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[8] We express T in units of GeV and recall that 1 eV $=11\,606$°K.

[9] Yang, Ref. 4; Benecke, Chou, Yang, and Yen, Ref. 4.

[10] We note that the temperature T estimated with the classical Bose distribution, i.e., $\lambda=1$, to fit the P_T distribution integrated over P_L yields a higher value: 0.145 ± 0.012 (see Ref. 7).

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Nonleptonic Hamiltonian, SU(3) Breaking, and the $K_2^0 \to \gamma\gamma$ Decay*

M. Moshe† and P. Singer
Department of Physics, Technion–Israel Institute of Technology, Haifa, Israel
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The inclusion of SU(3) symmetry breaking in the *PVV* vertices and the use of the "simplest" Hamiltonian consistent with $\Delta I=\frac{1}{2}$ for nonleptonic $\Delta S=1$ transitions bring the calculation of $K_2^0\to\gamma\gamma$ decay mode in agreement with experiment, *vis-à-vis* the disagreement of two orders of magnitude previously reported.

The decay mode $K_2^0\to\gamma\gamma$ has lately been measured by several groups,[1] and the reported average[2] for the branching ratio is

$$\Gamma_{K_2^0\to\gamma\gamma}/\Gamma_{K_2^0\to\mathrm{all}}=(5.6\pm0.5)\times10^{-4}. \qquad (1)$$

A careful theoretical treatment of this process by Rockmore[3] shows that the constructive interference between various parts of the amplitude leads to a calculated rate larger by 2 orders of magnitude than the experimental average (1). In his calculation Rockmore used a current-current weak Hamiltonian[4] behaving like a λ_6 vector of an SU(3) octet, as well as SU(3)-symmetric *PVV* vertices. Inclusion of various corrections to the main amplitudes,[5] such as η-π mixing as well as nonpole diagrams, does not change the theoretical value by more than 30%, and the large discrepancy between theory and experimental is thus essentially unaltered.

In this Letter we show that by using an SU(3)-broken form for the *PVV* vertices as suggested from radiative meson decays, as well as an alternative current-current Hamiltonian for the $\Delta S=1$ weak interaction, a theoretical value is calculated for the $K_2^0\to\gamma\gamma$ transition which is in very good agreement with the measured one.

The current-current weak Hamiltonian used by Rockmore,[3] which was introduced by Sakurai[4] and shown to account for the current-algebra results for $K\to2\pi$ and baryon nonleptonic parity-nonconserving transitions, has the form

$$H^W(\Delta I=\tfrac{1}{2},\Delta S=1)=(2/\sqrt{2})G_{\mathrm{nl}}d_{6ab}J_\mu{}^a(x)J_\mu{}^b(x), \qquad (2)$$

where

$$J_\mu{}^a=j_\mu{}^{Va}+j_\mu{}^{Aa}. \qquad (3)$$

Here a, b are SU(3) indices, and the vector and axial-vector currents are assumed to be dominated, respectively, by vector and pseudoscalar and pseudovector particles. If we use for G_{nl} a value close to the Fermi constant (i.e., $G_{\mathrm{nl}}=1.1\times10^{-5}/m_p{}^2$), the correct value for $K_1^0\to2\pi$ transition is reproduced.[6]

As an alternative to (2), which fails in the $K_2^0\to\gamma\gamma$ calculation by Rockmore, we suggest the following current-current weak Hamiltonian, which preserves all the good results obtained by Sakurai:

$$\overline{H}^W(\Delta I=\tfrac{1}{2},\Delta S=1)=(2/\sqrt{2})G_{\mathrm{nl}}(J_\mu{}^1J_\mu{}^4+J_\mu{}^2J_\mu{}^5-J_\mu{}^3J_\mu{}^6). \qquad (4)$$

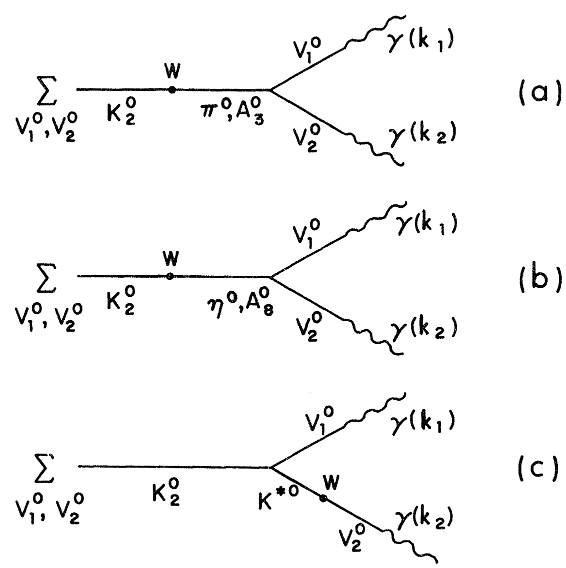

FIG. 1. The diagrams contributing to the amplitude $M_{K_2^0\to\gamma\gamma}$ in Sakurai's model. The diagram b is absent for the "simplest" Hamiltonian [Eq. (4)].

