Calculation of *N-d* Scattering with *S*-, *P*-, and *D*-Wave Forces*

S. C. Pieper
Department of Physics, Case Western Reserve University, Cleveland, Ohio 44106
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A two-potential formalism based on the equations of Alt, Grassberger, and Sanhas is used to generate elastic nucleon-deuteron scattering amplitudes by combing *P*- and *D*-wave nucleon-nucleon forces in first order with "exact" nucleon-deuteron scattering amplitudes for *S*-wave forces. Elastic differential cross sections at 14.1 MeV and nucleon polarizations at 10.04, 14.1, and 22.7 MeV are presented. The cross sections are very accurate while the polarizations are in qualitative agreement with experiment except at 22.7 MeV.

Since the initial computations of Aaron, Amado, and Yam[1] (AAY) in 1965, progress in the calculation of low-energy elastic nucleon-deuteron scattering has been slow. With the possible exception of fixed-scatterer approximations for the near-forward direction,[2] the AAY results and later calculations by Sloan[3,4] for the same interactions are the most accurate predictions presently available for the *N-d* differential cross section. Despite the wealth of data[5-7] available there have been no successful approximate or "exact" calculations of *N-d* polarization for energies beneath 100 MeV; the AAY model has zero polarization because of the simple *S*-wave forces involved. A computation[8] using the Sloan approximation[3,9] indicates that the nucleon-nucleon *P*-wave forces must be included in *N-d* polarization calculations and "exact" solutions of the Faddeev equations with such forces will be very difficult. Since the only reliable way to compute the effects of the *S*-wave *N-N* forces appears to be by exact solution of the Faddeev equations,[3] a two-potential–type formalism for the approximate incorporation of the *P*- and higher-wave *N-N* forces into an "exact" *S*-wave calculation seems desirable. Recently such a scheme based on the equations of Alt, Grassberger, and Sanhas[10] for three-particle scattering has been suggested[11] and our preliminary results from it for low-energy *N-d* scattering are very encouraging.

As used in this Letter, the "two-potential" scheme gives the following approximate form for the *N-d* elastic *T*-matrix elements:

$$\langle \vec{k}'|T(W)|\vec{k}\rangle = \langle \vec{k}'|T_{(1)}(W)|\vec{k}\rangle + 4\Big[\langle\varphi(1')| + \sum_{s_{23}'}\langle A_{s_{23}'}(1')v_{s_{23}'}(1')|G_0\Big]t_{(2)}(2',2)\Big[|\varphi(1)\rangle + G_0\sum_{s_{23}}|v_{s_{23}}(1)A_{s_{23}}(1)\rangle\Big]. \quad (1)$$

Here we have separated the *N-N* *T* matrices into two pieces,

$$t = t_{(1)} + t_{(2)}. \quad (2)$$

The $A_{s_{23}}(p)$ are the wave functions used in a variational calculation[12] of the "exact" *N-d* *T* matrix $T_{(1)}$ for the forces represented by $t_{(1)}$. The notation is the same as in Ref. 12. Equation (1) represents the addition to first order of $t_{(2)}$ to the *N-d* amplitudes.

The forces used in the "exact" calculation are *S*-wave separable potentials of the Yamaguchi[13] type for the singlet and triplet states. $t_{(2)}$ consists of rank-1 separable *N-N* amplitudes acting in the *P*- and *D*-wave channels. In order to reduce the computer time used, we find the two-potential amplitudes and wave functions only for *l*, the *N-d* orbital angular momentum, less than or equal to 2. Since we need many higher partial

