

perature independence of the excitation energies for $Q > 0.4$, 0.3 \AA^{-1} unexplained. Finally, nuclear specific-heat measurements⁷ confirm that the ordered moment originates in the $4f$ subshell so that explanations on the basis of conduction-electron polarization must be ruled out. Clearly some fresh theoretical insight is required; we hope that these results will serve to stimulate such theory.

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Exact Wave-Type Solution to f - g Theory of Gravity*

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The well-known plane-fronted waves with parallel rays, exact solutions of the vacuum Einstein gravitational field equations, are generalized to exact solutions of the equations of f - g gravity theory proposed by Isham, Salam, and Strathdee in regions free from matter.

Recently, Isham, Salam, and Strathdee¹ proposed to modify Einstein's equations of general relativity into a two-tensor theory of gravitation. In this modified theory one has, besides the usual $g^{\mu\nu}$ field, a second "metric" tensor $f^{\mu\nu}$ which is thought to describe the massive f -meson spin-2 field. It is assumed that the g field couples directly only to leptons (weak gravity), and the f field only to hadronic matter (strong gravity). A mixing term between f and g ensures that the g field interacts with hadronic matter indirectly via the f field (" f dominance of hadronic gravity"). However, so far no nontrivial, i.e., $g^{\mu\nu} \neq f^{\mu\nu}$, exact solution of the field equations has been found. We give here a class of exact solutions for the f - g field equations known from general relativity as plane-fronted waves with parallel rays (pp waves).

pp waves have been extensively studied in Einstein's theory. The search for pp waves in f - g theory was therefore strongly motivated by the following properties of pp waves (see Ehlers and

Kundt² and Bonnor,³ and references given there): (i) pp waves are pure radiation fields, for which the curvature and Einstein tensor are of a simple algebraic structure. (ii) Einstein's equations are linear for pp waves when written in a suitable coordinate system. (iii) As a consequence of (ii), pp waves propagating in the same direction can be superposed linearly. (iv) Nonvacuum pp waves can be superposed by superposing their sources. (v) Exact and linearized solutions of Einstein's equations coincide for pp waves.

The idea was to carry this over to f - g theory and look for pp-wave vacuum solutions. By vacuum we mean a space-time region without (leptonic and hadronic) matter and electromagnetic radiation, in which case the equations of f - g theory are of the form

$$\begin{aligned} \frac{G_{\mu\nu}(f)}{\kappa_f^2(-f)^{1/2}} + \frac{\partial \mathcal{L}_{fg}}{\partial f^{\mu\nu}} &= 0, \\ \frac{G_{\mu\nu}(g)}{\kappa_g^2(-g)^{1/2}} + \frac{\partial \mathcal{L}_{fg}}{\partial g^{\mu\nu}} &= 0, \end{aligned} \quad (1)$$

where

$$\begin{aligned} \mathfrak{L}_g &= \kappa_g^{-2}(-g)^{-1/2}R(g), \\ \mathfrak{L}_f &= \kappa_f^{-2}(-f)^{-1/2}R(f), \\ \mathfrak{L}_{fg} &= \kappa_f^{-2}(-f)^{-1/2}\frac{1}{4}M^2(f^{\alpha\beta}-g^{\alpha\beta})(f^{\kappa\lambda}-g^{\kappa\lambda}) \\ &\quad \times (g_{\alpha\kappa}g_{\beta\lambda}-g_{\alpha\beta}g_{\kappa\lambda}), \end{aligned} \tag{2}$$

and $R(g), R(f)$ are the Riemann curvature scalars belonging to the "metrics" g, f . (All symbols, conventions, and abbreviations are taken the same as in Ref. 1. To take advantage of properties (iii) and (iv) we made an *Ansatz* for the g and f fields in the form of pp waves moving in the same direction. It is convenient to choose radiation coordinates for which the *Ansatz* leading to pp waves is

$$\begin{aligned} g_{\mu\nu} dx^\mu dx^\nu &= 2G(dx^0)^2 + 2dx^0 dx^3 \\ &\quad - (dx^1)^2 - (dx^2)^2, \\ f_{\mu\nu} dx^\mu dx^\nu &= 2F(dx^0)^2 + 2dx^0 dx^3 \\ &\quad - (dx^1)^2 - (dx^2)^2, \end{aligned} \tag{3}$$

where G and F are of the following form:

$$G(x) = G(x^0, x^1, x^2), \quad F(x) = F(x^0, x^1, x^2). \tag{4}$$

For the above *Ansatz* the Einstein tensor has only one independent component, and because of the simple algebraic structure of Eqs. (3) the determinant of $g_{\mu\nu}$ and $f_{\mu\nu}$ is -1 . It is therefore easy to compute Eqs. (1), from which only the $\mu = \nu = 3$

components remain, leading to the equations

$$\begin{aligned} \Delta_2 G &= M^2(G - F), \\ \Delta_2 F &= -(\kappa_g^2/\kappa_f^2)M^2(G - F), \end{aligned} \tag{5}$$

where

$$\Delta_2 \equiv \partial^2/(\partial x^1)^2 + \partial^2/(\partial x^2)^2.$$

We see that the x^0 dependence of the functions G and F remains arbitrary. Clearly Eqs. (5) are linear and can be decoupled into

$$\begin{aligned} \Delta_2(G - F) &= M^2(1 + \kappa_g^2/\kappa_f^2)(G - F), \\ \Delta_2[F + (\kappa_g^2/\kappa_f^2)G] &= 0. \end{aligned} \tag{6}$$

Solutions of the decoupled equations can readily be written down.⁴ Physically these solutions are pp waves of the f and g field propagating in the same direction of space. The conformal tensor for both fields is of Petrov type N . Finally we would like to mention that the solutions of the linearized f - g equations lead to identical results in the case of pp waves.

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Isotropic Black-Body Boson Production and the Temperature of Hadronic Matter*†

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Inclusive π^+, π^- , and K^0 production from 11.8-GeV/c K^+p is isotropic in the center-of-mass system and obeys a Bose-Einstein distribution over a limited energy range. The values of the parameter kT are observed to be consistent with the pion mass.

We present a study of the energy spectra of bosons produced in certain inclusive reactions. We are particularly interested in the region of center-of-mass momentum less than several hundred MeV/c. Earlier¹ we reported an isotropic distribution of π^- mesons from the reaction

$$K^+ + p \rightarrow \pi^- + \text{anything} \tag{1}$$

when the c.m. momentum of the π^- is less than

320 MeV/c. We have now extended observations to π^+ and K^0 mesons produced via the reactions

$$K^+ + p \rightarrow \pi^+ + \text{anything} \tag{2}$$

and

$$K^+ + p \rightarrow K^0 + \text{anything}. \tag{3}$$

We find all three reactions exhibit isotropy (qualified in the K^0 case), and all three energy distributions are adequately described by Bose-