Storm, and T. A. Cahill, Phys. Rev. C 2, 2050 (1970).
[15]J. L. Durand, J. Arvieux, A. Fiore, and C. Perrin, J. Phys. Colloq. 2, C2-196 (1970).
[16]V. Valković, D. Rendić, V. A. Otte, and G. C. Phillips, Phys. Rev. Lett. 26, 394 (1971).
[17]J. Durisch, W. Neumann, and J. Rossel, Nucl. Instrum. Methods 80, 1 (1970).
[18]V. V. Komarov and A. M. Popova, Nucl. Phys. 54, 278 (1964).
[19]M. Durand, thesis, Institut des Sciences Nucléaires, Grenoble, 1971 (unpublished).
[20]R. Corfu, J. P. Egger, C. Lunke, C. Nussbaum, J. Rossel, E. Schwarz, J. L. Durand, and C. Perrin, Helv. Phys. Acta 43, 443 (1970).

Accurate Description of the Reaction ^{92}Mo$(d,n)^{93}$Tc to Unbound Analog States

H. T. Fortune*
Physics Department, University of Pennsylvania, Philadelphia, Pennsylvania 19104

and

C. M. Vincent*
Department of Physics, University of Pittsburgh, Pittsburgh, Pennsylvania 15213
(Received 7 October 1971)

We present the results of distorted-wave Born-approximation calculations for the reaction ^{92}Mo$(d,n)^{93}$Tc to unbound $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$ analog states. Unlike the results of previous authors, our calculated angular distributions agree with experiment (to within 20%) at forward angles. We conclude that the distorted-wave Born approximation with single-particle form factors describes the reaction with as much accuracy as for bound final states.

The literature[1-3] on the theoretical description of proton transfer to *unbound* analog ($T_>$) states near $A = 90$ leaves in doubt the status of the distorted-wave Born approximation (DWBA). The purpose of this Letter is to demonstrate that simple DWBA calculations can nevertheless give a good account of experimental results.

It is found empirically[4,5] that the ratio of s- to d-state cross sections is much smaller than for corresponding bound states. By simply taking the unbound nature of the states into account, early work showed that DWBA calculations could reproduce the observed cross sections at least in order of magnitude, and often more closely. However, later calculations[3] which used more elaborate form factors still showed large discrepancies in the s-state cross sections. Among possible causes of these discrepancies are the fine structure of the "microgiant" analog states and interference with background. Before concluding that these complications must be taken into account, we wished to examine the possibility that the discrepancies might be due to some combination of inadequate approximations and/or to inappropriate choice of parameters.

Accordingly, we have carried out calculations by the methods described in an earlier paper[6] for the reaction that was treated by Zaidi and Coker[3] (ZC), namely, ^{92}Mo$(d,n)^{93}$Tc. We used form factors obtained by solving the Schrödinger equation (with resonant boundary conditions) for a Woods-Saxon well. The depth of the well was adjusted to bring the resonance energy into agreement with experiment ("well-depth method"). For the well that was used to generate the form factor, and for the optical potentials, we used the same potentials that were used by ZC. The radial integrals involved in the DWBA were evaluated in zero-range approximation with $D_0^2 = 1.50 \times 10^4$ MeV^2 F^3, by using a contour-integration technique[6] to accelerate their convergence. The integration of the cross section over the energy of the unobserved proton introduces[7] a factor Γ_p, the proton width of the resonance. Unlike ZC, we used values of Γ_p extracted from proton elastic-scattering experiments.[8] Thus we avoided making assumptions about the structure and isospin purity of the resonant state.

In Table I the results of our calculation are compared with the experimental data and the results of ZC. For d states, the two calculations agree about equally well. For the s state, however, our calculation agrees much better with experiment. The observed reduction in the s-state cross section is therefore a feature of normal DWBA calculations. It is the combined result of kinematics, momentum matching, and "leaking out" of the resonant wave function for

TABLE I. Experimental and theoretical cross sections at 15° for the reaction ^{92}Mo(d, n)^{93}Tc to unbound analog states, for $E_d = 12$ MeV.

