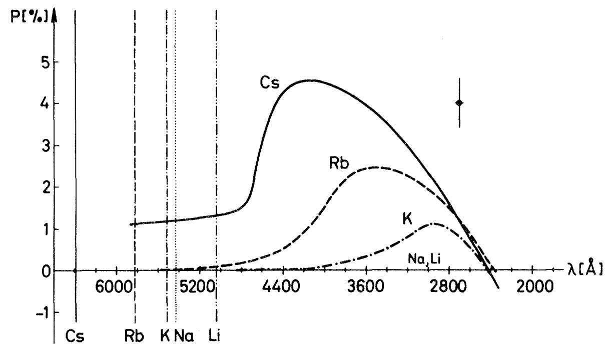

FIG. 2. Comparison of spin polarization obtained from solid Cs, Rb, K, Na, and Li targets. Dashed lines to the left, ionization thresholds. Error bar to the right applies to all curves.

It has been shown,[5,6] however, that the effective number of conduction electrons per atom decreases monotonically as one goes from sodium to potassium and rubidium to cesium.

A striking feature which will be displayed in a more detailed[7] paper is a close correlation between the wavelengths for producing plasma oscillations and the wavelengths at which the polarization maxima appear. We believe this correlation to be fortuitous, since we do not see a mechanism for its explanation.

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Macroscopic Quantum Uncertainty Principle and Superfluid Hydrodynamics

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By application of a macroscopic quantum uncertainty principle to the flow of superfluid helium, one can understand the onset of third sound in thin helium films, and the large excess absorption of third sound in thick helium films.

Liquid helium below $T_\lambda = 2.2°K$ (He II) is a remarkable fluid because Planck's constant h enters directly into the description of its hydrodynamic properties. That this should be the case was first proposed by Onsager[1] who claimed that the circulation of the superfluid component of He II must be quantized:

$$\oint \vec{v}_s \cdot d\vec{r} = nh/m, \qquad (1)$$

where m is the mass of a helium atom, n an integer, $\vec{v}_s$ the velocity of the superfluid component, and $d\vec{r}$ is an element of path length in the line integral. Equation (1) is clearly a macroscopic version of the Bohr-Sommerfeld quantization condition $\oint \vec{p}\cdot d\vec{q} = nh$ of the early quantum theory. In principle, the above quantization conditions are used to pick out a discrete set from the continuum of states obeying the classical dynamics.[2] At a higher level in the quantum theory, one must come to grips with the wave-particle duality and its consequence that the classical dynamics can no longer give the fundamental description of the motion. In the basic quantum theory these facts are most simply reflected by the uncertainty principle, $\Delta p_i \Delta q_i = \hbar$. One is naturally led to expect that at a higher level our description of superfluids will include a macroscopic uncertainty principle,

$$\Delta v_{s,i} \Delta r_i = \hbar/m. \qquad (2)$$

In this paper we propose to describe the onset of third sound in thin He II films and the large excess absorption of third sound in thick helium films by use of Eq. (2).

A solid surface immersed in helium vapor is coated by a film whose thickness is determined by the pressure of the vapor. The film is held to the substrate by the van der Waals force of attraction. Because of this force, surface waves (third sound), which are analogous to long gravity waves, can propagate through the film parallel to the substrate. The dynamical equation which describes the motion of the superfluid is given by Landau[2] as

$$\partial\vec{v}_s/\partial t = -\nabla(\mu + \tfrac{1}{2}v_s^2) + \nabla\mu'(y), \tag{3}$$

where μ is the chemical potential and $\mu'(y)$ is the external potential due to the van der Waals force of attraction; it is an explicit function of y, the perpendicular distance from the substrate.

In a film of thickness d the uncertainty Δy in the y location of a fluid particle is $d/\sqrt{12}$ if one assumes for simplicity that all locations are equally probable. From the macroscopic uncertainty principle (2) we find

$$\Delta v_{s,y} = \hbar\sqrt{12}/md. \tag{4}$$

It seems plausible to conjecture that third sound will no longer propagate when the film becomes of such a thickness that the uncertainty in $v_{s,y}$ becomes greater than the speed C_3 of third sound or [from (4)][3]

$$\hbar\sqrt{12}/m \geq C_3 d. \tag{5}$$

One might criticize comparing the uncertainty in the component of superfluid velocity perpendicular to the substrate with the sound velocity which is parallel to the substrate. The justification for making this comparison follows from noting that the restoring potential μ' responsible for third sound is a function only of the coordinate perpendicular to the substrate $[C_3^2 = 3(\rho_s/\rho)\mu'(d)$, where ρ_s/ρ is the superfluid fraction]. In fact, a condition very close to Eq. (5) could be obtained by assuming that onset occurs when the uncertainty in the contribution of $\frac{1}{2}v_s^2$ to the driving force becomes comparable to the contribution of μ' in Eq. (3) (or equivalently when the uncertainty in the kinetic energy per gram of the superfluid becomes comparable to the potential energy per gram).[4]

All quantities on the right-hand side of Eq. (5) can be independently measured. It has been found,[5] for instance, that at onset in thin films C_3d is (0.52 and 0.47)h/m at 1.3 and 1.586°K, respectively. These values are in good agreement with the predicted value of $0.55h/m$ from Eq. (5). If this is the correct manner in which to describe onset, then it is easy to understand how onset can take place at a finite value of ρ_s/ρ.

