Physical Processes in a Convergent Theory of the Weak and Electromagnetic Interactions*

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A previously proposed theory of leptonic weak and electromagnetic interactions is found to be free of the divergence difficulties present in conventional models. The experimental implications of this theory and its extension to hadrons are briefly discussed.

Several years ago I proposed a unified theory[1] of the weak and electromagnetic interactions of leptons, and suggested that this theory might be renormalizable. This theory is one of a general class of models which may be constructed by a three-step process[2]: (A) First write down a Lagrangian obeying some exact gauge symmetry, in which massless Yang-Mills fields interact with a multiplet of scalar fields[3] and other particle fields. (B) Choose a gauge in which all the scalar field components vanish, except for a few (in our case one) real scalar fields. (C) Allow the gauge group to be spontaneously broken by giving the remaining scalar field a nonvanishing vacuum expectation value. Redefine this field by subtracting a constant λ, so that the "shifted" field φ has zero vacuum expectation value. In the resulting perturbation theory, all vector mesons acquire a mass, except for those (in our case, the photon) associated with unbroken symmetries.

In the proposed theory, this procedure was applied to the gauge group $SU(2)_L \otimes Y$, and resulted in a model involving electrons, electron-type neutrinos, charged intermediate bosons (W_μ), neutral intermediate bosons (Z_μ), photons (A_μ), and massive neutral scalar mesons (φ), with an interaction of the form[1]

$$\begin{aligned}\mathcal{L}' = {} & \frac{ig}{(g+g'^2)^{1/2}}[gZ^\nu - g'A^\nu][W^\mu(\partial_\mu W_\nu{}^\dagger - \partial_\nu W_\mu{}^\dagger) - W^{\mu\dagger}(\partial_\mu W_\nu - \partial_\nu W_\mu) + \partial^\mu(W_\mu W_\nu{}^\dagger - W_\nu W_\mu{}^\dagger)] \\ & - \frac{g^2}{(g^2+g'^2)} W_\mu W_\nu{}^\dagger(gZ_\rho - g'A_\rho)(gZ_\sigma - g'A_\sigma)(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma}) + \frac{g^2}{2}[|W_\mu W^\mu|^2 - (W_\mu W^{\mu\dagger})^2] \\ & + F(\varphi) - \frac{m_e}{\lambda}\varphi\bar{e}e - \tfrac{1}{8}(\varphi^2 + 2\lambda\varphi)[(g^2+g'^2)Z_\mu Z^\mu + 2g^2 W_\mu W^{\mu\dagger}] \\ & + i(g^2+g'^2)^{-1/2}\bar{e}\gamma^\mu\left[\left(\frac{1-\gamma_5}{2}\right)g'^2 + \frac{1}{2}\left(\frac{1+\gamma_5}{2}\right)(g'^2 - g^2)\right]eZ_\mu + \frac{i}{2}(g^2+g'^2)^{1/2}\bar{\nu}\gamma^\mu\left(\frac{1+\gamma_5}{2}\right)\nu Z_\mu \\ & + \frac{igg'}{(g^2+g'^2)^{1/2}}\bar{e}\gamma^\mu e A_\mu + \frac{ig}{\sqrt{2}}\bar{\nu}\gamma^\mu\left(\frac{1+\gamma_5}{2}\right)eW_\mu{}^\dagger + \frac{ig}{\sqrt{2}}\bar{e}\gamma^\mu\left(\frac{1+\gamma_5}{2}\right)\nu W_\mu. \qquad (1)\end{aligned}$$

Here $F(\varphi)$ is a fourth-order polynomial in φ (chosen so that $\langle\varphi\rangle_0 = 0$), and g and g' are independent coupling constants. The electronic charge e, weak coupling constant G, and vector meson masses are given by the formulas

$$e = gg'/(g^2+g'^2)^{1/2}, \quad G/\sqrt{2} = \tfrac{1}{2}\gamma^2, \qquad (2)$$

$$m_W = \lambda g/2, \quad m_Z = \lambda(g^2+g'^2)^{1/2}/2. \qquad (3)$$

At the time that this theory was proposed, its renormalizability was still a matter of conjecture. It is well known[4] that the Yang-Mills theory with which we start in step A above is indeed renormalizable if quantized in the usual way. However, the shift of the scalar field performed in step C amounts to a rearrangement of the perturbation series, so that the S matrix calculated in perturbation theory corresponds to a representation of the algebra of field operators inequivalent to that with which we started in step A. There is no obvious way to tell that renormalizability is preserved in this shift.