E_X (MeV)	J^Π	nlj	E_p(c.m.)[a] (MeV)	Γ_p(exptl)[a] (keV)	σ_{exptl}[b] (mb/sr)	σ_{theor} (mb/sr) Present[c]	ZC[d]
8.40	$\frac{5}{2}^+$	$2d\frac{5}{2}$	4.32	1	2.3 ± 0.2	2.13	2.283
9.33	$\frac{1}{2}^+$	$3s\frac{1}{2}$	5.25	12	0.14 ± 0.04	0.178	0.425
9.91	$\frac{3}{2}^+$	$2d\frac{3}{2}$	5.83	3	0.48 ± 0.05	0.390	0.832

[a] From Ref. 8.

[b] From Ref. 5, as corrected by J. Horton and quoted by Ref. 3.

[c] See text. These calculations took the listed values of Γ_p as exact. Since σ_{theor} depends linearly on Γ_p in this work, a given percentage change in Γ_p will produce the same percentage change in σ_{theor}.

[d] From Ref. 3.

large penetrabilities. The experimental and theoretical angular distributions are shown in Fig. 1. Comparisons at 15° are clearly far more significant than those at 25°, especially for the s state, where the first *minimum* falls at an angle less than 25°. It can be seen that our calculations agree within the experimental errors for all three states.

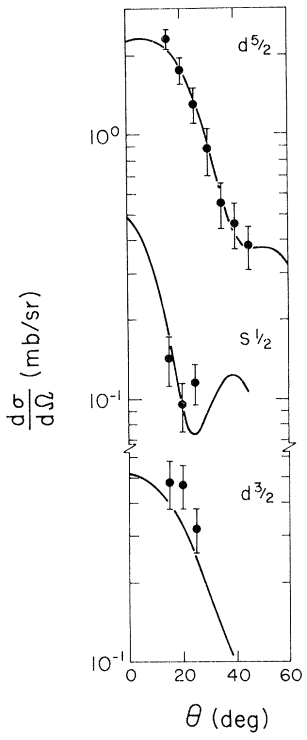

FIG. 1. Experimental (points) and theoretical (curves) angular distributions for the reaction ^{92}Mo(d,n)^{93}Tc at $E_d = 12$ MeV. The theoretical cross section has *not* been normalized to the data. The method described herein yields the absolute magnitude of the DWBA cross sections, provided that Γ_p is known from elastic scattering.

We cannot be sure of the reasons for the discrepancy of a factor of 3 between the two s-wave calculations (Table I). Although ZC used a less accurate technique for the radial integrals, this could not produce so large an error. ZC obtained their form factors by approximate solution of coupled equations with a $\vec{T}\cdot\vec{t}$ (Lane) term. The geometrical parameters involved were identical to ours. However, ZC have performed the calculation only for the case where the parent state is purely single particle. They correct for deviations from this limit by multiplying their results by S_n [the spectroscopic factor of the parent state, extracted from (d, p) data[9]]. This practice involves two related assumptions: first, that the width Γ_p is proportional to S_n, and second, that the shape of the form factor does not materially depend on the value of S_n. No justification of the first assumption seems to have been given.[10] Furthermore, it is possible that the experimental values[9] of S_n were extracted by using inappropriate geometrical parameters for the well that was used to generate the form factor. (These parameters are not listed in Ref. 9.) This would produce appreciable errors since an increase of r_0 by 10% can easily decrease the value of S_n by 40%. The second assumption (on which the well-depth method also depends) is used in both calculations. Because the surface region contributes most of the cross section, it is important for the form factor to behave correctly in the exterior (i.e., beyond the range of the nuclear forces). In the exterior, the ZC form factor should reduce to a multiple of the appropriate irregular Coulomb function G. However, by inspection it appears to be a linear combination of an exponentially decaying term and a G

function. This is due to the use of the Fano-Feshbach method, which treats the coupling as a perturbation that mixes the continuum into the bound mathematical analog of the parent state. Such an error in the shape of the form factor could have a large effect on the cross section, without greatly affecting the phase shifts.

