Chirality in Dual-Resonance Models*

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We present a dual-resonance model for N pions which satisfies the Adler self-consistency condition and provides a generalization of the Lovelace-Shapiro formula for four pions. Our model has difficulty with SU(3) since there is no candidate for the ω degenerate with the ρ. We also discuss briefly the problem of extending the model to include fermions.

A long-standing problem for dual-resonance models has been the consistent treatment of N-pion scattering. The four-pion dual amplitude, proposed long ago by Lovelace and Shapiro,[1] satisfied the Adler self-consistency condition provided that the ρ and π trajectories were separated by $\frac{1}{2}$ unit. Thus chirality seemed to place realistic constraints on the masses of the π and ρ. But since there are perfectly good dual chiral four-point functions which do not need this constraint,[2] the question of a consistent generalization to N pions becomes very important. One possible chiral N-pion amplitude has already been proposed by Brower.[3] Unfortunately, this model has a tachyon associated with the ρ trajectory and, furthermore, has an arbitrary ρ intercept, so that the constraints seen in the Lovelace-Shapiro formula do not appear. In this Letter we show that a simple generalization of the Neveu-Schwarz model has no such tachyon and, in fact reduces to the Lovelace-Shapiro model in the four-point case. We stress that, although there are no ghosts on leading or on the first subsidiary trajectories, there are ghosts on lower trajectories. Since similar problems occur in the ordinary dual-resonance model, we do not believe that this disease is connected with chirality.

Before we present our model we describe a few of its properties. Of course it has all the nice features of the ordinary dual models: crossing symmetry, factorization, and Regge behavior. The spectrum of resonances includes a pion at $m_\pi^2=0$, a ρ and σ at $m_\rho^2=m_\sigma^2=\frac{1}{2}$, and an ω at $m_\omega^2=1$. (We take the slope of all trajectories to be 1 BeV^{-2}.) In terms of trajectories there is the exchange-degenerate ρ-f_0 with intercept $\frac{1}{2}$ and an ω-A_2 degenerate pair with intercept 0 (degenerate with the pion trajectory). The breaking of the degeneracy between the ρ-f_0 and the ω-A_2 is probably the most serious deficiency of the model (aside from ghosts). There is an intrinsic breaking of SU(3). Although we choose the intercept of the pion to be 0 to ensure chiral symmetry, the model can, in fact, be written for arbitrary intercept. At $\alpha_\pi=\frac{1}{2}$ it is precisely the Neveu-Schwarz model.[4] This freedom of choice of pion intercept may be useful for studying the chiral breaking effects of a nonzero pion mass. We also have a method for introducing a single fermion line so we can describe N-pion–fermion scattering.

The simplest way to present the model is through the purely formal trick of increasing the dimensionality of the harmonic oscillators and the momenta to $N+4$ for the N-pion function. We assign the $(N+4)$-dimensional momenta $\hat{p}_k$ as follows:

$$\hat{p}_1=\begin{bmatrix} p_1 \\ c \\ -c \\ 0 \\ 0 \\ \cdots \\ 0 \\ 0 \end{bmatrix}, \quad \hat{p}_2=\begin{bmatrix} p_2 \\ 0 \\ c \\ -c \\ 0 \\ \cdots \\ 0 \\ 0 \end{bmatrix}, \quad \cdots, \quad \hat{p}_N=\begin{bmatrix} p_N \\ -c \\ 0 \\ 0 \\ 0 \\ \cdots \\ 0 \\ c \end{bmatrix}. \qquad (1)$$

To insure conformal symmetry we require $\hat{p}_k^2=p_k^2-2c^2=-\frac{1}{2}$ or $m_\pi^2=2c^2-\frac{1}{2}$ for zero-mass pions. Notice that this scheme shifts all masses by $2c^2$ in contrast to the model of Halpern and Thorn.[5] Our model can now be written

$$A_N=\langle 0|\hat{p}_1\cdot\hat{b}_{1/2}\hat{V}_{\hat{p}_2}(\hat{L}_0-1)^{-1}\hat{V}_{\hat{p}_3}(\hat{L}_0-1)^{-1}\cdots(L_0-1)^{-1}\hat{V}_{\hat{p}_{N-1}}\hat{p}_N\cdot\hat{b}_{-1/2}|0\rangle, \qquad (2)$$

where the hats mean simply to use the $(N+4)$-dimensional analogs of the operators in Ref. 4. It is easily seen that the extra oscillators have the following effects:

$$(L_0-1)^{-1}\to\int_0^1 du\,u^{L_0-2}(1-u)^{-2c^2-1}, \tag{3}$$

and a slight modification of the rules for contracting "nearest-neighbor" fields H:

$$\langle 0|\hat{k}_l\cdot\hat{H}(x_l)\hat{k}_m\cdot\hat{H}(x_m)|0\rangle \equiv \begin{cases}(k_l\cdot k_m+c^2)\dfrac{(x_lx_m)^{1/2}}{x_m-x_l}, & m=l+1,\\ k_l\cdot k_m\dfrac{(x_lx_m)^{1/2}}{x_m-x_l}, & \text{otherwise.}\end{cases} \tag{4}$$

