

# Treiman-Yang Criterion for Quasifree Scattering in Deuteron Breakup at Low Energy

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The Treiman-Yang criterion to test the dominance of the pole graph has been applied to the reaction  $H(d, 2p)n$  in the quasifree condition and at a deuteron energy of 20 MeV. The distribution obtained is not constant, thus indicating the need for further graphs.

The description of nuclear reactions, which are in fact many-body processes, is usually reduced to a two- or a three-body problem which is solved by the use of more or less phenomenological models. Many such models have been used to describe direct reactions (plane-wave impulse and distorted-wave Born approximations, etc.). Another approach suggested by Shapiro<sup>1</sup> is to apply to direct nuclear reactions the Feynman-graph technique, previously developed for elementary particle physics. In using this method there is, in general, no simple way to select the particular graphs which are dominant for a given reaction mechanism. Such a selection can only be made by the study of the proximity of the graphs' singularities. However, this requires the consideration of all possible graphs.

In the case of knock-out reactions, the structure of the scattering amplitude corresponding to the pole graph in the Feynman series, provided the matrix element can be factorized, is particularly simple and has experimentally observable consequences. For deuteron breakup and similar processes [Fig. 1(a)], provided the exchanged particle has spin 0 or  $\frac{1}{2}$ ,<sup>2</sup> the amplitude is invariant under rotation of the  $\vec{p}_2$ - $\vec{p}_3$  reaction plane around the transferred momentum  $\vec{p}_i$  in the proton ( $p$ ) rest frame [Fig. 1(b)]. This invariance (Treiman-Yang test),<sup>3</sup> which was already pointed out in the case of deuteron stripping,<sup>4</sup> is a necessary but not a sufficient condition for the dominance of the pole graph. As pointed out by Kolybasov,<sup>5</sup> under certain conditions and approximations some triangular graphs may satisfy this condition. However, the Treiman-Yang test provides a strong indicator for the pole-graph description of the reaction mechanism.

Measurements of deuteron breakup reactions have shown cross-section enhancements in two kinematical regions, one corresponding to a low

relative energy of a pair of nucleons (final-state interaction), the other to a small momentum transfer to one nucleon. This second enhancement, corresponding to the quasifree kinematical condition, is usually described by the impulse approximation.<sup>6</sup>

This approximation has been successful for incident energies greater than 150 MeV,<sup>6-9</sup> but fails below 100 MeV.<sup>10-16</sup> Some modifications have been proposed including off-shell and final-state effects but have not yet explained the low-energy data. In the Feynman-graph formalism, the pole graph also predicts an enhancement at low momentum transfer. However, the existence of such an enhancement in the experimental data does not necessarily imply that this graph rep-

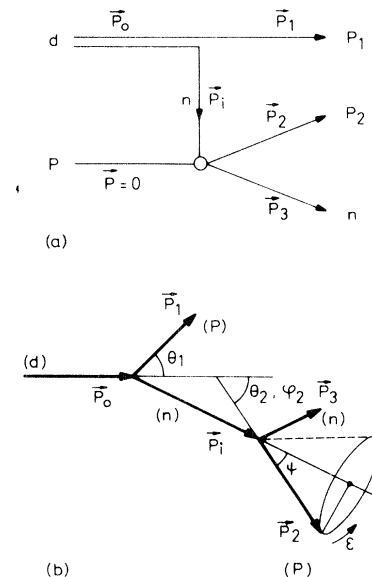


FIG. 1. (a) Pole graph for the reaction  $d + p \rightarrow p + p + n$ . Momentum of transferred particle in lab system,  $\vec{p}_i = \vec{p}_0 - \vec{p}_1$ ; momentum transfer,  $\vec{q} = \vec{p}_1 - \frac{1}{2}\vec{p}_0$ . (b) Momentum diagram in the lab system for the reaction  $d + p \rightarrow p + p + n$  corresponding to the pole graph.

resents a dominant contribution. The purpose of this experiment is to study the importance of the pole graph in the low-energy deuteron breakup by performing the Treiman-Yang test.