This Hamiltonian can be obtained from (2) by adding a suitable term belonging to a 27 representation, being the "simplest" Hamiltonian (i.e., with a minimum of neutral currents) preserving the $\Delta I = \frac{1}{2}$ rule.[7]

For the PVV vertices we take the SU(3)-octet-broken form suggested by Brown, Munczek, and Singer,[8,9] who used a phenomenological Langrangian with gauge vector fields and current mixing to account for the observed strong and electromagnetic decays. One of their main results is a predicted decrease of the $K^{*0}K^0\gamma$ vertex from the SU(3) value by 1 order of magnitude. Although not yet directly measured, indirect evidence from $\gamma p \to \Sigma^+ K^0$ (through K^* exchange) seems to confirm this prediction.[10] It is indeed the large calculated contribution of this vertex, assuming an SU(3)-symmetric PVV Lagrangian, which is partially responsible for the discrepancy discovered by Rockmore.

With the use of effective Lagrangians for PVV[8,9] and $V\gamma$[11] vertices and the form of Eqs. (2)–(3) for the weak transitions, the lowest-order contributions to the $K_2^0 \to \gamma\gamma$ process are depicted in Fig. 1.

The most general octet-broken form for the PVV vertices, as employed in Ref. 8, is given by

$$\mathcal{L}_{PVV} = \tfrac{1}{4}\epsilon^{\alpha\beta\mu\nu}\{h[d^{abc} + \sqrt{3}\epsilon_1 d^{abd}d^{d8c} + (\sqrt{3}/2)\epsilon_2(d^{acd}d^{d8b} + d^{bcd}d^{d8a}) + (\epsilon_3/\sqrt{3})\delta^{ab}\delta^{c8}]V_{\alpha\beta}{}^a V_{\mu\nu}{}^b P^c + \lambda(\delta^{ab} + \sqrt{3}\epsilon_4 d^{ab8})[V_{\alpha\beta}{}^a P^b V_{\mu\nu}{}^0]\}. \tag{5}$$

The coupling constants related to PVV and $V\gamma$ vertices were determined[8,9] from strong and electromagnetic meson decays as follows (for the ϵ_i's there are two possible solutions[12]):

$$\epsilon_1 = 0.85, \quad \epsilon_2 + \epsilon_3 = 2.16; \tag{6a}$$

$$\epsilon_1 = 1.18, \quad \epsilon_2 + \epsilon_3 = -2.54; \tag{6b}$$

$$g^2/4\pi = 3.2, \quad (m_\pi{}^2 h^2/4\pi)(1+\epsilon_1)^2 = 0.1. \tag{6c}$$

An explicit calculation of the three diagrams of Fig. 1 using Rockmore's procedure,[3] which exploits the hypothesis of partial conservation of axial-vector currents in order to combine the pseudoscalar and axial-vector intermediate contributions, gives the following matrix element:

$$M_{K_2{}^0 \to 2\gamma} = i g_{K_2{}^0\gamma\gamma}\epsilon^{\alpha\beta\mu\nu}k_\alpha{}^1\epsilon_\beta{}^1 k_\mu{}^2\epsilon_\nu{}^2,$$

$$g_{K_2{}^0\gamma\gamma} = -\frac{2G_{\rm nl}}{\sqrt{2}}C_K C_\pi \frac{m_\pi{}^2}{m_K{}^2 - m_\pi{}^2}\frac{4h}{3}(1+\epsilon_1)\left(\frac{e}{g}\right)^2 - \frac{2G_{\rm nl}}{\sqrt{2}}\frac{C_K C_\eta}{\sqrt{3}}\frac{m_\eta{}^2}{m_K{}^2 - m_\eta{}^2}\frac{4h}{3\sqrt{3}}[1+\epsilon_1+2(\epsilon_2+\epsilon_3)]\left(\frac{e}{g}\right)^2 + \frac{2G_{\rm nl}}{\sqrt{2}}\frac{8}{3}h\left(1-\frac{\epsilon_1}{2}\right)\frac{4}{3}\frac{m_\rho}{f_\rho}\frac{m_{K^*}}{f_{K^*}}\left(\frac{e}{g}\right)^2. \tag{7}$$

Numerically, this gives

$$|g_{K_2{}^0\gamma\gamma}| = \frac{10^{-9}}{m_{K^0}}\frac{1}{1+\epsilon_1}|-0.18(1+\epsilon_1) + 4.94[1-\epsilon_1+2(\epsilon_2+\epsilon_3)] + 11.2(1-\tfrac{1}{2}\epsilon_1)|, \tag{8}$$

where we used

$$C_\eta = 1.13C_\pi, \quad C_K = 1.08C_\pi, \quad (m_{K^*}/f_{K^*})^2 = (m_\rho/f_\rho)^2 = (m/g)^2. \tag{9}$$