Cyclic symmetry is proven by performing the usual manipulations on (2) and then observing that making a cyclic change in the assignment of the extra components of the $\hat{p}_k$'s will have no effect on the amplitude. To see the decoupling of the tachyon and ancestors we use the techniques of Neveu, Schwarz, and Thorn[6] to rewrite Eq. (2) as

$$A_N=\langle 0|\hat{V}_{\hat{p}_2}(\hat{L}_0-\tfrac{1}{2})^{-1}\hat{V}_{\hat{p}_3}(\hat{L}_0-\tfrac{1}{2})^{-1}\cdots(\hat{L}_0-\tfrac{1}{2})^{-1}\hat{V}_{\hat{p}_{N-1}}|0\rangle. \tag{5}$$

The Adler zero is most easily seen when we write (5) as the integral representation

$$\int_0^1 du_2\cdots du_{N-2}u_2^{-(\hat{p}_1+\hat{p}_2)^2-3/2}(1-u_2)^{-2\hat{p}_2\cdot\hat{p}_3}(1-u_2u_3)^{-2\hat{p}_2\cdot\hat{p}_4}\cdots(1-u_2u_3\cdots u_{N-2})^{-2\hat{p}_2\cdot\hat{p}_{N-1}}I$$
$$\times\langle 0|\hat{p}_2\cdot\hat{H}(1)\hat{p}_3\cdot\hat{H}(u_2)\cdots\hat{p}_{N-1}\cdot\hat{H}(u_2u_3\cdots u_{N-2})|0\rangle,$$

where I does not depend on u_2. Letting $p_2\to 0$ we get

$$\int_0^1 du_2 u_2^{-3/2+2c^2}(1-u_2)^{-2c^2}[u_2^{1/2}/(1-u_2)]\int_0^1 du_3\cdots du_{N-2}IJ,$$

with J independent of u_2. But

$$\int_0^1 du_2 u_2^{-1+2c^2}(1-u)^{-2c^2-1}=B(2c^2,-2c^2)=0.$$

We observe that this mechanism for the Adler zero is essentially the same as that of Brower's model.

At first sight our model does not seem to factorize since the number of extra dimensions depends on N. This is illusory, however. It is a very easy exercise to show that, because of the nearest-neighbor coupling, in the form (5) only one of the extra sets of oscillators contributes at any pole: e.g., at a pole in the $(1\cdots l)$ channel only $a_{l+1}{}^m$, $b_{l+1}{}^k$ and $a_\mu{}^m$, $a_\mu{}^k$ contribute. Furthermore, $\langle\lambda|a_{l+1}{}^{k_1+\cdots+k_m}$ and $\langle\lambda|a_{l+1}{}^{k_1}a_{l+1}{}^{k_2}\cdots a_{l+1}{}^{k_m}$ couple in exactly the same way as seen by direct calculation; in the same way $\langle\lambda|b_{l+1}{}^{k+1/2}$ couples just like $\langle\lambda|b_{l+1}{}^{1/2}a_{l+1}{}^k$. Therefore, only states containing $a_{l+1}{}^1$ and $b_{l+1}{}^{1/2}$ can be linearly independent: All other states can be written as linear combinations of these. Finally, the gauges $\hat{G}_{1/2}$ and $\hat{L}_1$ effectively remove the time component of $b_\mu{}^{1/2}$ and $a_\mu{}^1$, respectively. The gauges $\hat{G}_{3/2}$ and $\hat{L}_2$ do not remove states since they involve linear relations among all the $a_k{}^m$'s and $b_k{}^m$'s including those already removed.

It is instructive to write Eq. (5) in a form which does not involve the extra oscillators. We claim that an alternative form for (5) is

$$A_N=\langle 0|G_{1/2}V_{p_2}{}^0DG_{1/2}V_{p_3}{}^0D\cdots DG_{1/2}V_{p_{N-1}}{}^0|0\rangle, \tag{6}$$

where $D=\int_0^1 du\,u^{L_0-3/2+2c^2}(1-u)^{-2c^2}$, and $G_{1/2}$ and V^0 are defined in Ref. 4. To see this, start by moving the left-most $G_{1/2}$ to the right. When it passes through a $V_p{}^0$ it picks up the commutator $[G_{1/2},V_p{}^0]=-\sqrt{2}\,V_p$. Left over is the term

$$V_p{}^0G_{1/2}B(L_0-\tfrac{1}{2}+2c^2,-2c^2+1)G_{1/2}=V_p{}^0B(L_0+2c^2,-2c^2+1)(L_1-L_0)-2c^2V_p{}^0B(L_0+2c^2,-2c^2).$$

The first term on the right is proportional to a gauge which decouples, whereas the second term corresponds precisely to one of the extra terms required by the modified contractions (4). By continuing this process one easily sees that (6) and (5) differ at most by a multiplicative constant. To study factorization in the $(1\cdots k)$ channel, (6) must be symmetrized:

$$A_N=\langle 0|V_{p_2}{}^0G_{-1/2}DV_{p_3}{}^0G_{1/2}\cdots V_{p_k}{}^0(G_{-1/2}DG_{1/2}+2c^2\bar{D})V_{p_{k+1}}{}^0DG_{1/2}\cdots G_{1/2}V_{p_{N-1}}{}^0|0\rangle, \tag{7}$$

where

$$\bar{D}=\int_0^1 du\, u^{L_0+2c^2-1}(1-u)^{-2c^2-1}.$$

The proof of (7) is quite analogous to that of (6). The form (7) is useful for studying the factorization properties of A_N, but cyclic symmetry is not transparent as it is in the form (2).