The momentum diagram in the laboratory system corresponding to the pole graph is shown on Fig. 1(b). As the test has to be made in the rest frame of proton  $p$ , it is convenient to have the deuterons incident on the protons to facilitate the analysis. We detected in coincidence the two protons  $p_1$  and  $p_2$ . Counter 1 is maintained in a fixed position  $(\theta_1, \varphi_1)$ , and counter 2  $(\theta_2, \varphi_2)$  is rotated around the direction of  $\vec{p}_i$  on a cone of opening angle  $\psi$  and  $0^\circ \leq \epsilon \leq 110^\circ$ . Then, if the criterion is fulfilled and, as in the present case, if the phase space is invariant, then the cross section  $d^3\sigma/d\Omega_1 d\Omega_2 dE_1$  should be independent of  $\epsilon$  at a particular value  $E_1$  corresponding to the chosen momentum transfer  $\vec{q}$ . The configurations have to satisfy four simultaneous requirements: (i)  $q^2/2m < E_1 = 2.225$  MeV in order to be close to the quasifree peak; (ii) the other two momentum transfers have to be large; (iii) the relative energies of the three pairs of nucleons have to be larger than 1 MeV to minimize final-state interaction effects; (iv) angles  $\theta_1$  and  $\theta_2$  must be sufficiently large in order that the detectors are not saturated by elastically scattered particles. For  $q^2/2m = 0.9$  MeV the above requirements dictated the following configurations  $(\theta_1 = 16.5^\circ$  and  $\varphi_1$

$= 180^\circ)$ :

$\epsilon$	$\theta_2$	$\varphi_2$
$0^\circ$	$36.5^\circ$	$0^\circ$
$25^\circ$	$36.0^\circ$	$14.5^\circ$
$45^\circ$	$34.0^\circ$	$26.5^\circ$
$60^\circ$	$31.5^\circ$	$34.5^\circ$
$90^\circ$	$26.0^\circ$	$51.5^\circ$
$110^\circ$	$21.0^\circ$	$64.0^\circ$

The experiment was performed with 20-MeV deuterons from the isochronous cyclotron at the Institut des Sciences Nucléaires in Grenoble. The  $4.3\text{-mg-cm}^{-2}$   $\text{CH}_2$  target was placed in a 1.20-m-diam spherical reaction chamber.<sup>17</sup> Protons were detected by two solid-state detector telescopes ( $\Delta E$ - $E$ ) each subtending a solid angle of 0.2 msr. The experiment was monitored by detecting recoil protons in a third similar detector. For each coincident event the energy in the four detectors of the telescopes plus the time-of-flight difference of the two particles were recorded on magnetic tape by a PDP-9 computer for subsequent analysis.

After off-line particle identification and background subtraction, events were displayed in a  $E_1$ - $E_2$  matrix. The events in the kinematical curve were then projected onto the  $E_1$  axis. The spectra thus obtained are shown in Fig. 2. Absolute cross sections were calculated using the monitor spectra and the known  $p$ - $d$  elastic cross

