Flute Stabilization Via Electrostatically Confined Cold Electrons

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(Received 18 October 1971)

Flute instabilities driven by unfavorable magnetic field curvature can be stabilized by a positive ambipolar potential which confines cold electrons electrostatically. This mechanism does not depend on line tying in the conventional sense. We derive stability criteria and discuss their relation to experimental observations.

The nonoccurrence of flute instabilities in many plasmas confined in simple magnetic mirror traps has frequently been attributed to various types of "line tying."[1] The theoretical models advanced to explain the observations usually postulate a current flow to conducting end walls which can neutralize the space charge of the incipient flute. In other models the vanishing of the fluctuating potential on grounded conducting end plates serves to couple cold electrons to the unstable waves so that the modes are stable for sufficiently large cold electron density.

One mechanism which we believe has received inadequate attention in the past arises if cold electrons are at least partially confined to the central region of the trap by positive ambipolar electric potentials. This circumstance generally obtains in neutral injection experiments[2] and can also occur in electron-cyclotron–heated plasmas[3] provided the ambient gas pressure is above a critical value. If cold electrons are electrostatically confined then neutralization of incipient flute instabilities can take place *independently of the end walls*. To see how this is possible, consider a simple mirror machine in which plasma is continuously being created (by neutral injection or electron-cyclotron heating) and also continuously being lost through the mirrors. In equilibrium the production and loss of plasma are equal. We assume that there is a positive ambipolar potential which confines a cold electron component. Now, suppose that the potential on a flux tube is momentarily raised. Fewer of the electrons being produced will escape, so the number of electrons in the flux tube will increase and consequently the potential will decrease. Similarly, if the potential is decreased, more electrons escape and the potential rises. It is clear that this mechanism will act to suppress the growth of a flute instability; as will be shown, it can lead to stabilization under rather reasonable conditions.

We shall now discuss these ideas more quantitatively and derive a sufficient condition for stability. Following this we shall return to a discussion of experimental evidence bearing on this effect and its significance for fusion research in some flute-unstable traps.

Let us recall briefly the conventional picture of flute instabilities and their stabilization by infinitely conducting end walls. The electrons and ions drift perpendicular to the magnetic field with the velocity

$$v_j = g_j/\Omega_j, \quad j = i, e \tag{1}$$

where $\Omega_j = q_j B/m_j c$ is the cyclotron frequency, and the "effective gravitational field" is given by

$$g_j = v_{tj}^2/2R_c, \tag{2}$$

where $v_{tj} = (2T_j/m_j)^{1/2}$ is the thermal velocity and R_c is radius of curvature of the field line. In the presence of a density gradient and a perturbation, space charge builds up because of the oppositely directed drifts of electrons and ions. The resultant electric field is in such a direction that the drift of the electrons and ions with velocity cE/B causes the perturbation to grow exponentially with maximum growth rate given by

$$\gamma = \left(\frac{g}{N}\frac{dN}{dr}\right)^{1/2}. \tag{3}$$

If there are perfectly conducting end walls, then the field lines which are tied to the plasma and to the end walls must be bent.[4] If the displacement of a flux tube in the center of the plasma is ξ then the restoring force $\vec{F} = \vec{B}\cdot\nabla\vec{B}/4\pi$ may be estimated to be

$$F \simeq (B^2/4\pi)\xi/L^2,$$

where L is the distance between the mirrors. If this force is greater than the force $\rho\gamma^2\xi$ which drives the instability, then the system is stable. This gives a criterion for stabilization by infinitely conducting end walls:

$$\gamma^2 < \frac{B^2}{4\pi\rho}\frac{1}{L^2} = \frac{V_a^2}{L^2}, \tag{4}$$

where V_a is the Alfvén speed.

