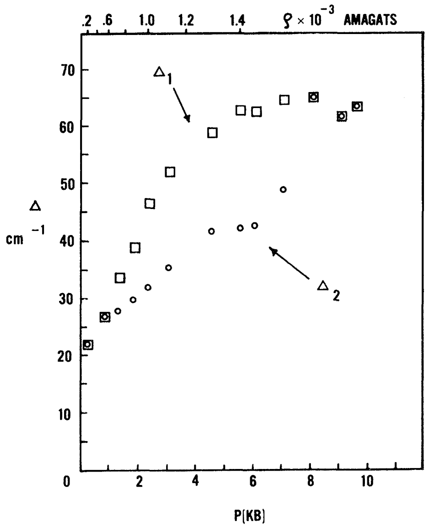

FIG. 4. Pressure dependence of spectral exponents in neon in the high-temperature regime (300°K). Note that where they differ, Δ_1 is greater than Δ_2. The incomplete density scale at the top of the figure is a result of a corresponding-states calculation based on the 1020°K isotherm for argon.

which is particularly simple for reduced temperatures $kT/\epsilon \lesssim 3$. In this range the temperature and density effects on the spectral shape are separable and individually very simple. For $kT/\epsilon > 3$, the simple temperature dependence fails and the intermolecular polarizability giving rise to high frequencies (short times) saturates and eventually decreases.

We have avoided associating the observed spectra with a specific model for the short-time dynamics of the fluid, although several aspects of the spectra can be given quite interesting interpretations within the framework of such models. It is our hope that as dynamic theories of dense fluids are developed, the information contained in these spectra will provide useful guidelines.

We are grateful to D. H. Olson and H. L. Carter for excellent technical assistance in these experiments.

*Work at Princeton supported in part by the National Science Foundation under Grant No. GP18573.

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Collisional Drift Instability Driven by an Axial Current*

R. F. Ellis† and R. W. Motley
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08540
(Received 12 October 1971)

The current-driven collisional drift instability is observed in a double-ended Q-machine cesium plasma. The instability is of large amplitude and leads to significant density decrease at onset. The characteristics of the instability are in good agreement with a linearized two-fluid theory which includes electron heat flow and electron temperature oscillations.

Plasma instabilities driven by currents which flow parallel to a magnetic field are of importance because such currents are utilized for Ohmic heating and plasma production in many devices. Low-frequency drift instabilities driven by current would be particularly interesting since drift instabilities in the *absence* of current[1-5] are known to cause enhanced plasma losses across the magnetic field. Moreover, Coppi and Mazzucato[6,7] attribute anomalous resistivity in Ohmic-heated stellarator discharges to the current-driven collisional drift instability. In this Letter we report the observation of this drift instability in a cesium plasma created in a double-ended Q machine. Certain important instability features are shown to agree with a linearized two-fluid theory which includes electron-heat-flow effects and electron temperature oscillations. It is important to stress that the theory which assumes isothermal electrons, such as in Refs. 1–3, predicts no interaction between the axial current and the collisional drift mode. The present experiments are interpreted as strong evidence that electron temperature oscillations and heat flow are important elements when considering collisional drift instabilities. These features were first pointed out clearly by Coppi and Mazzucato[6] and investigated in more detail by Tsai, Perkins, and Stix.[8]

As our theoretical model we consider a low-β collisional plasma slab with a static magnetic field B in the z direction and a density $n_0(x)$ varying exponentially in the x direction. An electron drift u_0 is assumed in the z direction with $u_0 \ll (kT_e/m_e)^{1/2}$; ion motion parallel to B is ignored: The plasma is described by the two-fluid equations,[9] where the electron-heat-flow equation is considered and the electron temperature is allowed to oscillate. The equilibrium ion and electron temperatures are equal. We consider electrostatic small-amplitude perturbations of the form $\tilde{A}\exp(-i\omega t + ik_x x + ik_y y + ik_z z)$, where the tilde denotes perturbed quantities. Electron inertia and Larmor-radius effects are retained. Modes localized in x are treated, and we assume $k_z \gg (1/n_0)dn_0/dx$.

The basic electron equations for the case of a current-carrying plasma are presented in Ref. 6. One may write the equation of heat transfer by electrons as

$$\tfrac{3}{2}n\frac{dT_e}{dt} - T_e\frac{dn}{dt} + \nabla\cdot\vec{q}_e = neE_zV_z - V_z\nabla_z(nT_e), \quad (1)$$

where $\vec{q}_e$, the electron-heat-flux vector, is defined in Ref. 6.

