Observation of Quantum Effects in the Spectrum of Piezoelectrically Amplified Acoustic Flux in GaAs

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(Received 19 May 1971)

Using x-ray scattering, we have made the first observations of the spectrum of acoustoelectrically amplified phonons in the quantum limit, where the conduction-electron system obeys quantum statistics and the observed phonon wave vector q is comparable to the electron Fermi wave vector k_F. In GaAs samples with degenerate carrier distributions, we observe a dramatic quantum anomaly consisting of an abrupt decrease in phonon gain for q larger than $2k_F$.

Using x-ray scattering, we have made the first observations of the spectrum of acoustoelectrically amplified phonons in the quantum limit, i.e., where the electron system obeys quantum or degenerate statistics and where the phonon wave vector is comparable to the wave vector of an electron on the Fermi surface.

We observe a Kohn-type anomaly,[1] i.e., a dramatic decrease in acoustic gain for acoustic wave vectors q larger than twice the radius of the electron Fermi sphere, k_F. This occurs because, in an electron gas obeying degenerate statistics, phonon emission with simultaneous conservation of energy and momentum between the initial and final electron states is not possible for $q \geq 2k_F$.

We observe this acoustic gain anomaly in GaAs with a dc electric field present; in the absence of the dc field, the electron-phonon coupling causes acoustoelectric attenuation, and the anomaly appears as an abrupt decrease in attenuation for $q \geq 2k_F$. This type of anomaly occurs in any crystal with degenerate conduction electrons, i.e., in piezoelectric semiconductors, nonpiezoelectric semiconductors, and metals. (In nonpiezoelectric crystals, the coupling is via the deformation potential.) For a general Fermi surface, the gain anomaly occurs at the same point as the Kohn[1] velocity anomaly. As discussed below, the gain anomaly is much stronger than the previously reported velocity anomaly.[2]

Thermal shear waves were acoustoelectrically amplified[3,4] by applying a pulsed dc electric field of 150 V/cm along the [011] direction of a GaAs epitaxial layer. The epitaxial layer, which was n type and doped with selenium, was 10 μm thick and was grown on a semi-insulating GaAs substrate. The sample temperature was 20°K and the free-electron concentration was 1.35×10^{17} cm^{-3}; hence, the free electrons obeyed degenerate statistics. The Hall mobility of the epitaxial layer was 5000 cm^2/V sec. The sample resistance was Ohmic during the first microsecond of the pulse; the current then decreased[5] 20% as a result of the acoustoelectric interaction with the intense acoustic flux.

The acoustic intensity as a function of q was obtained by measuring the scattered x-ray intensity as a function of sample orientation[6]; this measurement involved scanning over the (400) GaAs reciprocal lattice point, along the [011] direction, by rotating the crystal through the Bragg angle θ and keeping the detector fixed at 2θ. Cu $K\alpha_1$ x rays were used, and were incident on the sample 1 mm from the positive contact.

Figure 1 shows a typical spectrum. The sharp drop in intensity at $q = 3.2 \times 10^6$ cm^{-1} (indicated by A) occurs at a phonon wave vector equal to twice the radius of the electron Fermi sphere. The primary peak at $q = 1.85 \times 10^6$ cm^{-1} occurs at the wave vector of maximum net gain; the net gain is the acoustoelectric gain minus the non-

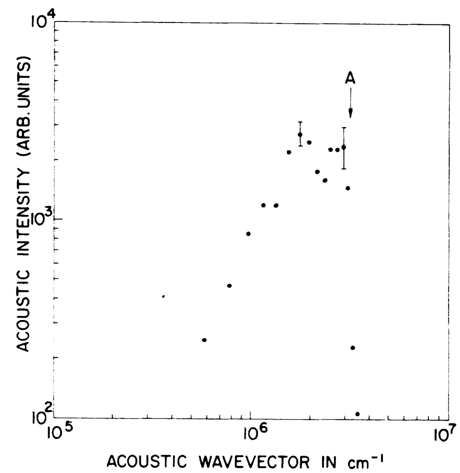

FIG. 1. Experimentally observed acoustic intensity versus acoustic wave number for fast transverse waves in the [011] direction. The symbol A indicates the point where $q = 2k_F$.

electronic acoustic losses $\alpha_{\rm loss}$. The secondary peak at $q=2.7\times10^6$ cm^{-1} is attributed to parametric upconversion, and subsequent linear amplification, of lower frequency modes.

