Production Cross Section of Lee-Wick Hypothetical Massive Electromagnetic Bosons by Muons at High Energies*

Ralph Linsker
Institute for Space Studies, Goddard Space Flight Center, National Aeronautics and Space Administration, New York, New York 10025, and Department of Physics, Columbia University, New York, New York 10027
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The total cross section is computed for the resonant process $\mu^{\pm}+Z\rightarrow\mu^{\pm}+Z+B^0\rightarrow\mu^{\pm}+Z$ + (hadrons or leptons), where B^0 is the hypothetical massive spin-1 boson of electromagnetic interactions, proposed by Lee and Wick, and where Z is either a proton or Fe. Only those diagrams in which the B^0 is emitted by the muon are calculated.

This Letter reports on computations of production cross sections for the hypothetical massive spin-1 boson of electromagnetic interactions (the B^0 particle) proposed by Lee and Wick.[1,2] The resonant process

$$\mu^{\pm}+Z\rightarrow\mu^{\pm}+Z'+B^0\rightarrow\mu^{\pm}+Z'+(\text{hadrons or leptons}) \tag{1}$$

is considered in lowest order for two cases of Z and Z': (a) elastic scattering off a proton ($Z=Z'=p$) and (b) coherent scattering off a nucleus ($Z=Z'=$ Fe is used). The case of inelastic scattering off a proton ($Z=p$, Z' is a nucleon resonance or an hadronic system in the continuum) will be treated elsewhere. Possible contributions from diagrams in which the B^0 is emitted by Z or Z' (rather than by the muon) are neglected. It should be noted that the neglect of such diagrams is consistent with gauge invariance.[3]

The interaction Lagrangian of the B^0 with the muon is anti-Hermitian,[1,2] and is given by[4]

$$\mathcal{L}_{\text{int}}=e\psi^{\dagger}\gamma_4\gamma_{\lambda}\psi\varphi_{\lambda}, \tag{2}$$

where φ and ψ denote the B^0 and muon fields. The two lowest-order Feynman diagrams in which the B^0 is emitted by the muon are given in Fig. 1 in which the cross denotes the electromagnetic interaction with the nucleon or nucleus. The four-momenta of the incoming muon, the target Z, the outgoing muon, the B^0 particle, and the final nucleon or nucleus Z', shall be denoted by k, p, k', B, and p', respectively. All these momenta are on the mass shell; thus we have $k^2=k'^2=-m_{\mu}^2$, $p^2=p'^2=-M^2$, and $B^2=-m_B^2$. Defining the four-momentum transfer $q=p'-p$, the matrix element $\mathfrak{M}$ for the lowest-order process is then

$$\mathfrak{M}=-ie^2\bar{u}'\{[(k-q)^2+m_{\mu}^2]^{-1}(-q_{\nu}\gamma_{\lambda}\gamma_{\nu}\gamma_{\sigma}+2k_{\sigma}\gamma_{\lambda})+[(k'+q)^2+m_{\mu}^2]^{-1}(q_{\nu}\gamma_{\sigma}\gamma_{\nu}\gamma_{\lambda}+2k_{\sigma}'\gamma_{\lambda})\}u\varphi_{\lambda}V_{\sigma}. \tag{3}$$

Here u and u' are the respective spinors of the incoming and outgoing muons, normalized so that $\bar{u}u=\bar{u}'u'=2m_{\mu}$. The amplitude of the B^0, denoted by φ_{λ}, satisfies

$$\sum\varphi_{\lambda}\varphi_{\kappa}=\delta_{\lambda\kappa}+B_{\lambda}B_{\kappa}/m_B^2, \tag{4}$$

where the sum extends over the three polarization states of the B^0.

The factor V_{σ} is calculated for each of the Z, Z' cases as follows.

(a) For the case of elastic scattering off a proton, or incoherent scattering off a nucleus (when the lepton elastically scatters off a nuclear proton which can be regarded as essentially free), the familiar dipole fit[5] is used:

$$F_1(q^2)=G_E[1+\kappa(1+4M^2/q^2)^{-1}],\quad F_2(q^2)=\tfrac{1}{2}G_E\kappa(1+q^2/4M^2)^{-1}, \tag{5}$$

where $G_E=(1+q^2/0.71)^{-2}$, q in GeV, and $\kappa=1.7928$. Then

$$V_{\sigma}=(ie/q^2)\bar{u}_{p'}[F_1\gamma_{\sigma}+\tfrac{1}{2}iF_2M^{-1}(\gamma_{\sigma}\gamma_{\beta}-\gamma_{\beta}\gamma_{\sigma})q_{\beta}]u_p, \tag{6}$$

where u_p and $u_{p'}$ are the spinors of the initial and final protons, with $\bar{u}_pu_p=\bar{u}_{p'}u_{p'}=2M$.