In conclusion, we have demonstrated that correct DWBA calculations can adequately describe the data on proton stripping to unbound analog states, even when well-depth form factors are used—in analogy with common practice for bound analog states. This result is of obvious practical significance. Much worse results have been obtained with a form factor obtained by solving the Lane equations. This may be a warning that the usual rules of DWBA must be respected, even if the calculation of the form factor is in some way improved.

We thank Dr. W. R. Coker for providing us with the data in tabular form.

*Work supported in part by the National Science Foundation.

[1]D. H. Youngblood, R. L. Kozub, R. A. Kenefick, and J. C. Hiebert, Phys. Rev. C 2, 477 (1970).

[2]B. J. Cole, R. Huby, and J. R. Mines, Phys. Rev. Lett. 26, 264 (1971).

[3]S. A. A. Zaidi and W. R. Coker, Phys. Rev. C 4, 236 (1971).

[4]R. L. McGrath, N. Cue, W. R. Hering, L. L. Lee, B. L. Liebler, and Z. Vager, Phys. Rev. Lett. 25, 682 (1970).

[5]S. A. A. Zaidi, C. L. Hollas, J. L. Horton, P. J. Riley, J. C. L. Ford, and C. M. Jones, Phys. Rev. Lett. 25, 1503 (1970).

[6]C. M. Vincent and H. T. Fortune, Phys. Rev. C 2, 782 (1970).

[7]Vincent and Fortune, Ref. 6, Eq. (30). Note that $d\sigma_{lj}{}^{F}/d\Omega_p$ represents the output of DWBA calculations in which the form factor is normalized like $(kr)^{-1}\cos kr$ at large distances. The normalization of the theoretical cross section is then automatically determined.

[8]C. F. Moore, P. Richard, C. E. Watson, D. Robson, and J. D. Fox, Phys. Rev. 141, 1166 (1966).

[9]J. B. Moorhead and R. A. Moyer, Phys. Rev. 184, 1205 (1969).

[10]Nevertheless, the assumption emerges in most treatments of the extraction of S_n from Γ_p. For examples, see N. Auerbach, J. Hüfner, A. K. Kerman, and C. M. Shakin, to be published; W. J. Thompson, J. L. Adams, and D. Robson, Phys. Rev. 173, 975 (1968).

Relativistic Dynamics with Noncanonical Positions

Asher Peres

Department of Physics, Technion—Israel Institute of Technology, Haifa, Israel

(Received 1 September 1971)

For any Hamiltonian H having vanishing Poisson brackets with the total momentum $\vec{P}$ and angular momentum $\vec{J}$, it is always possible to construct boost operators $\vec{K}$ having the correct Poisson brackets with all the other generators of the Poincaré group. Thus the existence of dynamical variables satisfying the algebra of the Poincaré group generators is not in itself a guarantee of Lorentz invariance.

Traditionally, the first step in the quantization of a classical system has been to write its equations of motion in canonical form. The conditions for the compatibility of these canonical equations of motion with the requirements of special relativity were investigated long ago by Dirac.[1] In essence, Dirac's argument was that if a canonical formulation is possible in one Lorentz frame, then it should be possible in every Lorentz frame (by the principle of relativity). Therefore, a Lorentz transformation must be a canonical transformation of the dynamical variables. In particular, an infinitesimal Lorentz transformation has ten generators, namely the Hamiltonian H (translation in time), the total momentum $\vec{P}$ (translations in space), the total angular momentum $\vec{J}$ (rotations), and the boost $\vec{K}$ (pure Lorentz transformations). The Poisson bracket relations characteristic of the Poincaré group are

$$[H, P_m] = 0, \tag{1}$$

$$[H, J_m] = 0, \tag{2}$$

$$[P_m, P_n] = 0, \tag{3}$$

$$[J_m, P_n] = \epsilon_{mns} P_s, \tag{4}$$

$$[J_m, J_n] = \epsilon_{mns} J_s, \tag{5}$$

$$[J_m, K_n] = \epsilon_{mns} K_s, \tag{6}$$

$$[H, K_m] = -P_m, \tag{7}$$

$$[P_m, K_n] = -\delta_{mn} H, \tag{8}$$

$$[K_m, K_n] = -\epsilon_{mns} J_s. \tag{9}$$