The equation for particle and momentum conservation are given in Refs. 6 and 8. For application to the present experiment, the Ohmic heating term $-neE_zV_z$ will be omitted from Eq. (1), allowing a stationary equilibrium which is uniform in the z direction. This omission is rigorously valid for low currents and experimental support for the omission at higher currents will be presented in a future paper. The electron equations are perturbed about the assumed equilibrium and after some algebra can be reduced to[10]

$$\frac{\tilde{n}}{n_0} = \frac{e\tilde{\varphi}}{kT_0}\frac{(\omega^* + i\nu_\parallel)(\omega - 2.71\omega_1 + 3i\nu_\parallel) - 1.14i\nu_\parallel(1.71i\nu_\parallel - 1.5\omega_1)}{(\omega + i\nu_\parallel)(\omega - 2.71\omega_1 + 3i\nu_\parallel) - (1.71i\nu_\parallel - 1.5\omega_1)(0.67\omega_1 + 1.14i\nu_\parallel)}, \quad (2a)$$

where $\nu_\parallel = k_z^2T_0/0.51m_e\nu_{ei}$, $\omega_1 = k_zu_0$, and $\omega^* = k_y(cT_0/eB)[-(1/n_0)(dn_0/dx)]$, ν_{ei} being the electron-ion collision frequency.

The ion motion is treated as in Ref. 3, yielding another relation between $\tilde{n}$ and $\tilde{\varphi}$,

$$\frac{\tilde{n}}{n_0} = \frac{e\tilde{\varphi}}{kT_0}\frac{\omega^*(1-b) - \omega b - i\nu_\perp}{\omega + i\nu_\perp}, \quad (2b)$$

with $b = k_\perp^2(kT_0/M_i)$ and $\nu_\perp = \frac{3}{10}b^2\nu_{ii}$, ν_{ii} being the ion-ion collision frequency. Equations (2a) and (2b) are combined to give the linear dispersion relation

$$a_3\omega^3 + a_2\omega^2 + a_1\omega + a_0 = 0. \quad (3)$$

The values of the a's are somewhat lengthy and will be given in the more complete work.

Unlike the isothermal approximation, this dispersion relation predicts a strong current-driven drift instability for the parameter range of experimental interest. Many features are perhaps best illustrated by the marginal-stability curves of Fig. 1, which are drawn for typical conditions with the assumption $k_x = k_y$. Note that the mode is stable at low B/k_y and small u_0/u_{th}^i and destabilizes as these parameters are increased. In addition, the critical value of axial current, i.e., u_0/u_{th}^i, is a strong function of B and decreases as B is increased. Finally, the instability exists for a wide range of λ_z, and for $\lambda_z > 200$ cm stabili-

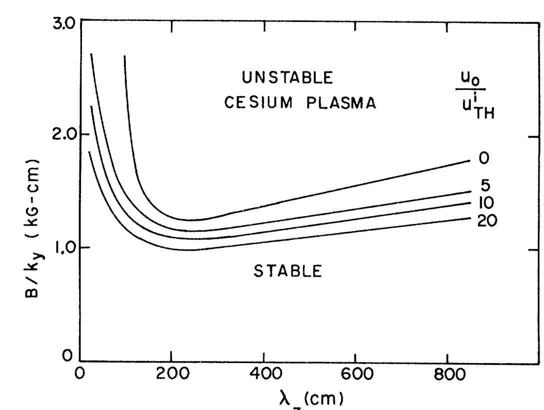

FIG. 1. Theoretical marginal-stability curves with u_0/u_{th}^i as parameter $[u_{th}^i = (kT_0/M_i)^{1/2}]$ calculated from Eq. (3) with $n_0 = 10^{11}$ cm^{-3}, $(1/n_0)(dn_0/dx) = -1$ cm^{-1}, and $T_0 = 2500$°K.

ty is insensitive to small changes of λ_z.