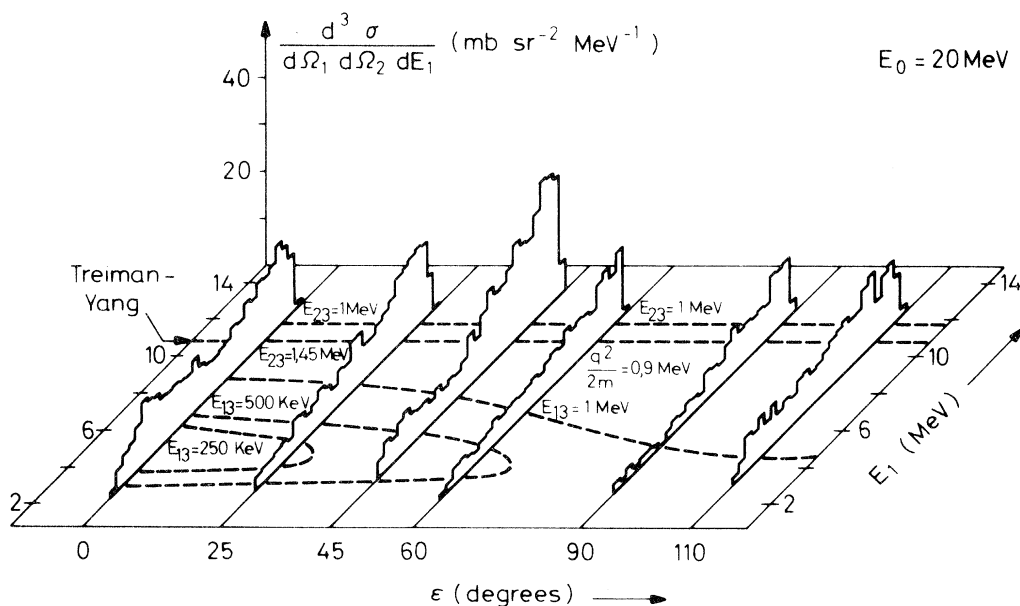


FIG. 2. Display of cross section versus angle  $\epsilon$  and energy of spectator. Dashed lines indicate values of relative energies. At the Treiman-Yang points,  $E_{23} = 1.45$  MeV and  $q^2/2m = 0.9$  MeV for all  $\epsilon$ .

section. Within a 2-MeV band about an energy  $E_1$  of 10.9 MeV, which corresponds to the selected Treiman-Yang axis and momentum transfer, this momentum transfer varies very little ( $0.82 < q^2/2m < 0.98$  MeV). Therefore, the cross section was obtained by summing all events in this band; it is plotted as a function of the azimuthal angle  $\epsilon$  in Fig. 3. If one attempts to fit these points with a horizontal line as demanded by the criterion, one obtains a mean cross section of  $13.0 \text{ mb sr}^{-2} \text{ MeV}^{-1}$ . However the  $\chi^2$  of such a fit is 20.4, which corresponds to a confidence level of  $5 \times 10^{-3}$ . It therefore seems unlikely that the Treiman-Yang criterion is fulfilled for deuteron breakup with 20-MeV incident deuterons.

At low energies the nucleon-deuteron wavelength is comparable to the size of the deuteron, and the transit time of the projectile across the interaction region is of the order of the period of the target internal motion. Therefore the basic requirements for the validity of the impulse approximation are not fulfilled. Nevertheless, the enhancement corresponding to the quasifree kinematic condition is still important at low energy. Calculations have been attempted in this energy region. The first is based on the Feynman-graph technique as presented by Komarov and Popova<sup>18</sup> for the deuteron breakup. The cross section obtained from the pole graph was  $68 \text{ mb sr}^{-2} \text{ MeV}^{-1}$  at the Treiman-Yang point (independent of  $\epsilon$ ). The second, made by Durand,<sup>19</sup> is based on an iteration of the Faddeev

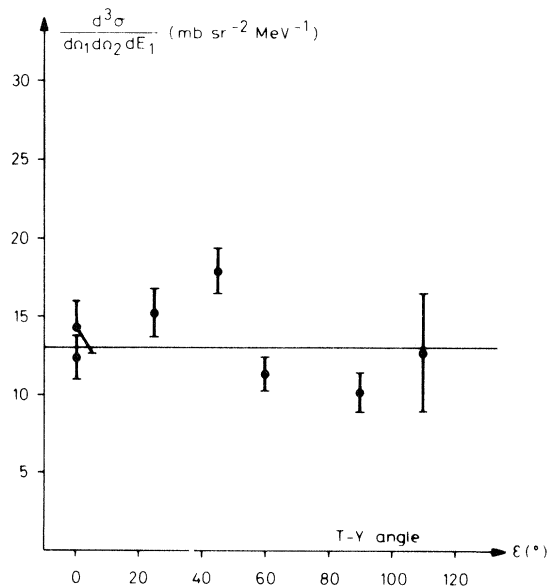


FIG. 3. Variation of cross section with Treiman-Yang angle  $\epsilon$ .

equations using a spin-dependent  $s$ -wave separable Yamaguchi potential. The cross section obtained here was  $26 \text{ mb sr}^{-2} \text{ MeV}^{-1}$  for the first term (independent of  $\epsilon$ ) and 24 to  $30 \text{ mb sr}^{-2} \text{ MeV}^{-1}$  (depending on  $\epsilon$ ) for the first plus second term. Both calculations give the correct shape for the energy spectra.