We now discuss a quantum calculation of the acoustic attenuation due to piezoelectric coupling to a degenerate gas of electrons in the absence of a dc electric field. A dc field when present serves to displace the electron distribution in momentum space, causing the sign and/or magnitude of the attenuation to change. However, we expect the shape of a curve of acoustoelectric attenuation versus wave number to be relatively unchanged by the application of a dc field for the range of fields and material parameters used in our experiments. Several semiclassical Boltzmann-equation analyses[7,8] of the acoustoelectric gain have shown that, for $ql\gg1$, where l is the electron mean free path, the electric field changes the magnitude and/or sign of the gain, but not its dependence on q.

The acoustoelectric attenuation is found from the electrical conductivity $\sigma'(q,\omega)$[4]:

$$\alpha=-qK^2\,{\rm Im}(1-i\sigma'/\omega\epsilon_0)^{-1}, \qquad (1)$$

where K is the electromechanical coupling constant,[4] ϵ_0 is the real dielectric constant of the medium, and ω is the acoustic angular frequency. The conductivity $\sigma'(q,\omega)$ is found from a single-particle density-matrix calculation. Plane-wave electron eigenfunctions are assumed. Electron collisions with impurities and lattice imperfections are described by a relaxation time τ. The diffusion current due to spatial bunching of the electrons is included. The treatment of the diffusion current follows that of Greene *et al.*[9] in their calculation of the magnetoconductivity tensor. Several previous density-matrix calculations neglected the diffusion-current contribution to the conductivity.[10,11] Only the results are given here; a more detailed analysis will be the subject of a later publication. The total conductivity $\sigma'(q,\omega)$ is given by

$$\sigma'=\sigma+\delta, \qquad (2)$$

where

$$\sigma=-\sigma_0 L_1\frac{1+i\omega\tau}{(qc\tau)^2},\qquad \delta=-\frac{\sigma L_1}{L_1+i\omega\tau L_2},$$

and

$$L_1=(3m^2c^2/4\hbar^2qk_{\rm F}^3)[A(q_m')-A(q_p')],$$

$$L_2=-\frac{3m^2c^2}{2\hbar^2qk_{\rm F}^3}\left\{(k_{\rm F}^2-\tfrac14q^2)\ln\frac{k_{\rm F}+\frac12q}{|k_{\rm F}-\frac12q|}+qk_{\rm F}\right\},$$

and where

$$A(x)=(k_{\rm F}^2-\tfrac14x^2)\ln[(k_{\rm F}+\tfrac12x)/(\tfrac12x-k_{\rm F})]+xk_{\rm F},$$

$$q_m'=-2m\omega/\hbar q-q+i2m/\hbar q\tau,\quad q_p'=q_m'+2q.$$

In the above equations σ_0 is the dc electrical conductivity and m is the electron effective mass.

Figure 2 shows the linear acoustoelectric attenuation constant as a function of acoustic wave vector calculated from Eqs. (1) and (2). The acoustic intensity, which is plotted in Fig. 1, depends exponentially on the gain constant. The material parameters used to calculate Fig. 2 are those of the sample discussed in connection with Fig. 1; no adjustable parameters were used. The calculated attenuation shows the marked decrease at the point marked A, where $q=2k_{\rm F}$ $=3.18\times10^6$ cm^{-1}. The wave vector of maximum attenuation occurs at $q=2.4\times10^6$ cm$^{-1}\approx\sqrt3q_{\rm FT}$, where $q_{\rm FT}$ is the reciprocal screening length[8] for an electron gas obeying quantum statistics: $q_{\rm FT}=\sqrt3\omega_p/v_{\rm F}$, with ω_p the plasma frequency and $v_{\rm F}$ the Fermi velocity.

