at once.