(b) For coherent scattering off a nucleus of charge Z we take

$$V_{\sigma}=(eZ/q^2)(p+p')_{\sigma}F^Z(q^2), \tag{7}$$

which is exact for a spin-0 nucleus. The form factor used is the same as that assumed by Lee, Markstein, and Yang[6]:

$$F^Z(q^2) = (1 + \tfrac{1}{12}a^2q^2)^{-2}, \tag{8}$$

with

$$a^2 = \tfrac{3}{5}(1.3\times10^{-13}A^{1/3})^2\ \mathrm{cm}^2. \tag{9}$$

The differential cross section is given by

$$d\sigma = \frac{|\mathfrak{M}|^2 d^3k'\, d^3B\, d^3p'\, \delta^{(4)}(k-k'-B-q)}{(4\pi)^5 M|\vec{k}|k_0'B_0p_0'} \tag{10}$$

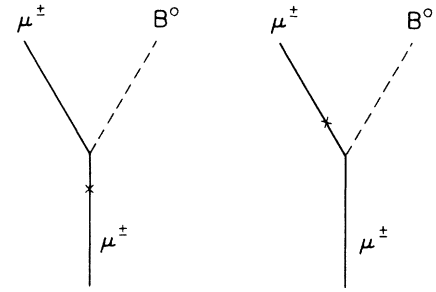

FIG. 1. Feynman diagrams.

in the lab frame. To calculate the total cross section (assuming the incoming beam to be unpolarized and summing over spins of all outgoing particles) we express $\sum|\mathfrak{M}|^2$ (summed over all spins) as the product of a leptonic factor $L_{\sigma\tau}$ and an hadronic factor $W_{\sigma\tau}$, where $L_{\sigma\tau}$ and $W_{\sigma\tau}$ are symmetric, and $q_\sigma L_{\sigma\tau} = q_\sigma W_{\sigma\tau} = 0$. The five variables of integration are chosen to be q^2, $k\cdot q$, $\varphi_{p'}$, β, and φ', where $\beta = m_B^2 + 2k\cdot B$, $\varphi_{p'}$ is the azimuthal angle of $\vec{p}'$ in the lab frame, and φ' is the azimuthal angle of $\vec{k}'$ in

TABLE I. Total cross section in units of 10^{-38} cm^2 for the process $\mu^{\pm} + Z \to \mu^{\pm} + Z + B^0 \to \mu^{\pm} + Z +$ (hadrons or leptons) for $Z = p$ and $Z = \mathrm{Fe}$.

m_B (GeV)	$E_{\mu,\,\mathrm{in}}$ (GeV)	$\sigma_{p,\,\mathrm{elastic}}$	$\frac{1}{26}\sigma_{\mathrm{Fe,\,coherent}}$	$\frac{1}{26}\sigma_{\mathrm{Fe,\,total\ elastic}}$
4	14	0.0498	0.0171	0.0662
	40	874	52.7	925
	120	6830	8870	15400
	200	11900	34900	45500
	400	20700	121000	137000
7	40	0.112	0.00613	0.118
	80	97.7	1.43	99.1
	120	390	21.9	411
	160	768	116	880
	200	1170	358	1510
	240	1580	806	2360
	280	1980	1490	3410
	320	2370	2410	4690
	400	3100	4900	7810
	700	5360	19100	23700
	1000	7100	36000	41700
10	80	0.116	0.00342	0.119
	120	12.2	0.0898	12.3
	160	57.1	0.749	57.8
	200	129	3.52	132
	240	218	11.4	229
	280	318	28.8	346
	320	423	60.7	481
	400	642	188	823
	700	1430	1720	3080
	1000	2120	4780	6720
13	120	0.00772	0.000900	0.00859
	160	1.26	0.00941	1.27
	200	8.30	0.0540	8.35
	240	23.4	0.213	23.6
	280	45.7	0.648	46.3
	320	73.5	1.63	75.1
	400	141	6.98	148
	700	443	145	582
	1000	747	630	1350

the frame $\vec{k}'=-\vec{B}$. (In each frame the polar axis is chosen to be $\vec{k}$.) The $\varphi_{p'}$ integration is trivial. The double integral

$$N_{\sigma\tau}=\iint L_{\sigma\tau}\,d\beta\, d\varphi' \tag{11}$$

depends only upon k and q, and can be written as

$$N_{\sigma\tau}=N_A(\delta_{\sigma\tau}-q^{-2}q_\sigma q_\tau)+N_B(k_\sigma-q^{-2}k\cdot qq_\sigma)(k_\tau-q^{-2}k\cdot qq_\tau), \tag{12}$$

where N_A and N_B are simply related to $N_{\sigma\sigma}$ and $k_\sigma N_{\sigma\tau}k_\tau$.

The expressions for $L_{\sigma\sigma}$ and $k_\sigma L_{\sigma\tau}k_\tau$ are calculated in invariant form, using Veltman's symbolic manipulation program SCHOONSCHIP[7] to perform some of the tedious algebra. Since $\beta^2 L_{\sigma\sigma}$ and $\beta^2 k_\sigma L_{\sigma\tau}\times k_\tau$ are independent of φ' and are polynomials in β, N_A and N_B are easily calculated. The remaining variables of integration are q^2 (in which variable the integrand varies most rapidly) and $k\cdot q$. These integrations are carried out by a multiple adaptive Simpson integration scheme.

