Spatial Distribution of Superfluid Vortices*

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We report accurate measurements of the increase, due to rotation, of the attenuation of second sound in an annulus. The measurements can be explained in detail by a vortex-free–region model in close agreement with theory, assuming a vortex-core parameter of 10 Å at 1.65 K. The range of agreement extends from the vorticity threshold to relatively high angular velocities.

The spatial distribution of quantized vortices in superfluid helium has been the subject of considerable theoretical[1-3] and experimental[4-8] work. A commonly used geometry is an annulus rotating about its axis of symmetry. In such a geometry, the vortices are expected to be parallel to the walls of the annulus, and distributed uniformly within the annulus except for vortex-free regions near the walls.[1-3] Bendt and Donnelly,[4] using a second-sound technique, experimentally confirmed a predicted[2] angular-velocity threshold Ω_0 for the stability of vortex lines in a narrow annulus. More recently, Northby and Donnelly,[5] using an ion technique and working with a wide annulus at angular velocities Ω well above Ω_0, demonstrated the existence of the vortex-free regions. Their measurements showed the correct dependence of ΔR, the thickness of the vortex-free region, on Ω, but appeared to yield a numerical value for ΔR which was roughly double the predicted theoretical value.

In this paper, we report accurate measurements of the increase, due to rotation, of the attenuation of second sound in a rotating narrow annulus. The measurements can be accurately accounted for by a vortex-free–region model in close agreement with theory, assuming a vortex-core parameter of 10 Å. The range of agreement extends from angular velocities just above the vorticity threshold Ω_0 to relatively high values, where ΔR is reduced to about 10% of the annulus width.

Theoretical work shows that in the limit of many vortices,[9] the free energy of the liquid is minimized by a uniform distribution of vortices with the vortex density given by

$$n_0 = 2\Omega/\kappa, \tag{1}$$

where $\kappa = h/m_{\mathrm{He}}$ is the quantum of circulation. In the next order of approximation,[1,3] with vortex self-energy terms included, a vortex-free region near the boundaries is predicted, with the thickness of the region given by

$$\Delta R = s[(1/2\pi)\ln(b/a)]^{1/2}, \tag{2}$$

where $s = n_0^{-1/2} \propto \Omega^{-1/2}$ is the mean vortex spacing, a is the vortex core parameter, and b is a length comparable to s at high Ω. At sufficiently low angular velocities, ΔR is larger than the annulus width D, and no vortices are expected. Vortices first become stable when $\Delta R = D/2$, or at an angular velocity

$$\Omega_0 = (\kappa/\pi D^2)\ln(b/a). \tag{3}$$

Fetter[2] has given an exact treatment for the case of a ring of vortices in a narrow annulus. He calculates a value of Ω_0 in agreement with (3) if b takes the value $b = 2D/\pi$. The ring of vortices quickly fills as Ω is increases slightly above Ω_0.

The apparatus used in the present work consists of an annulus with a mean radius of 1.82 cm, a width D of 0.18 cm, and a height of 3.5 cm. The walls are made of Pyrex with a precision of 2 μm. Boron nitride spacers are used to position the cylindrical walls and to close the ends of the annular cavity. Thin slots are cut in the glass walls near the ends of the annulus to allow heat to escape by thermal counterflow. A carbon film with a resistance of a few hundred ohms applied to the inner wall serves as an emitter of second sound when heated by an ac current. A similar film, with a resistance of several thousand ohms, is applied to the outer wall. When biased with a dc current, this film serves as a bolometer to detect the second sound, which typically has an amplitude on the order of 10–100 μK. The dependence of all measured quantities on bolometer and emitter power levels is checked to insure the absence of nonlinear effects. The cavity is enclosed in a brass cylinder which has small holes at the top to communicate with the helium bath. The apparatus is supported by a small turntable at the top of the Dewar. Rotary transformers are used to transfer signals between stationary electronics and the rotating system. A phase-sensitive signal-averaging system is used to observe the second-sound signals. Electronic components with good linearity and low drift were selected to permit accurate attenuation measurements. The temperature of the helium bath was controlled with an electronic regulator capable of holding drifts to less than 1 μK/h.