In conclusion, contrary to indications of preliminary data,<sup>20</sup> our more complete measurement of the Treiman-Yang distribution shows that at low energy the pole graph does not give a sufficient description of the enhancement associated with the quasifree kinematic condition and that more exact calculations must be performed. This is consistent with the discrepancy between the magnitude of the experimental cross section and the cross section calculated with the pole graph or the first two terms of a perturbation series.

We wish to thank all our friends at Grenoble for their support and Dr. Beveridge for his help in the final stage of the experiment; we also express our gratitude to Mr. Fermé who spared no effort to have the cyclotron working.

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## Accurate Description of the Reaction $^{92}\text{Mo}(d,n)^{93}\text{Tc}$ to Unbound Analog States

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We present the results of distorted-wave Born-approximation calculations for the reaction  $^{92}\text{Mo}(d,n)^{93}\text{Tc}$  to unbound  $d_{5/2}$ ,  $s_{1/2}$ , and  $d_{3/2}$  analog states. Unlike the results of previous authors, our calculated angular distributions agree with experiment (to within 20%) at forward angles. We conclude that the distorted-wave Born approximation with single-particle form factors describes the reaction with as much accuracy as for bound final states.

The literature<sup>1-3</sup> on the theoretical description of proton transfer to *unbound* analog ( $T_{\gamma}$ ) states near  $A=90$  leaves in doubt the status of the distorted-wave Born approximation (DWBA). The purpose of this Letter is to demonstrate that simple DWBA calculations can nevertheless give a good account of experimental results.

It is found empirically<sup>4,5</sup> that the ratio of  $s$ - to  $d$ -state cross sections is much smaller than for corresponding bound states. By simply taking the unbound nature of the states into account, early work showed that DWBA calculations could reproduce the observed cross sections at least in order of magnitude, and often more closely. However, later calculations<sup>3</sup> which used more elaborate form factors still showed large discrepancies in the  $s$ -state cross sections. Among possible causes of these discrepancies are the fine structure of the "microgiant" analog states and interference with background. Before concluding that these complications must be taken into account, we wished to examine the possibility that the discrepancies might be due to some combination of inadequate approximations and/or to inappropriate choice of parameters.

Accordingly, we have carried out calculations by the methods described in an earlier paper<sup>6</sup> for the reaction that was treated by Zaidi and Coker<sup>3</sup> (ZC), namely,  $^{92}\text{Mo}(d,n)^{93}\text{Tc}$ . We used

form factors obtained by solving the Schrödinger equation (with resonant boundary conditions) for a Woods-Saxon well. The depth of the well was adjusted to bring the resonance energy into agreement with experiment ("well-depth method"). For the well that was used to generate the form factor, and for the optical potentials, we used the same potentials that were used by ZC. The radial integrals involved in the DWBA were evaluated in zero-range approximation with  $D_0^2 = 1.50 \times 10^4 \text{ MeV}^2 \text{ F}^3$ , by using a contour-integration technique<sup>6</sup> to accelerate their convergence. The integration of the cross section over the energy of the unobserved proton introduces<sup>7</sup> a factor  $\Gamma_p$ , the proton width of the resonance. Unlike ZC, we used values of  $\Gamma_p$  extracted from proton elastic-scattering experiments.<sup>8</sup> Thus we avoided making assumptions about the structure and isospin purity of the resonant state.

In Table I the results of our calculation are compared with the experimental data and the results of ZC. For  $d$  states, the two calculations agree about equally well. For the  $s$  state, however, our calculation agrees much better with experiment. The observed reduction in the  $s$ -state cross section is therefore a feature of normal DWBA calculations. It is the combined result of kinematics, momentum matching, and "leaking out" of the resonant wave function for