Using Eq. (8), with SU(3) symmetry ($\epsilon_i = 0$), or with broken SU(3) [Eqs. (6)], one obtains

$$|g_{K_2{}^0\gamma\gamma}{}^{\rm SU(3)}| = 15.9\times 10^{-9}/m_{K^0}, \quad |g_{K_2{}^0\gamma\gamma}{}^A| = 14\times 10^{-9}/m_{K^0}, \quad |g_{K_2{}^0\gamma\gamma}{}^B| = 9.5\times 10^{-9}/m_{K^0}, \tag{10}$$

to be compared with

$$|g_{K_2{}^0\gamma\gamma}|_{\rm exp} = 1.7\times 10^{-9}\, m_{K^0}{}^{-1}. \tag{11}$$

The three results obtained above [the SU(3)-symmetric solution agrees with Rockmore's[3]] lead to a calculated decay width that is 2 orders of magnitude larger than the experimental value.[13]

At this point one is led to suspect that the above result has to do with the form of the nonleptonic weak Hamiltonian. Starting with a nonleptonic Hamiltonian of current-current type built from charged currents, one can ask for the minimal addition which would secure the $\Delta I = \frac{1}{2}$ rule. As is well known, it turns out that suitable addition of a J^3J^6 term is sufficient. In this way one arrives at the "simplest" Hamiltonian suggested in Eq. (4).[14] Recalculating the matrix element for $K_2^0 \to \gamma\gamma$ by using Eq.

(4) instead of Eq. (2), we obtain [a bar denoting quantities calculated with (4)]

$$|\bar{g}_{K_2{}^0\gamma\gamma}| = \frac{10^{-9}}{m_{K^0}} \frac{1}{1+\epsilon_1} |-0.18(1+\epsilon_1) + 8.4(1-\tfrac{1}{2}\epsilon_1)|. \qquad (12)$$

Only diagrams (a) and (c) of Fig. 1 contribute, and the sum over V_2 in (c) does not contain the 6–8 transition. With $\epsilon_i = 0$, one still has a calculated rate approximately 20 times larger than experiment. Using the SU(3)-broken solutions of Eqs. (7), we obtain the satisfactory results

$$|\bar{g}_{K_2{}^0\gamma\gamma}{}^A| = 2.4\times10^{-9}\, m_{K^0}{}^{-1}; \quad |\bar{g}_{K_2{}^0\gamma\gamma}{}^B| = 1.4\times10^{-9}\, m_{K^0}{}^{-1}. \qquad (13)$$

Although experiment slightly favors solution B, it cannot yet unambiguously rule out solution A, as the values of ϵ_i we used have uncertainties of up to 20%.

In view of the very large discrepancy between experiment and the calculated decay width with the previous approach,[3] we consider the agreement we obtain here as strong evidence favoring SU(3) breaking in the $K^{*0}K^0V$ vertex and the correctness of the weak nonleptonic Hamiltonian[15] for $\Delta S = 1$ transitions suggested in Eq. (4). This framework has also been used to investigate[16] various other weak radiative decays of K mesons, and it is found to provide a satisfactory picture, consistent with the available experimental data.[17]

In concluding we should like to stress that the measuring of the transition $K^{*0} \to K^0\gamma$ will provide a sensitive test of our model. With the ϵ_i's of Eq. (6), it is predicted to be 24 keV (6a) or 13 keV (6b), approximately 1 order of magnitude less than the SU(3) prediction.

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†Presently at the Department of Physics, Tel-Aviv University, Ramat-Aviv, Israel.

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[6]For the hyperon decays a value 25% higher is required. See also I. Kimel, Nucl. Phys. B23, 574 (1970).

[7]R. Rockmore [Phys. Rev. 185, 1847 (1969)] has also pointed out difficulties with the Hamiltonian (2) in connection with the $K_1{}^0$-$K_2{}^0$ mass difference. As the form suggested in (4) does not contain J^6J^8, Eq. (9) of Rockmore should be modified by dropping the η contribution, and hence the $K_2{}^0$ self-mass will be restored to a positive value.

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[11]N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967).

[12]We use the same notation as Ref. 8, to which the reader is referred for more details; in particular, current mixing for the vector mesons is used in Ref. 8, with $m = 847$ MeV and $\theta = 27.5°$.

[13]We have also included η-π mixing, whose magnitude was determined so as to account for the observed $K^+ \to \pi^+\pi^0$ decay. This leads to changes in the figures of Eq. (10) of a few percent only.

[14]C. H. Albright and R. Oakes, Phys. Rev. D 2, 1883 (1970), and 3, 1270 (1971). These authors also show from an analysis of various experimental data that the existence of a J^6J^8 term in the Hamiltonian is not required.

[15]It is interesting to point out the equivalence of this Hamiltonian with the schizon model proposed by Lee and Yang [Phys. Rev. 119, 1410 (1960)]. We are indebted to Professor R. J. Oakes for this remark.

[16]M. Moshe and P. Singer, to be published.

[17]After completing this manuscript, we received a preprint by R. F. Sarraga and H. J. Munczek, who also discuss the $K_2{}^0 \to \gamma\gamma$ problem. Although their conclusions concerning the weak Hamiltonian are similar to ours, we disagree with their procedure of modifying the Lee-Kroll-Zumino formalism.