In what follows we shall assume that Eq. (4) is always satisfied so that the system would be stable if there were infinitely conducting walls. Therefore, we may neglect any bending of the field lines by the force driving the instability. We assume that the magnetic field is uniform and in the z direction. The effective gravitational field $\vec{g}_j$ is in the x direction and the density gradient is in the negative x direction. We write the particle densities and the potential as $N = N_0 + n$ and $\Phi = \Phi_0 + \varphi$, where n and φ are small perturbations which are assumed to vary in space and time as $\exp(iky - i\omega t)$. The drift velocity of a particle of species j is the sum of its gravitational, electric, and polarization drift:

$$\vec{v}_j = \frac{m_j\vec{g}_j \times \vec{B}}{q_j B^2} + c\frac{\vec{E}\times\vec{B}}{B^2} + \frac{mc^2}{q_j B^2}\frac{d\vec{E}}{dt}.$$

This velocity is to be used in the continuity equation $\partial n/\partial t + \nabla\cdot(N\vec{v}) = S$, where the source term S will be chosen to account for the current that flows along the field lines in response to the fluctuating potential. For the moment we write it as $S = \eta\varphi$, where the coefficient η is still to be specified.

We assume that there are three species of particles in the plasma: ions, hot electrons, and cold electrons. The continuity equation is linearized and solved for n. The result is then used in Poisson's equation, and the dispersion relation is found to be

$$1 + \sum_j \frac{\omega_{pj}^2}{\Omega_j^2} = \sum_j \frac{1}{\omega - \omega_j}\left(-\frac{N_j'}{kN_j}\frac{\omega_{pj}^2}{\Omega_j} + \frac{i4\pi q_j\eta_j}{k^2}\right), \tag{5}$$

where ω_{pj} is the plasma frequency of the jth species and $\omega_j = kg_j/\Omega_j$ is the frequency associated with the drift in the effective gravitational field. We will write $N_j'/N_j = N_j^{-1}\, dN_j/dx = R_p^{-1}$, where R_p is the radius of the plasma.

It is worthwhile to look briefly at some special illustrative cases. First, assume $\eta_j = 0$ and that there is only one electron species. Equation (5) simplifies to

$$1 = -\frac{\Omega_i}{kR_p}\left(\frac{1}{\omega - \omega_i} - \frac{1}{\omega - \omega_e}\right)$$

which has the solution

$$\omega = \tfrac{1}{2}(\omega_i + \omega_e) \pm \left[\tfrac{1}{4}(\omega_i + \omega_e)^2 - \frac{\Omega_i}{kR_p}(\omega_i - \omega_e)\right]^{1/2}.$$

For large k there are two real roots, $\omega = \omega_i$ and $\omega = \omega_e$. Below a critical value of k the roots become complex indicating an instability. In the limit of small k we find

$$\omega \xrightarrow[k\to 0]{} \pm\left[-\frac{\Omega_i}{kR_p}(\omega_i - \omega_e)\right]^{1/2} = \pm i\left[\frac{1}{R_p}\left(g_i + \frac{m}{M}g_e\right)\right]^{1/2}.$$

This is in agreement with Eq. (3). By setting $\eta_j = 0$ we have neglected the end effects which are expected to stabilize the system.

Next, let us assume that there are resistive end walls. Then the current J that flows along the field line will be proportional to φ and we can write $\partial n/\partial t \sim J \sim \varphi$ so the appropriate source term is $S = \eta\varphi$ with η a real number. For this case we can also assume that there is only one electron species, that the electrons are hot and the ions are cold (so that $\omega_i = 0$), and that $\eta_i = 0$. In the limit $k \to 0$ the roots are

$$\omega = \frac{i}{2}\frac{V_a^2 4\pi e\eta_e}{k^2c^2} \pm \left[-\frac{m}{M}g_e\frac{1}{R_p} - \left(\frac{V_a^2 4\pi e\eta_e}{k^2c^2}\right)^2\right]^{1/2}.$$

In the limit $\eta_e \to \infty$ corresponding to an infinitely conducting wall, the unstable root reduces to $\omega = 0$. For any finite conductivity the growth rate of the instability is finite. This is in agreement with the result of Kunkel and Guillory[5] who analyzed the effect of a finite conductivity of a wall sheath.

