In the course of the experimental work, two problems were encountered. First, Rb vapor reacts with Pyrex and leads to cell yellowing and uv opacity after several hours at temperatures above about 300°C. Second, the low Rb diffusion rate in Xe combined with the Rb reaction or clean-up problem limited the amount of Rb which was uniformly obtainable over our cell length and prevented experiments at higher Rb-Xe pressures. We believe that both of these problems can be solved by employing a heat-pipe oven of the type recently described by Vidal and Cooper.[5]

In summary, we have measured $\chi_{\mathrm{Rb}} = 1.4\times10^{-32}$ esu for tripling 1.064 to 0.3547 μm, and have demonstrated that metal vapors may be phase matched via the addition of inert gases. Based on these measurements and on previous calculations,[1] we believe that if the Rb vapor pressure could be increased to 16 Torr (415°C) and the cell length extended to 50 cm, 50% conversion efficiency to 0.35 μm should be obtainable with an input power of about 10 MW. Peak powers exceeding this are now readily available with picosecond lasers, and calculations have shown that subpicosecond pulses are acceptable in a system of this type.[1]

Since metal vapors are often nearly transparent for wavelengths above their ionization potentials,[6] this technique should allow tripling of 6943 Å in Na and also tripling of tunable dye lasers. It should also be possible to cascade several similar systems to extend this technique through the vacuum ultraviolet. For example, Cd-He is of interest for 3547 → 1182 Å generation.

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Computer Simulation of Anomalous dc Resistivity

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The current-driven ion-sound instability is investigated by two-dimensional computer experiments. It is found that, in contrast to the one-dimensional case, the instability produces an anomalous resistivity.

The two-stream instability excited by a strong electric current in a plasma has been the subject of numerous investigations, since it is believed to cause anomalous resistivity and turbulent heating in many plasma devices. Though a linear stability analysis is rather simple, little is known so far about the nonlinear behavior. Theoretical methods are rather limited in the case of a strongly turbulent system, while experimental observations normally do not admit of simple interpretation since various effects interfere. Here numerical simulation experiments can be very useful.

Recently, the two-stream instability driven by a constant external electric field E_0 has been studied by numerical simulation of a one-dimensional plasma.[1,2] The main result was that under the combined influence of the instability and the driving field the electrons are accelerated and heated in such a way that the drift velocity v_d tends to be close to the thermal velocity $v_{\mathrm{th},e}$, $v_d \simeq v_{\mathrm{th},e}$, which means that the current increases linearly with time with half the free acceleration rate, and that there is no resistivity in the usual sense. This behavior seems to be in clear contrast to observations in many experiments, implying that the model of a one-dimensional plasma submitted to a constant electric field is inadequate.

We have extended and changed the numerical model in a twofold way: (a) We treat a two-dimensional plasma to study the importance of modes propagating obliquely to the current direction. Theory predicts that in the presence of a broad angular spectrum of waves the number of resonant electrons is much larger than in the case of a very narrow (one-dimensional) spectrum and hence the resistivity should be larger. (b) Instead of applying a constant electric field and measuring the current we impose a constant current and observe the resulting voltage. In real experiments the current must first rise above the instability threshold, but in general rapid changes of the current are prohibited by, for instance, the plasma inductance, so that it seems to be essentially the current which is controlled.

In this Letter we investigate a particular aspect of the turbulent resistivity problem, viz., the effect of the ion-sound instability in a plasma with $T_e \gg T_i$.

The numerical model used in the computations is the standard clouds-in-cell or particle-in-cell method.[3,4] The computer experiments were performed on a grid of 128×128 mesh points with $\Delta x = 0.4\lambda_D$ (λ_D is the initial Debye length, $\lambda_D^2 = T_{e0}/4\pi ne^2$) using periodic boundary conditions. The number of simulation particles was $N \simeq 2 \times 10^5$ per species, so that initially $n\lambda_D^2 \simeq 80$. The initial temperature ratio was $T_{e0}/T_{i0} = 50$, and the initial ratio of the drift to thermal velocity, $v_d/v_{\mathrm{th},e0} = 1$ ($m_e v_{\mathrm{th},e}^2 = T_e$). The mass ratio was varied, $m_i/m_e = 100$, 400, 1600.