The observed wave vector of maximum intensity, $q_{\rm max}$, of Fig. 1 is about 25% less than the calculated wave vector of maximum acoustoelectric attenuation. However, $q_{\rm max}$ represents the wave vector of maximum net gain; since $\alpha_{\rm loss}$ increases with wave vector, $q_{\rm max}$ must be less than the wave vector of maximum acoustoelectric gain.

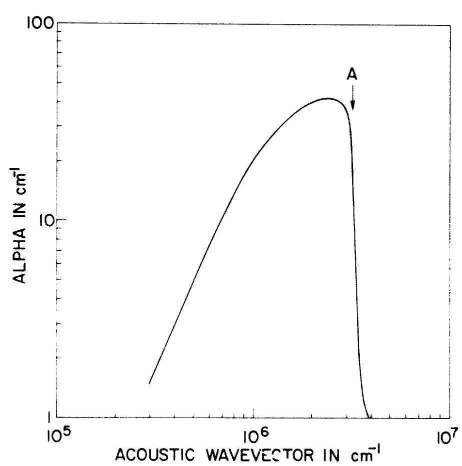

FIG. 2. Calculated acoustic attenuation, alpha, due to acoustoelectric coupling versus acoustic wave number for the fast transverse waves in the [110] direction of GaAs. The symbol A indicates the point where q $=2k_{\rm F}$. The carrier density and electron collision time are assumed to be 1.35×10^{17} cm^{-3} and 0.02×10^{-11} sec; zero electric field and degenerate electron statistics are assumed.

The decrease in acoustic gain for $q \geq 2k_F$ is related to the acoustic velocity anomaly seen in metals by Brockhouse, Rao, and Woods,[2] and predicted by Kohn.[1] However, the gain anomaly is much stronger than the velocity anomaly. The velocity and gain depend on $\mathrm{Re}\epsilon(q, \omega)$ and $\mathrm{Im}\epsilon(q, \omega)$, respectively, where $\mathrm{Re}\epsilon(q, \omega)$ is the real part of the dielectric response function $\epsilon(q, \omega)$[12]; $\epsilon(q, \omega)$ is related to the conductivity by $\epsilon(q, \omega) = 1 + \sigma(q, \omega)/4\pi i\omega$. The velocity anomaly appears (in the absence of collisions and at zero temperature) as a logarithmic singularity in $\partial\mathrm{Re}\epsilon(q,\omega)/\partial q$ at $q = 2k_F$; the gain anomaly appears as a discontinuity in $\mathrm{Im}\epsilon(q, \omega)$ at $q = 2k_F$, and hence the singularity in $\partial\,\mathrm{Im}\epsilon/\partial q$ is stronger than that in $\partial\,\mathrm{Re}\epsilon/\partial q$. When collisions are included, the derivatives of $\epsilon(q, \omega)$ are large but finite at $q = 2k_F$, but the gain anomaly is still found to be much larger than the velocity anomaly.

The large value of $\partial\epsilon(q, \omega)/\partial q$ at $q = 2k_F$ can be traced to the sharpness of the Fermi sphere coupled with the requirement of conservation of energy and momentum. Anything that reduces the sharpness of the Fermi sphere (such as finite temperature or carrier concentration inhomogeneities) or that relaxes the conservation requirements (such as collisions) will reduce $\partial\epsilon(q, \omega)/\partial q$ at $q = 2k_F$; this in turn reduces the rate of change, with respect to q, of the acoustoelectric gain near $q = 2k_F$. The anomalies are increasingly difficult to see as the temperature is increased or the scattering time is decreased.

For the range of parameters found in degenerate semiconductors, the attenuation or gain anomaly at $q = 2k_F$ is a sensitive measure of the Fermi surface; the attenuation anomaly is also a sensitive probe to determine the Fermi surface of metals but the difficulties of observing attenuation or gain at $q = 2k_F$ in metals are considerable.

It is a pleasure to acknowledge the help on the density-matrix calculation given us by T. D. Schultz and J. W. Woo. We are also indebted to N. S. Shiren, E. Burstein, M. Pomerantz, J. A. Armstrong, and J. F. Woods for helpful discussions, and to D. R. Vigliotti for expert experimental assistance.

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