Recently several studies have indicated that various models of this general class actually are renormalizable. By choosing a different gauge in step B, 't Hooft[5] derived effective Lagrangians which appear manifestly renormalizable, but which involve fictitious massless scalar mesons of both positive and negative norm. Subsequently, Lee[6] showed in one case that the renormalization program does actually work in this gauge, and that the spurious singularities associated with the fictitious particles all cancel. I would suggest, as an explanation of these results, that, although the shift of fields in step C really does generate an inequivalent representation of the field operators, the choice of gauge in step B does not, so that the S matrix calculated in the manifestly renormalizable gauges of 't Hooft should agree with the S matrix calculated in the "manifestly unitary" gauge used to derive Eq. (1).

If this is correct, then it ought to be possible to carry out calculations of higher-order weak interactions using the interaction (1) directly. This has obvious advantages over the use of Lagrangians of the 't Hooft–Lee form, because (1) involves only physical particles. This paper will explore the results of this model in certain physical processes, both in order to test its renormalizability, and also to gain some insight into its general properties.

First, it is necessary to derive the Feynman rules for this theory. The interaction Hamiltonian here is given by $-\mathcal{L}'$ plus noncovariant terms which are canceled by the noncovariant parts of the propagators of the vector meson fields and their derivatives. However, after this cancelation, there is left over a covariant effective interaction[7]

$$\delta\mathcal{L} = -6i\delta^4(0)\ln(1+\varphi/\lambda).$$

The correct Feynman rules are thus generated by using $-\mathcal{L}' - \delta\mathcal{L}$ as an effective interaction Hamiltonian, keeping only the "naive" covariant parts of the various propagators. No "ghost loops"[4] appear here.

Now let us consider some specific physical processes:

$\nu+\bar{\nu}\to W^+ + W^-$.—This is the reaction used by Gell-Mann, Goldberger, Kroll, and Low[8] to exhibit the difficulties associated with conventional intermediate boson theories. In such theories, the amplitude[9] for production of zero-helicity $W^\pm$ is given in lowest order by

$$f_{\mathrm{GGKL}} = \frac{-iGp^{3/2}\sin\theta e^{-i\varphi}}{2\pi(2E)^{1/2}}\left\{\frac{2E^2[1-(E/p)\cos\theta]-m_W^2}{2E^2[1-(p/E)\cos\theta]-m_W^2+m_e^2}\right\}, \tag{4}$$

where E, p, θ, and φ are the energy, momentum, and scattering angles, respectively, of the W^+ in the center of mass system. For $E\to\infty$ this is dominated by a pure $J=1$ term which grows like E, so that in order to save unitarity it is necessary to introduce a cutoff[8] at energies of order $1/\sqrt{G}$. In the theory proposed here, there is an additional term produced by Z exchange in the s channel:

$$f_Z = \frac{iGp^{3/2}\sin\theta e^{-i\varphi}}{2\pi(2E)^{1/2}}\left[\frac{4E^2+2m_W^2}{4E^2-m_Z^2}\right]. \tag{5}$$

Inspection of (4) and (5) shows that the total scattering amplitude now grows only near threshold and falls off like $1/E$ for $E\gg m_W$. This natural cutoff at $E\sim m_W$ obviates the need for any special unitarity cutoff. To the extent that this is a general phenomenon, we can expect that the perturbation series in G is really an expansion in powers of $Gm_W^2\sim g^2$, which may be as small as e^2.

$\nu+\nu\to\nu+\nu$.—This is a good process to use as a test of the performance of our theory in loop diagrams, because, as pointed out by Low,[10] the exchange of a pair of W bosons generates a quadratic divergence, related to the failure of unitarity bounds in Eq. (4). In our present theory, there are two additional fourth-order diagrams in which a pair of Z's is exchanged, plus a large number of fourth-order diagrams in which a single Z is exchanged with second-order self-energy or vertex insertions. The former diagrams contain quadratic divergences which cancel among themselves. The latter diagrams contain quartic divergences, which cancel among themselves, plus a large number of quadratically divergent terms. Some of these quadratic divergences can be grouped together as renormalizations of m_Z and the Z-ν coupling constant (and probably cancel) but there remain quadratic divergences in the Z self-energy proportional to $(t-m_Z^2)^2$, and in the Z-ν vertices proportional to $(t-m_Z^2)$. These terms generate an effective quadratically divergent neutrino Fermi interaction, *which turns out to cancel the quadratic divergence found by Low.* (I have not yet checked what happens to the logarithmically divergent terms.)