Measurements are made by setting up a standing wave of second sound in the annulus. Typically, in our apparatus, the Q of a fundamental radial resonance will be about 5000 near 1.65 K. An important source of experimental difficulty is the close spacing of the various radial modes of the cavity, a difficulty compounded by the inevitable geometric imperfections of such a narrow annular cavity. We found that relative rotation of the inner and outer walls of the cavity, performed as a preliminary adjustment, helped us select out a single mode and avoid problems of overlapping resonances.

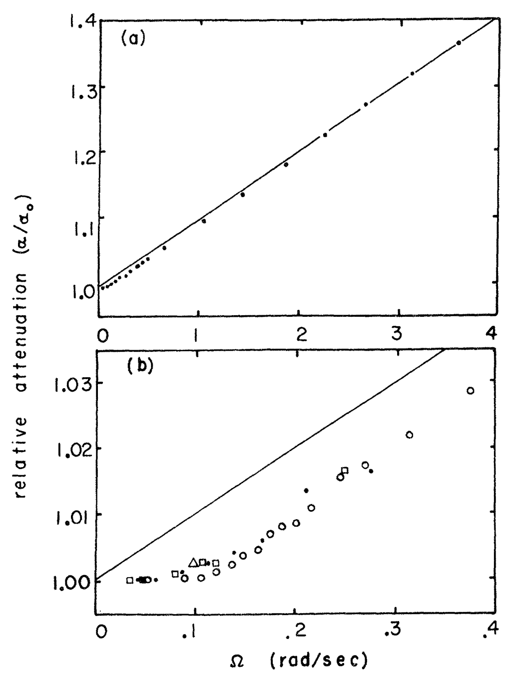

FIG. 1. The relative attenuation of second sound in the rotating annulus versus Ω. The straight lines are fitted to the high-Ω data. For clarity, most of the points in (b) are omitted from (a). In (b) the sequence of measurements is indicated by the symbols: solid and open circles, increasing Ω; solid and open squares, decreasing Ω; triangles, increasing Ω. The open symbols represent data taken several hours after completion of the measurements represented by the solid symbols. All measurements were made at 1.65 K.

Vortices can be detected using the technique of Hall and Vinen.[10] The amplitude of the second sound at resonance in a high-Q cavity is strongly dependent on the attenuation of second sound, which is increased by the presence of vortices. Figure 1 shows the relative attenuation α/α_0 as a function of Ω, for a fundamental radial mode at 1.65 K. The data were derived from measurements of the amplitude of second sound, read from a digital voltmeter at the output of the phase-sensitive detector. The estimated accuracy of the readings is about 0.1%. No hysteresis was observed. The attenuation is constant at very low Ω and begins to increase above $\Omega \approx 0.12$ rad/sec. As Ω increases the attenuation becomes approximately linear in Ω.

The transition from threshold to uniform vorticity is shown more clearly in Fig. 2, which shows α_r/Ω as a function of Ω, where $\alpha_r = \alpha - \alpha_0$ is the increase in attenuation due to rotation. The uncertainty in the low-Ω points is very high because of the fact that α_r represents the difference of relatively large numbers. The scatter of the points is well within estimates of experimental uncertainty. When ΔR is negligible, α_r is proportional to the total number of vortex lines

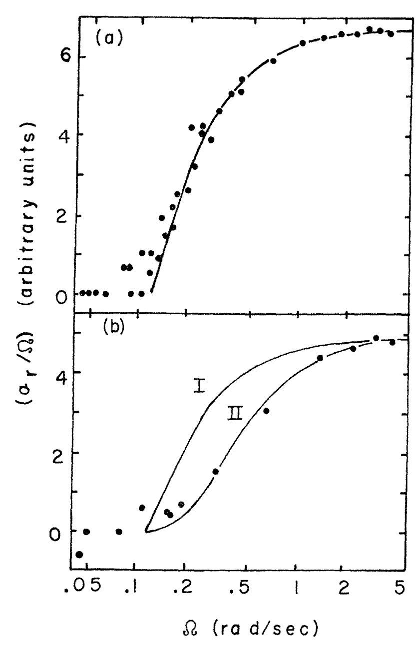

FIG. 2. Plots of α_r/Ω vs Ω, where $\alpha_r = \alpha - \alpha_0$ is the increase in attenuation due to rotation: (a) fundamental mode, (b) second harmonic. The curves are theoretical, normalized to fit the high-Ω data, and assume a vortex-core parameter $a = 10$ Å. In (b) curves I and II were calculated for the fundamental and second harmonic, respectively.