Finally, we consider the case of an electrostatically confined cold-electron component of the plasma. We shall assume that $N_c = N_{c0}\exp(e\Phi/T_c)$ so that

$$dN_c/dt = -i\omega e N_c T_c^{-1}\varphi,$$

from which

$$\eta_{ce} = -i\omega e N_c/T_c.$$

We assume $\eta_{he} = \eta_i = 0$ and, for simplicity, $\omega_i = \omega_{ce} = 0$; Eq. (5) becomes

$$1 + \frac{V_a^2}{\lambda_c^2 k^2 c^2} = -\frac{\Omega_i}{kR_p}\frac{N_{he}}{N_i}\left[\frac{1}{\omega} - \frac{1}{\omega - \omega_{he}}\right], \tag{6}$$

where

$$\lambda_c = (T_{ce}/4\pi N_c e^2)^{1/2}$$

is the Debye length of the cold electron component. We have used $N_i = N_{ce} + N_{he}$. The roots of

Eq. (6),

$$\omega = -\frac{1}{2}\frac{kg_e}{\Omega_e} \pm \left[-\frac{m}{M}g_e\frac{N_{he}}{N_i}\frac{1}{R_p}\left(1+\frac{V_a^2}{\lambda_c^2k^2c^2}\right)^{-1}+\frac{1}{4}\frac{k^2g_e^2}{\Omega_e^2}\right]^{1/2},$$

will be real for all values of k if

$$\frac{1}{4}\frac{g_e^2}{\Omega_e^2}\frac{V_a^2}{\lambda_c^2c^2} > g_e\frac{N_{he}}{N_i}\frac{1}{R_p}\frac{m}{M},$$

or equivalently, if

$$\frac{N_{ce}}{N_{he}} > 4\frac{T_{ce}}{T_{he}}\frac{R_c}{R_p}. \tag{7}$$

For two-component plasmas having energetic ions and colder electrons, this criterion becomes

$$1 > 4\frac{T_e}{T_i}\frac{R_c}{R_p}. \tag{8}$$

In typical electron-cyclotron–heated plasmas, cold ion-electron pairs are created by ionization of background gas. Since the cold electrons would flow out of the trap more rapidly than the ions, a positive ambipolar potential will arise to retard the loss of the electrons. For quasineutrality,

$$N_{ce}+N_{he} \cong N_i = N_{he}N_0\langle\sigma v\rangle_{\text{ionization}}\bar{\tau},$$

whence, for stability,

$$\frac{N_{ce}}{N_{he}} \cong N_0\langle\sigma v\rangle\bar{\tau} - 1 > 4\frac{T_{ce}}{T_{he}}\frac{R_e}{R_p},$$

that is,

$$(N_0)_{\text{crit}} \cong \left(1+\frac{4T_{ce}}{T_{he}}\frac{R_c}{R_p}\right)\frac{1}{\langle\sigma v\rangle\bar{\tau}}, \tag{9}$$

where $\bar{\tau}$ is the average particle lifetime in the plasma.

The present mechanism may be inadequate to stabilize perturbations of sufficiently large amplitude that

$$\varphi/\Phi_0 > \delta N_{ce}/N_{ce},$$

where δN_{ce} is the number of new cold electrons created in a wave period. For electron-cyclotron–heated plasmas we estimate this critical level to be

$$\frac{\delta N_{ce}}{N_{ce}} = \frac{N_{he}N_0\langle\sigma v\rangle\tau_{\text{wave}}}{N_{ce}} \lesssim N_0\langle\sigma v\rangle\frac{2\pi}{\omega_{\text{wave}}}\left(\frac{4T_{ce}}{T_{he}}\frac{R_c}{R_p}\right)^{-1},$$

$$\frac{\delta N_{ce}}{N_{ce}} \lesssim \frac{\pi}{2}N_0\langle\sigma v\rangle\frac{eBR_p^2}{T_{ce}}.$$

Typical experimental parameters[3] give a critical pressure from Eq. (9) of around 2×10^{-5} Torr and a limiting perturbation $\varphi/\Phi_0 \sim 1$, that is, tens of volts. A critical pressure stability of (2–5) $\times10^{-5}$ Torr was reported in Ref. 3 and, in later experiments,[6] found to be unchanged when water-cooled glass plates were substituted for the original copper end walls.