It should be noted that any modification of the photon propagator due to the possible existence of B^0 enters also into the empirical determination of the hadronic form factors, and therefore need not be considered separately.

Total running time on the CDC 6600 at New York University was about ten minutes.

The integration scheme was checked in part by using it to calculate the cross section for spin-1 W-boson production by neutrinos. The results for elastic scattering agreed with those of Brown and Smith[8] (using the same form factors) to within about 1%, and those for coherent scattering off Fe agreed with the results of Lee, Markstein, and Yang[6] to within 3%.

The total elastic cross section for production off iron is calculated using the approximate expression[6]

$$\sigma_{\text{total elastic}}=Z\sigma_{p,\text{ elastic}}+(1-Z^{-1})\sigma_{\text{Fe, coherent}}. \tag{13}$$

At energies below, or very close to, the threshold for elastic production off protons and at sufficiently large energies (where essentially all production occurs via the coherent process) σ_{coherent} exceeds the right-hand side of Eq. (13). In that case we replace Eq. (13) by

$$\sigma_{\text{total elastic}}=\sigma_{\text{Fe, coherent}}. \tag{14}$$

Eqs. (13) and (14) neglect the effect of higher-order graphs, neutron contributions, Fermi statistics of the nucleons, details of the proton wave functions within the nucleus, and contributions from the graphs in which the B^0 is emitted by Z or Z'. The cross sections for incoming muon energies $E_{\mu,\text{ in}}\equiv(k_0)_{\text{lab}}$ up to 1000 GeV, and B^0 masses of 4, 7, 10, and 13 GeV, are presented in Table I. The results for $m_B=7$ and 10 GeV are also plotted in Fig. 2.

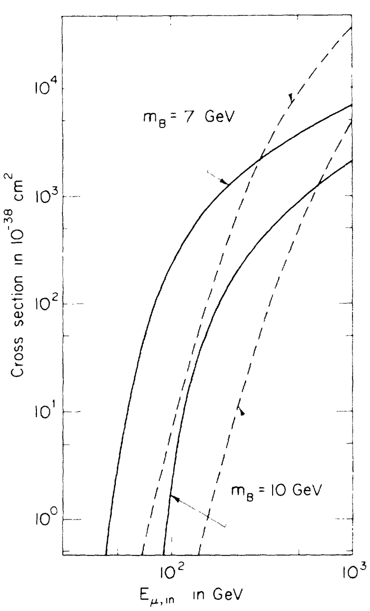

FIG. 2. Total cross section, per proton, for the resonant processes $\mu^\pm+Z\rightarrow\mu^\pm+Z+B^0\rightarrow\mu^\pm+Z+$(hadrons or leptons) at B^0 masses of 7 and 10 GeV, for Z=proton (solid curves), and for the coherent process with Z=Fe (broken curves).

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[1]T. D. Lee, in *Topical Conference on Weak Interactions, CERN, Geneva, Switzerland, 14-17 January 1969* (CERN Scientific Information Service, Geneva, Switzerland, 1969), p. 427.

[2]T. D. Lee and G. C. Wick, Phys. Rev. D 2, 1033 (1970).

[3]For kinematic configurations in which $(k-k')^2$ is large, the neglect of these diagrams is a reasonably good approximation. Because the lower limit on $(k-k')^2$ is $O(m_\mu^2)$, contributions from these diagrams at small $(k-k')^2$ could significantly affect the total cross section. In any case, the magnitude of the interference term between the diagrams in which the muon emits the B^0 and those in which the B^0 is emitted by the nucleon or nucleus is experimentally measurable by comparing cross sections for incident μ^- and μ^+ beams.

[4]Throughout the paper, $\hbar = c = 1$ and $e^2 \cong 4\pi/137$. The metric used is such that $a \cdot b = \vec{a} \cdot \vec{b} + a_4 b_4$, $a_4 = i a_0$. All γ matrices are Hermitian.

[5]D. H. Coward *et al.*, Phys. Rev. Lett. 20, 292 (1968); W. K. H. Panofsky, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, Austria, September 1968,* edited by J. Prentki and J. Steinberger (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 23.

[6]T. D. Lee, P. Markstein, and C. N. Yang, Phys. Rev. Lett. 7, 429 (1961).

[7]M. Veltman, unpublished.

[8]R. W. Brown and J. Smith, Phys. Rev. D 3, 207 (1971).

ERRATA

THEORY OF FLUCTUATION-INDUCED DIAMAGNETISM IN SUPERCONDUCTORS. Patrick A. Lee and Marvin G. Payne [Phys. Rev. Lett. 26, 1537 (1971)].

The manuscript receipt date as printed is erroneous; it should read 4 March 1971.

DISPERSION OF SURFACE PLASMONS IN InSb. N. Marschall, B. Fischer, and H. J. Queisser [Phys. Rev. Lett. 27, 95 (1971)].

A typographical error resulted in the chemical formula in the title appearing as "InSp."