in the cavity and hence to Ω, so α_r/Ω approaches a constant at high Ω. The value of this constant can be used to determine an experimental value of the Hall-Vinen parameter B. Values of B determined from our measurements in this way are in good agreement with accepted values.[11]

The dependence of α_r/Ω on Ω can be calculated assuming the following model: As Ω is increased above Ω_0, the ring of vortices is formed. Then, as additional vortices enter the liquid, the ring breaks up and there is a gradual transition to a state where the vortices are uniformly distributed over a region near the center of the annulus. As Ω increases this region broadens as the vortex-free region shrinks in accordance with the Ω dependence of ΔR.

The vortex-line–second-sound interaction can be described by the mutual friction forces $F_{sn} \propto v_s - v_n$, where v_s and v_n are the normal fluid and superfluid velocities.[10] Since this force acts on both the normal fluid and superfluid, but with opposite sign, the rotational dissipation is proportional to $(v_s - v_n)^2$. For the fundamental radial modes, $v_s - v_n$ has a sinusoidal dependence on the radial coordinate and nodes at the annulus walls. Averaging $(v_s - v_n)^2$ over the region of vorticity near the center of the annulus gives the Ω dependence of α_r/Ω. The results depend on the values of b and a in (2). We assume $b = 0.27s$, a result valid for a triangular vortex lattice[12] and a reasonable approximation for high Ω. It is reasonable to assume that b should approach $2D/\pi$ near Ω_0, but the errors made by assuming $b = 0.27s$ over the full range are not important. The theoretical curves are scaled to agree with the high-Ω data. Once this is done, the only free parameter is the vortex-core parameter a. We tried values of 0.1 to 100 Å. The theoretical curve in Fig. 2(a) was drawn assuming $a = 10$ Å. The fit was visibly poorer if a was increased or decreased by a factor of 2. This value of a is consistent with an early estimate by Hall of 8 Å, based on vortex-wave experiments.[1] Our value is of course dependent on the assumption $b = 0.27s$. The quality of the fit does not seem sensitive to the assumed Ω dependence of b. If b is assumed constant, an equally good fit can be obtained with an appropriate value of b/a.

Further confirmation of the correctness of the ideas discussed here comes from the attenuation measurements for a second-harmonic radial mode, shown in Fig. 2(b). Since this mode has a node in $v_s - v_n$ at the center of the annulus, the attenuation will not be increased by the ring of vortices which appears just above Ω_0, and will increase relatively slowly as the vortices spread radially. The data in Fig. 2(b) clearly show that α_r/Ω increases more slowly with Ω for the second harmonic than for the fundamental, and that the increase is in agreement with our theoretical model.

Measurements have been repeated several times at temperatures of 1.40, 1.65, and 1.80 K, using both the fundamental and second-harmonic radial modes. Good fits to the data at 1.65 and 1.80 K and from one run at 1.40 K were obtained with a value of a of about 10 Å. Attempts to fit two other sets of measurements taken at 1.40 K were not successful. The reasons for this are unknown and are under investigation.

A theoretical value of Ω_0 can be calculated from (3) assuming $b = 2D/\pi$ and $a = 10$ Å. The result, 0.14 rad/sec, is somewhat larger than the experimentally determined value of 0.12 ± 0.01 rad/sec from Fig. 1(b). The results of all determinations of Ω_0 with this apparatus were consistently less than the predicted value. It is hoped that measurements in other cavities will lead to an understanding of this disagreement.