Figures 1(a) and 1(b) show plots of the fluctuating electric field energy $W = \langle E^2 \rangle / 8\pi$, the average value of the local ion temperature $T_i = \langle T_i(x) \rangle$, both on a logarithmic scale, and the current-conserving dc field E_0 for a run with $m_i/m_e = 400$. It can be seen that the phase of exponential growth of W is terminated and W starts to saturate at the point where ion heating sets in rather abruptly. This is connected with the onset of ion trapping, as becomes evident from phase-space plots. The maximum value of the field energy over thermal energy is small, $W/nT_e \simeq 0.006$. Thus the occurrence of ion trapping seems surprising since a rough estimate shows the trapping value of W/nT_e to be of the order of 1. It is easy, however, to evaluate the trapping condition exactly for a typical thermal ion with $m_i v_{\mathrm{th},i}^2 = T_i$ by a sinusoidal ion wave of amplitude φ_0 and with wavelength and

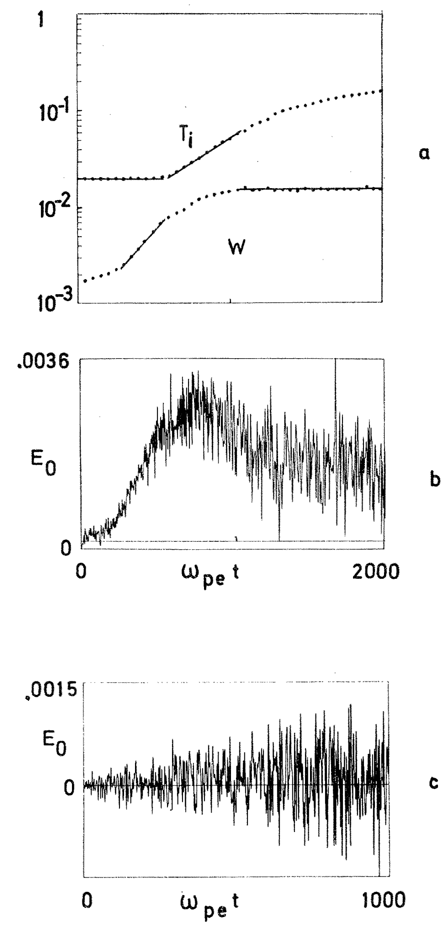

FIG. 1. Results of a run with $m_i/m_e = 400$. (a) $\ln W$ and $\ln T_i$ (W in units of nT_{e0}, T_i in units of T_{e0}), (b) E_0 [in units of $(m_e/e)\omega_{pe} V_{\mathrm{th},e0}$], (c) E_0 of the corresponding one-dimensional system.

phase velocity corresponding to the fastest growing mode, $k\lambda_D = \frac{1}{2}\sqrt{2}$, $v_{\mathrm{ph}} = (\frac{2}{3})^{1/2} c_s$, $c_s^2 = T_e/m_i$. From

$$2e\varphi_0/m_i - [(\tfrac{2}{3})^{1/2} c_s - v_{\mathrm{th},i}]^2 = 0 \quad (1)$$

one obtains

$$W/nT_e = \tfrac{1}{72}[1 - (3T_i/2T_e)^{1/2}]^4 \quad (2)$$

which is in fact small, $\lesssim 1\%$, consistent with values observed in the numerical experiments. Thus saturation of the instability seems to be essentially due to strong interaction of ion waves with ions and not to "quasilinear" relaxation (plateau formation) of the electron distribution f_e.

There is nevertheless a distortion of f_e [see Fig. 2]. Those electrons which do not resonate with (are not trapped by) the waves are freely accelerated by the dc field E_0, producing a "run-