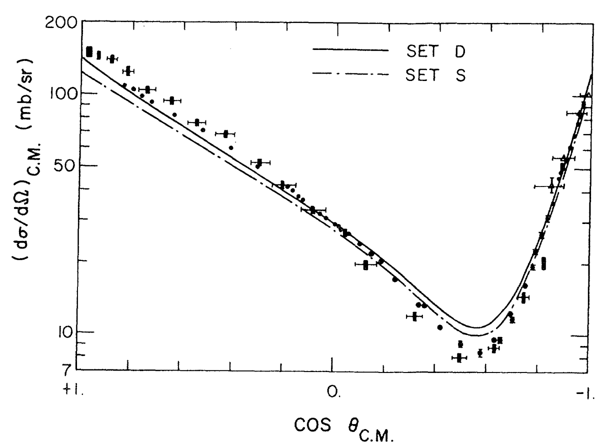

FIG. 1. Elastic *N-d* differential cross sections in the c.m. system at 14.1 MeV. The dots are from Ref. 15 (13.93-MeV *pd*), triangles from Ref. 16 (14.3-MeV *nd*), and rectangles from Ref. 17 (14.1-MeV *nd*).

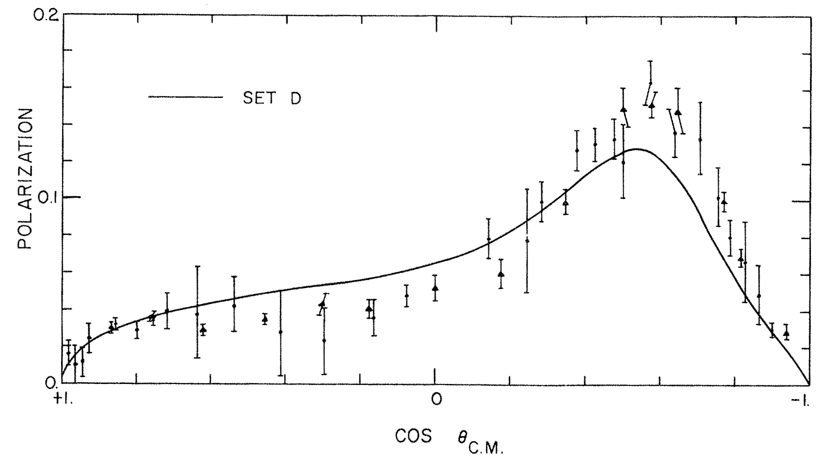

FIG. 2. Polarization at 10.04 MeV. The dots are from Ref. 5 (11.0-MeV *pd*) and triangles from Ref. 6 (10.0-MeV *pd*).

waves to ensure convergence of the polarization,[14] we use the Sloan approximation[3,9] to compute $\langle S', l' | T^J | S, l \rangle$ for $l' > 2$ or $l > 2$ up to $J = \frac{19}{2}$. In order to verify that these partial waves are accurate,[3] we made one computation for which two-potential results were obtained for $l', l \leq 4$. Similarly we have varied the number of trial functions in the variational calculation, the maximum value of J ($\frac{19}{2}$), and the numbers of Gaussian points used in the various numerical integrals. These tests show that our results are stable to better than 1%. Thus the only possible source of substantial error in our results for a given set of N-N interactions is the higher-order contributions of $t_{(2)}$ that are left out of Eq. (1). Unfortunately the computation of the second-order corrections to (1) appears prohibitatively difficult.[11]

Figure 1 shows the measured[15-17] and theoretical elastic differential cross sections at 14.1 MeV. The detailed properties of the potentials used are given by Pieper and Kowalski (PK).[8] Curve S includes just the S-wave potentials labeled 1*SA*, 3*SC* by PK and is very similar to the results that would be obtained with the AAY parameters.[1,3] Curve D contains the P-wave forces $P2$ and the D-wave forces $D1$ (note that there is no ${}^3S_1$-${}^3D_1$ coupling). In both the forward and backward directions the P- and D-wave forces have raised the cross sections, thus considerably improving the fit with the data. Since the very deep minimum in the cross section is due to strong cancelations of the partial waves, it is not surprising that the theory overestimates this region.