What is remarkable is that the uncertainty in $v_{s,y}$ becomes greater than C_3 also when the film becomes too thick! This is made clear from the graph of C_3d versus thickness d for $T = 1.3$°K in Fig. 1. There we see that for a thickness $d > 16$ atomic layers, one has again the value of C_3d at onset. Third sound can be observed in these thicker films, but the attenuation is much greater than can be accounted for by the hydrodynamics.[6,7] Perhaps the reason for not seeing a sharp onset in thick films is that C_3d is a much more slowly varying function of d than it is in the thin films. Also plotted in Fig. 1 is the attenuation of third sound in excess of that expected from the Landau two-fluid theory.[8] It seems as though the uncertainty in v_s accounts completely for the qualitative behavior of the excess attenuation.[7] For instance, when the uncertainty in v_s is smallest compared to C_3 (i.e., when C_3d is a maximum), the excess attenuation is a minimum, and on either side of this maximum the attenuations are comparable for comparable values of C_3d.

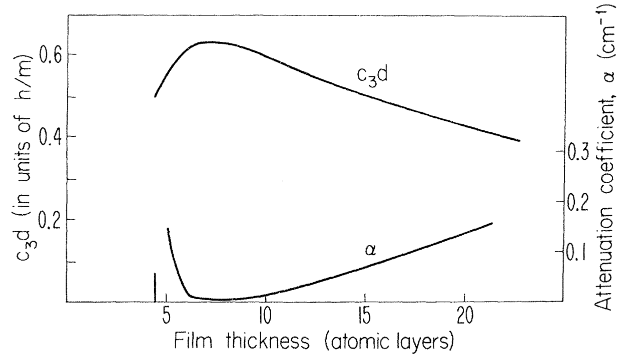

FIG. 1. Graph of C_3d and excess attentuation α at 200 Hz versus film thickness ($T = 1.3$°K). C_3 is the velocity of third sound; d is the film thickness minus one atomic layer. The superfluid onset thickness is 4.4 atomic layers and is indicated in the figure.

Although the films under consideration are quite thin compared to geometries usually discussed in ordinary hydrodynamical theories, we consider the experiments and discussion presented here as being on the macroscopic level. To appreciate this one need only realize that the third sound is generated and detected through the *temperature* variations associated with its propagation.

If one applies the above reasoning to the propagation of long gravity waves in He II, then one finds that below a thickness

$$d = (6\hbar^2/m^2g)^{1/3} = 5.3 \times 10^{-4} \text{ cm}$$

the propagation will be seriously impeded. We know of no experimental evidence with which this consequence of the macroscopic uncertainty principle can be checked. (Experiments are now underway to remedy this situation.)

It might be that the uncertainty in v_s (and hence the excess absorption of third sound) is realized as a result of the complicated nonstationary velocity field generated by the presence of quantized vortices, and that as one passes through onset, the density of these vortices increases rapidly. However, in the thin films the entrance of vortices seems to be energetically unfavored, and it may be that in this case the uncertainty in V_s is related to a macroscopic zero-point motion.

While some success has perhaps been achieved by grafting various (macroscopic) quantum ideas onto the Landau two-fluid equations, the following basic question is forced upon us: What is the single macroscopic Schrödinger theory which contains the macroscopic quantum ideas such as (1) and (2), and in some limit reduces to the Landau two-fluid theory (macroscopic correspondence principle)?

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[8]Still another way of obtaining this condition is to equate the van der Waals potential experienced by a single particle with the quantum fluctuation energy given to this particle by localizing it in the shallow superfluid layer. It is important, however, to notice the following difference between this condition and a corresponding condition that could be written, for instance, for the propagation of long gravity waves in an ordinary fluid. In the latter case we expect that the waves will still propagate even when the quantum fluctuation energy of the single particles is greater than the potential energy. This is so because for an ordinary fluid the individual particles move incoherently, whereas for He II the superfluid particles move coherently in a macroscopically occupied quantum state.

Raman Scattering from Condensed Phases of He^3 and He^4

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Inelastic light scattering with frequency shifts from 10 to 150 cm^{-1} has been measured in liquid and solid He^3 and He^4 under pressures up to 150 atm. In the hcp solids a transverse optic phonon was observed and resolved. In addition a broad peak was observed which is remarkably similar in both liquids and solids. In the solid this peak is interpreted as scattering from two or more damped, coupled phonons. Scattering intensities and spectral forms are compared with recent theoretical calculations.

Light scattering is a useful and often unique probe of density fluctuations in quantum crystals and fluids. It is complementary to neutron scattering but is capable of greater resolution. Light scattering is particularly important for He^3 since neutron scattering is made impractical because of the large nuclear absorption. Since the light interacts with the density fluctuations only via the weak polarizability of the bound electrons, the scattered signal-to-noise ratios in helium are small. In spite of experimental difficulties, a number of interesting results have been observed, such as light scattering from first and second sound and from two rotons.[1-4] We report here the scattering of laser light from liquid He^3 and liquid He^4 under pressure, from the bcc and hcp