In concluding, we compare our results with those of Northby and Donnelly.[5] Our results support a value for the vortex-free gap width ΔR in close agreement with expectations, while the experiment of Northby and Donnelly appears, at face value, to yield a gap width of roughly double the predicted value. This apparent contradiction cannot be resolved by any reasonable choice for the core parameter. Rather, we believe that our results support the suggestion of Northby and Donnelly,[5] that in their experiment, the time-averaged density of the vortices nearest the wall was equal to the equilibrium value, but the vortices were not observed because of lifetime effects. Their detection technique relied on the ability of a vortex to trap a negative ion and deliver its charge to a collector at the upper end of the apparatus, about 5 cm from the trapping region. If the lifetime of a vortex near the outer wall was less than the ion transit time,[13] the vortex would not deliver the charge to the collector and thus would not be detected. The dissipative effects observed in our experiment, on the other hand, depend only on the time average of the vortex distribution, and are insensitive to vortex lifetimes.

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[13] J. A. Northby (private communication) has informed us that it is difficult to estimate a good value for this transit time, but that it is probably less than one minute.

Dynamics of Concentration Fluctuations Near the Critical Mixing Point of a Binary Fluid

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The decay rate Γ of the concentration fluctuations near the critical mixing point of 3-methylpentane-nitroethane has been measured as a function of the correlation length ξ and the wave number k in the range $0.1 \leq k\xi \leq 20$. We have confirmed the theoretical prediction that Γ varies as k^3 in the critical regime. An analysis of Γ as a function of $k\xi$ reveals deviations from the theory of Kawasaki, which are discussed.

A system near a critical point exhibits large fluctuations in the order parameter, which decay very slowly in time. A central assumption in the current theoretical descriptions of this phenomenon is that the anomalous behavior of both static and dynamic properties is governed by a single correlation length ξ. It is, therefore, desirable to measure the relaxation time of the fluctuations and the equilibrium correlation length simultaneously. For this purpose we have measured the total intensity and the spectral width of light scattered by concentration fluctuations near the critical mixing point of 3-methylpentane-nitroethane. Details of the experimental arrangement will be published elsewhere.

The scattering angle θ is related to the wave number k of the fluctuations by the Bragg relation $k = 2k_0 \sin\frac{1}{2}\theta$, where k_0 is the wave number of the incident light in the medium. In the hydrodynamic regime ($k\xi \ll 1$), the decay rate Γ of the concentration fluctuations is given by $\Gamma = Dk^2$, where D is the binary diffusion coefficient.[1] The modern theories of dynamical scaling postulate that for all $k\xi$ the decay rate Γ should be a homogeneous function of k and ξ^{-1},[2]

$$\Gamma = \varphi(k, \xi^{-1}) = k^z \varphi(1, 1/k\xi) = k^z \Phi(k\xi), \tag{1}$$

where $z = 3$ according to the mode-mode coupling theory.[3,4] A specific form for the function $\Phi(k\xi)$ has been derived first by Kawasaki[4] and subsequently by Ferrell[5]:

$$\Gamma/k^3 = A(2/\pi)\{(k\xi)^{-3} + (k\xi)^{-1} + [1 - (k\xi)^{-4}]\arctan k\xi\}, \tag{2}$$

with $A = k_B T/16\bar{\eta}$, k_B being Boltzmann's constant and $\bar{\eta}$ a shear viscosity which is assumed to be independent of k and ξ in the theory.

We have measured the decay rate Γ from the linewidth of the central component in the spectrum of scattered light by the method of self-beat spectrometry.[6] A total of 88 data points were obtained at the critical concentration in the temperature interval $0.001 \leq T - T_c \leq 0.3$°C for seven different scattering angles. The scaling hypothesis (1) implies that in the critical regime ($k\xi \gg 1$), Γ should vary as k^z. To test this prediction we have plotted the linewidth data obtained at the seven scattering angles at $\Delta T = 0.001$ and 0.003°C as a function of k in Fig. 1. For the exponent z we find 2.99 ± 0.05 at $\Delta T = 0.001$°C