W-photon interactions.—The first term in Eq. (1) gives the W an "anomalous" magnetic moment, with gyromagnetic ratio $g_W=2$. This is just the value required[11] if the amplitude for Compton scattering on a W behaves well enough at high energies to satisfy a Drell-Hearn sum rule.

$e^+ + e^- \to W^+ + W^-$.—Because the W has an "anomalous" magnetic moment, the electromagnetic pair-production amplitude[9] grows like E as $E \to \infty$. Further, the neutrino t-channel and Z s-channel exchange diagrams do *not* cancel here, so that the weak pair-production amplitude also grows like E. However, *the weak and electromagnetic amplitudes cancel each other as* $E \to \infty$, leaving a scattering amplitude which vanishes like $1/E$ as $E \to \infty$, as required by unitarity bounds. This cooperation between the weak and electromagnetic interactions in solving each other's problems is one of the most satisfying features of this theory.

The weak and electromagnetic interactions of the leptons appear to be in good shape, so let us consider how to incorporate the hadrons. In order to preserve renormalizability, it is necessary to couple Z, W, and A to the currents of an *exact* $SU(2)_L \otimes U(1)$ symmetry of the strong-interaction Lagrangian. This poses a problem, because, apart from any spontaneous symmetry-breaking mechanisms responsible for the baryon masses, it is usually presumed that the nonzero masses of the π and K arise from an *intrinsic* breaking of $SU(2) \otimes SU(2)$ or $SU(3) \otimes SU(3)$. The only way that I can see to save renormalizability is to suppose instead that the π and K masses arise from the same purely spontaneous symmetry-breaking mechanism responsible for the W and Z masses. The problem then is whether it is natural for the strong interactions to conserve parity and isospin.

Leaving aside strange particles, the simplest way to couple the scalar doublet[3] (φ^+, $\varphi^0 + \lambda$) of our model to the hadrons is to find some $(\frac{1}{2}, \frac{1}{2})$ $SU(2) \otimes SU(2)$ multiplet $(\sigma, \vec{\pi})$ of hadronic field operators, and write an $SU(2)_L$-invariant interaction,

$$-if\varphi^{+\dagger}(\pi_1 - i\pi_2) + f(\varphi^{0\dagger} + \lambda)(\sigma + i\pi_3) + \text{H.c.}$$

The rest of the strong-interaction Lagrangian is assumed to conserve $SU(2)_R$ as well as $SU(2)_L$, so by an $SU(2)_R$ rotation we can define σ and $\vec{\pi}$ so that f is real. After eliminating φ^+ and $\mathrm{Im}\varphi^0$ in step B, the only remaining symmetry-breaking term is $2f(\lambda + \varphi)\sigma$, which does conserve parity and isospin. Thus the spontaneous symmetry breaking of the weak interactions can act as a seemingly intrinsic symmetry-breaking mechanism for the strong interactions, which in turn is amplified if σ develops a large vacuum expectation value. A particularly attractive aspect of this approach is that the requirement of renormalizability provides the rationale for the conservation or partial conservation of the hadronic weak currents.

The most direct verification of this theory would be the discovery of W's and Z's with the predicted properties. However, the lower limits on m_W and m_Z are, respectively, $\lambda e/2 = 37.3$ GeV and $\lambda e = 74.6$ GeV, so this discovery will take a while.[12] The most accessible effect of the Z's is to change the cross sections for scattering of neutrinos and antineutrinos on electrons. We know nothing about the mass of the scalar meson φ, but its field might contribute to the level shifts in muonic atoms. Higher-order weak interactions produce various "radiative" corrections, including a change of order Gm_μ^2 in the gyromagnetic ratio of the muon.[13] The extension of this theory to strange particles appears to require both strangeness-changing and strangeness-conserving neutral hadronic currents, but the former can be eliminated in an $SU(4) \otimes SU(4)$–invariant model.[14] These matters will be dealt with at greater length in future papers.

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