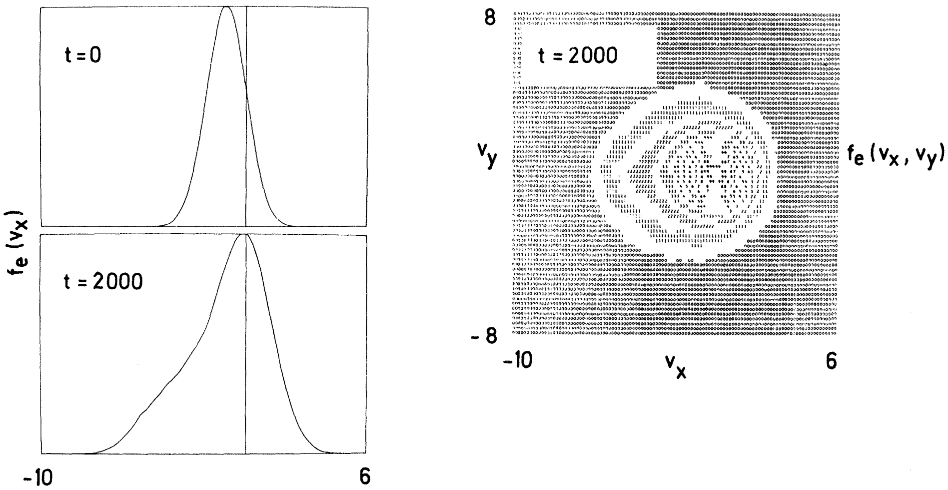

FIG. 2. Relaxation of the electron distribution function. (a) $f_e(v_x)$ for $t=0$, $t=2000\omega_{pe}^{-1}$, (b) contour plot of $f_e(v_x, v_y)$.

away" tail, while the bulk of the electron distribution remains fairly isotropic [Fig. 2(b)] but with smaller drift, $v_{\text{bulk}} < v_d$. The runaway electrons, however, do not switch off the instability by taking over the total current and reducing v_{bulk} to zero. In fact, after the runaway tail has been formed, the fraction of current it carries changes very slowly; the rate of acceleration by the mean field E_0 is of the same order as the increase of the thermal velocity of the electron bulk due to turbulent heating, so that the total anisotropy (including the runaway part of f_e) does not noticeably change.

To see more clearly the effect of the change of f_e, the mass ratio was varied, since $\dot{f}_e \sim \omega_{pe}$ is independent of m_i, while $\dot{W} \sim \omega_{pi}$ is reduced with increasing m_i. Some results are given in Table I. Comparing runs with $m_i/m_e = 100$, 400, and 1600, we find that the final level $W_{\max}$ does not decrease with increasing m_i/m_e. This confirms the result that saturation of the instability is brought about by interaction of sound waves with ions and not by "quasilinear" relaxation of f_e.

However, the maximum value of W/nT_e decreases when the ion mass increases, and so does E_0. In fact, we find a rather strong correlation between W/nT_e and the effective collision frequency ν_{eff}, given in terms of the applied field $E_0(t)$ by $\nu_{\text{eff}}/\omega_{pe} = E_0$ (in the units adopted), which can be explained qualitatively by a simple Fokker-Planck model of electron scattering by a turbulent field $\vec{E}$. As seen in Table I, the m_i/m_e dependences of W/nT_e and ν_{eff} are not very strong, $\nu_{\text{eff}} \sim (m_i/m_e)^{-a}$, $a \simeq \frac{1}{4}-\frac{1}{3}$. This dependence is caused by the electron heating being relatively faster for larger m_i, and has apparently nothing to do with the linear growth rate $\gamma_L \sim (m_e/m_i)^{1/2}$ which disproves the simple "theoretical" formula $\nu_{\text{eff}} \sim \gamma_L$.

The angular distribution of the turbulence spectrum, $W(\theta) = \int W(k, \theta) k\, dk$, is given in Fig. 3 for the case shown in Figs. 1(a) and 1(b). Since the

TABLE I. Comparison of three runs with $m_i/m_e = 100$, 400, 1600; W in units of nT_{e0}, E_0 in units of $(m_e/e)\omega_{pe}v_{\text{th},e0}$. In these units $\nu_{\text{eff}}/\omega_{pe} = E_0$.

m_i/m_e	$W_{\max}$ (%)	$(W/nT_e)_{\max}$ (%)	$E_{0\,\max}$ (10^{-3})
100	1.4	0.8	3.2
400	1.55	0.64	2.4
1600	1.7	0.5	1.6

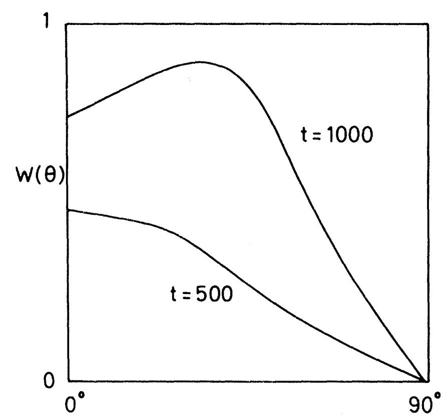

FIG. 3. Angular dependence of the turbulence spectrum for $m_i/m_e = 400$.

growth rate of modes in the direction of the current is most strongly affected by the change in f_e, oblique modes are more pronounced.