Figures 2 and 3 show nucleon polarizations at 10.04 and 14.1 MeV. These curves both diaplay the peak in the backward hemisphere and predict its magnitude with less than 20% error. Figure 4

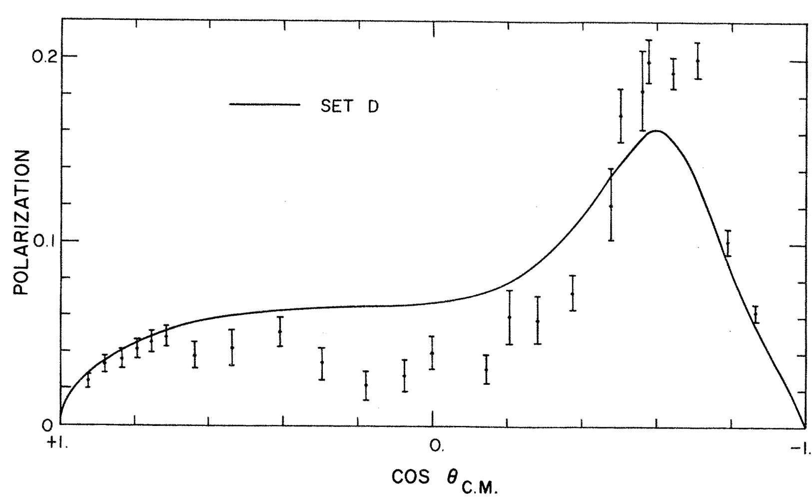

FIG. 3. Polarization at 14.1 MeV. The dots are from Ref. 5 (14.5-MeV *pd*).

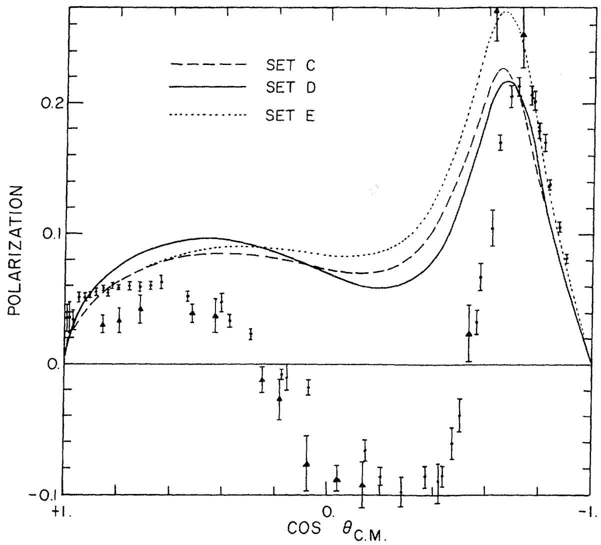

FIG. 4. Polarization at 22.7 MeV. The dots are from Ref. 5 (22.7-MeV *pd*) and triangles from Ref. 7 (22.0-MeV *pd*).

shows the polarization at 22.7 MeV. Here although the theory has developed a slight dip at intermediate angles, the discrepancy that was beginning to develop at 14.1 MeV has become very pronounced. Nevertheless the backward peak is still well reproduced. A calculation at 40 MeV does produce negative polarization at intermediate angles, but the magnitude is too small. The additional curves in Fig. 4 show the dependence of the polarization on different P-wave N-N interactions. Curves C and E contain the potential sets $P2$ and $P1$, respectively, of PK; neither curve has D-wave forces. We see that, at least in this approximation, the N-d polarization is fairly sensitive to the details of the N-N interactions. In particular, although it would at present be difficult to choose between sets C and E based on their N-N properties (see PK), they produce significantly different N-d polarizations. This difference is larger than the change caused by adding the D waves in set D. The differential cross sections at all of these energies are in the same excellent agreement with the data as the results at 14.1 MeV.

The two-potential formalism appears to be a useful technique for adding the effects of more complicated interactions to "exact" three-body computations. The differential cross sections presented here are, except in the vicinity of the minimum at 125°, the most accurate to have been calculated to date. The polarization results are the first in this energy region to reproduce the data even qualitatively and suggest that computation of the other spin-dependent observables is possible. Calculations with higher-rank separable T matrices for $t_{(2)}$, or possibly even T matrices derived from local potentials, should also be carried out to determine if the failure of the theory to predict negative polarization at 22.7 MeV is due to the potentials used or is an inadequacy of the approximation. A complete description of this calculation will be published elsewhere.

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