Our attempt to generalize our model to include a single fermion line runs into difficulties with chirality. The amplitude suggested by the formalism developed by Ramond,[7] Neveu and Schwarz,[8] and Thorn[9] is

$$A_N^{\text{fermion}}=\bar{u}(-p')\langle 0|V_{p_1}{}^0\Gamma^5(F^0-m)DV_{p_2}{}^0\Gamma^5(F^0-m)D\cdots(F^0-)DV_{p_N}{}^0\Gamma^5|0\rangle u(p). \tag{8}$$

where $V_p{}^0$ is the ordinary vertex,

$$\Gamma^5=(-1)^{\sum_n d_n{}^\dagger d_n}\gamma_5,\quad D=\int_0^1 du\, u^{m^2+L_0-1}(1-u)^{-2c^2},$$

and F^0 is the gauge introduced by Ramond. One can show by direct calculation that the residue of the pole at m_π^2 in the fermion-antifermion channel $(f\bar{f})$ of A_{2N-1} factorizes into $u(-p')\gamma_5 u(p)$ times our chiral amplitude for $2N$ pions. But, unfortunately (8) fails to satisfy the Adler self-consistency condition for one soft pion. The problem is that the external line insertions give a Yukawa-type coupling rather than a gradient-type coupling. In fact, for pion-nucleon scattering the A amplitude is identically zero while the B amplitude approaches a nonzero limit as one of the pions becomes soft. Of course, the $I_t=1$ part of the amplitude is linear in the soft momentum as it should be. The fermion problem clearly requires more study.

Isospin is incorporated into our model via the usual Chan-Paton factors. That is, one multiplies A_N by $\mathrm{Tr}(\tau_{a_1}\tau_{a_2}\cdots\tau_{a_N})$ and then sums over all inequivalent cyclic permutations as required by Bose statistics. For the fermion amplitude the isospin factor is $\varphi'^\dagger\tau_{a1}\tau_{a2}\cdots\tau_{aN}\varphi$, where φ' and φ are isospinors for the fermions. With these assignments one obtains the correct isospin assignments for the mesons.

So far we have only described how our model obeys the Adler self-consistency condition for a single soft pion. Other aspects of chiral invariance are also true in our model. For example, the p-wave coupling of two soft pions is universal, which Mandelstam has shown is a general consequence of the Adler condition.[10] Furthermore this universality is not trivial since there are soft poles arising from external line insertions. These points have been discussed by Brower for his model, but the same considerations apply to our model. The present difficulty with fermions is disappointing, but we are hopeful that the problems can be overcome with more sophisticated models.

The really difficult problem to solve is the incorporation of SU(3) into the model. In a sense we have imposed G parity in an artificial way which requires all odd-G-parity trajectories to be split by $\frac{1}{2}$ unit from all even-G-parity trajectories. Clearly, a really satisfactory model must have odd-G-parity states degenerate with even states. Our model may be a useful starting point but we really need some new ideas. Despite all its shortcomings, our model is interesting both as an example of a chiral dual model and as a potentially useful phenomenological tool.

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Note added in proof.—After we completed this work, we received a preprint by John H. Schwarz in which he discussed some of the properties of this model using a slightly different operator formalism.

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ERRATA

MEASUREMENT OF THE VARIANCE OF THE NUMBER OF NEUTRONS EMITTED IN FISSION OF ^{252}Cf AS A FUNCTION OF THE FRAGMENT MASS AND TOTAL KINETIC ENERGY. Avigdor Gavron and Zeev Fraenkel [Phys. Rev. Lett. 27, 1148 (1971)].

Equation (4) should read

$$\langle \nu(A,E_K)[\nu(A,E_K)-1]\epsilon^2(A,E_K)\rangle = \langle \nu(A,E_K)[\nu(A,E_K)-1]\rangle \epsilon^{-2}(A,E_K).$$

Equation (11) should read

$$\sigma_\nu'^2(A,E_K) \simeq \int P_{A,E_K}(E_T)\left[\left(\frac{d\bar{\nu}}{\partial E_K}\right)(A,E_K)(E_T-\bar{E}_T)\right]^2 dE_T = \left(\frac{\partial\bar{\nu}}{\partial E_K}\right)^2 (A,E_K)\sigma_T^2(R,E_K).$$

NEUTRON RADIUS OF ^{208}Pb FROM SUB-COULOMB PICKUP. H. J. Körner and J. P. Schiffer [Phys. Rev. Lett. 27, 1457 (1971)].

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