The situation is quite different in two-component, hot-ion experiments, in which the average particle lifetimes are two to three orders of magnitude greater than in the electron-cyclotron–heated plasmas. In particular, the critical fluctuation level becomes very small in experiments such as[2] DCX-1:

$$(\varphi/\Phi_0)_{\text{crit}} \sim \delta N/N \sim \tau_{\text{wave}}/\bar{\tau} \lesssim 10^{-3}.$$

The present stabilization would not be significant in such plasmas, in agreement with observations of end-wall–dependent flute thresholds. Note, however, that in a larger mirror experiment which more nearly approaches thermonuclear conditions, this critical fluctuation level can be significantly higher:

$$(\varphi/\Phi_0)_{\text{crit}} \sim \tau_{\text{wave}}/\bar{\tau} \sim \pi eBR_pR_cNT^{-5/2}C_{\text{Coul}},$$

where

$$C_{\text{Coul}} \cong \frac{2}{25.8\sqrt{\pi}}\left(\frac{e^2}{\epsilon_0}\right)^2\frac{\ln\Lambda}{\sqrt{M}}.$$

If $B = 50$ kG, $R_p \sim R_c \sim 1$ m, $N \sim 10^5$ cm^{-3}, and $T \sim 100$ keV, we have

$$(\varphi/\Phi_0)_{\text{crit}} \sim 10^{-2}.$$

Since $\Phi_0 \sim \tau_e \sim \tau_i/10 \sim 10^4$ V, this mechanism could stabilize perturbations of around 100 V. Should this prove inadequate, it seems likely that the surface of a large plasma could be sustained against more rapid loss rates corresponding to a lower temperature with increased ambient gas pressure. The strong temperature dependence of φ_{crit} then permits $(\varphi/\Phi_0)_{\text{crit}} \sim 1$ for T(surface) $\sim 10^4$ eV. The technological and economic burdens associated with minimum-B traps add considerable incentive to a test of the present stability criterion.

This research was sponsored by the U. S. Atomic Energy Commission under contract with the Union Carbide Corporation. We acknowledge with pleasure stimulating discussions with R. A.

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Selection Rules for Tunneling into Single-Crystal Superconductors*

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The first conclusive evidence for a tunneling selection rule for single-crystal superconductors is presented. Using tunneling data for the (001) plane of gallium obtained in two different laboratories, we show that the *zero transverse k-vector rule* of Dowman, MacVicar, and Waldram is superior to the usual group-velocity rule. The dominant tunnel direction in *real* space is the normal to the tunnel barrier and does not appear to be affected by barrier orientation or structure.

The problems involved in interpreting tunneling data obtained from single-crystal specimens were summarized recently by Dowman, MacVicar, and Waldram[1] (DMW). There are two questions involved: (1) What tunneling direction dominates across the barrier? For many reasons pointed out in Ref. 1, the barriers produced on single-crystal substrates might have a highly ordered structure, giving rise to an anisotropic tunneling probability that is not a maximum perpendicular to the barrier surface, as is usually assumed. (2) Which electrons are associated with the tunneling process? The usual rule has been to choose electrons with a group velocity perpendicular to the tunneling barrier, but application of this rule has met with little success.[2,3]

In this Letter, we will examine these problems using our data on gallium[4] and similar data of Yoshihiro and Sasaki[5] (YS).

Effect of barrier structure.—Figure 1(a) shows the reduced energy gap, $2\Delta(0)/kT_c$, as a function of crystal orientation in the (001) plane of gallium obtained by our group and by YS. (Note that the gallium phase we are dealing with, the one stable near room temperatures and atmospheric pressure, is orthorhombic.) In several directions, indicated by the dashed lines, we have identified multiple energy gaps. This identification is *not* made by YS, probably because of a difference in data analysis. A complete analysis of these data will appear in a full length article.

Aside from the difference of interpretation of multiple gaps, our experiments and those of YS were essentially the same with one exception: The tunneling barriers used in our work were untouched naturally grown oxides while YS bombarded the single-crystal surfaces with ions, to reduce barrier impedance, before the tunneling probe was applied.[6] As a result, their barriers were undoubtedly disordered while the barriers used in our work had some chance to develop an ordered structure. (However, both barriers were probably somewhat disordered.) As one can see from Fig. 1(a), there is good agreement between the two sets of measurements, with only slightly more scatter in the data obtained by YS. Some scatter in their data might be expected since the bombardment process would also produce shorts and damage in the barrier region and this might give rise to slightly imperfect tunneling characteristics.

The fact that data on two types of barriers yield the same energy-gap data is an indication that tunneling occurs in the direction perpendicular to the barrier for all orientations measured. This point can be checked further by assuming perpendicular tunneling and comparing features of the energy-gap curve with the features of the