To compare these results with the behavior of a one-dimensional plasma the same run as shown in Figs. 1(a) and 1(b) was performed for a one-dimensional system. Here the saturation field energy is found to be much smaller than in the corresponding two-dimensional case, and so is the *average* field $E_0 = \nu_{\rm eff}/\omega_{pe}$ plotted in Fig. 1(c). This is because in a one-dimensional system the ion sound instability is switched off by formation of a small plateau in f_e, since only a few electrons are resonant. In the two-dimensional case where a broad spectrum in θ is excited, a "plateau" cannot stabilize oblique modes—the change of f_e actually taking place is illustrated in Fig. 2—which makes the ion-sound instability more effective.

Finally, we compare the results of our two-dimensional computations with experimental values of the turbulent resistivity. Since we adopted periodic boundary conditions, the comparison has to be made with toroidal experiments. For instance, Hamberger and Jancarik[5] obtain $\nu_{\rm eff}/\omega_{pe} \simeq (1/2\pi)(m_e/m_i)^{1/3}$ for the Buneman instability and typically a factor of 5 less in the ion-sound regime with $v_d \simeq 0.2 v_{{\rm th},e}$.

Assuming $\nu_{\rm eff} \sim v_d/v_{{\rm th},e}$, we can extrapolate this result to the regime of the computer experiments, $v_d \simeq v_{{\rm th},e}/2$, obtaining $\nu_{\rm eff}/\omega_{pe} \simeq 0.007$ for $m_i/m_e = 1600$. This value is larger by a factor of 4 than the numerical result given in Table I.

The discrepancy seems to be due to the two-dimensional character of the computations, where a large "runaway" contribution reduces the effective drift of the bulk, $v_{\rm bulk}$, considerably. We expect on simple geometrical grounds that in a fully three-dimensional computer experiment the number of runaway electrons will be appreciably smaller.

By way of summary, the two-dimensional computations have shown that the ion-sound instability for $v_d < v_{{\rm th},e}$ can produce an effective turbulent resistivity. Saturation of the instability is achieved by ion trapping, which cannot be described adequately by any weak-turbulence approach. The relaxation of the electron distribution gives rise to a dependence of $\nu_{\rm eff}$ on m_i/m_e, which, however, is weaker than the dependence obtained from the *ad hoc* argument $\nu_{\rm eff} \sim \gamma_L \sim (m_e/m_i)^{1/2}$. The numerical value of $\nu_{\rm eff}$ is smaller than that found in real experiments, which might be a result of the two-dimensional nature of the computed plasma.

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Correlation Range and Compressibility of Xenon near the Critical Point*

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From the magnitude and angular dependence of the intensity of light scattered from xenon near its critical point we have measured the Ornstein-Zernike long-range correlation length ξ and the reduced compressibility $(\partial\rho/\partial\mu)_T$. We find $\xi = (3.0 \pm 0.10)\epsilon^{-0.58 \pm 0.05}$ Å and $(\partial\rho/\partial\mu)_T = (1.43 \pm 0.06)\times 10^{-9}\epsilon^{-1.21 \pm 0.03}$ g^2/erg cm^3 along the critical isochore, and $(\partial\rho/\partial\mu)_T = (0.346 \pm 0.01)\times 10^{-9}\epsilon^{-1.21 \pm 0.02}$ g^2/erg cm^3 along the coexistence curve, with $\epsilon \equiv |T - T_c|/T_c$. Using our results, we have compared the predictions of the Kawasaki-Kadanoff-Swift mode-mode coupling theories with measurements of the thermal diffusivity in xenon.

Using essentially the apparatus of Giglio,[1] we have measured the absolute magnitude and angular anisotropy of the intensity of light scattered from xenon near its critical point. The light-scattering cell is of beryllium copper with a hollow, cylindrical, optical-quality glass window