The experiments were performed on the Princeton Q-3 device with a plasma column having the following characteristics: a length of 110 cm, a diameter of 4 cm, $T_e \simeq T_i \simeq 2500°$K, $n_0 = 10^{10}$–3×10^{11} cm^{-3}, and $(1/n_0)(dn_0/dr) \sim 1$ cm^{-1}. Axial currents in the range 20–250 mA were driven by a dc voltage applied between the hot end plates. Langmuir probes were used for the measurement of all quantities except the absolute value of the plasma density, which was measured with an 8-mm microwave interferometer.

For small values of B/k_y the *current-free* collisional drift mode is stabilized by ion collisional viscosity[1-3] (the curve $u_0/u_{\rm th}^i = 0$ of Fig. 1). In this regime an axial current was applied. At a critical current the onset of the current-driven mode was detected as a large-amplitude, nearly sinusoidal, single-mode, low-frequency oscillation of probe ion saturation current and floating potential (φ_f). Concomittant with mode onset there occurred a significant decrease (10–30%) in the average density of the plasma and a reduction of the average density gradient, as illustrated in Fig. 2(a). Oscillation amplitudes and average plasma decrease at onset were considerably greater than those observed in the *current-free* drift mode.[2] With respect to plasma confinement the current-driven drift mode is much more "dangerous" than its current-free counterpart. The observed oscillations had the following general characteristics: (1) $\tilde{n}/n_0 \sim e\tilde{\varphi}_f/kT_0$, (2) oscillation amplitude radially localized in the region of maximum density gradient [see Fig. 2(b)], (3) modes propagated azimuthally with azimuthal mode numbers (m) from one to six observed under varying conditions, (4) an axial-wave structure which exhibited features of both a standing wave and traveling wave and which could be approximated by $\lambda_z = 2\times$(machine length). These features are consistent with those expected for a current-driven collisional drift mode in a double-ended Q machine. More detailed comparisons with the theory are present below. To apply the slab-model calculations to the cylindrical plasma we make the following identifications: $x \to r$, $y \to \theta$, $k_y \to m/r$, and we assume $k_r \approx m/r$. Wherever possible, experimental quantities are evaluated at the radius of maximum wave amplitude.

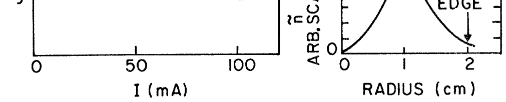

FIG. 2. (a) Recorder trace of ion saturation current (J_+) versus axial current I for three radial positions of probe. (b) Measured radial profiles of equilibrium density n_0, equilibrium potential φ_0, and amplitude of fluctuating density $\tilde{n}$. $I = 75$ mA, $B = 3.17$ kG, $m = 2$, $\tilde{n}/n_0 = 0.12$ at peak.

We consider now the measured frequency and its variation with plasma parameters. The presence of an equilibrium radial electric field [see curve of φ_0 of Fig. 2(b)] causes the plasma to undergo an $\vec{E}\times\vec{B}$ rotation which leads to a Doppler shift of the frequency by an amount $\omega_E \equiv -(m/r) \times (cE_{\theta r}/B)$, where $E_{\theta r} = -\partial\varphi_0/\partial r$.[2,3,11] If we designate by ω_R the frequency in the nonrotating plasma *rest* frame and by $\omega_{\rm obs}$ frequency observed in the laboratory then $\omega_R = \omega_{\rm obs} - \omega_E$ and ω_R is the value to be compared with theory. Note from Fig. 2(b) that $\varphi_0(r)$ closely approximates $\varphi_0 \propto r^2$ over most of the plasma so that the plasma rotates as a rigid body, i.e., ω_E is nearly independent of radius; this eliminates effects which might be caused by radial shear in the plasma rotation. In Fig. 3(a) we show the variation of ω_R with magnetic field for three azimuthal modes and typical values of density and current. The theoretical lines in this figure have been drawn for the measured value of $(1/n_0)dn_0/dr$ and are subject to an error of $\pm 20\%$. Note also that the value of $(1/n_0)dn_0/dr$ is different for each azimuthal mode. Within the limits of experimental error the agreement is quite good for the absolute value of ω_R and the dependence on magnetic field and azimuthal mode number. The variation of ω_R with axial current was very slight and is not displayed; this point was in excellent agreement with the theory.

The variation of stability with certain parameters is indicated in Fig. 3(b) which displays the critical current for instability onset versus the magnetic field for three azimuthal modes at a density $n_0 = 2\times10^{11}$ cm^{-3}. The theoretical value of magnetic field in this figure has been scaled by a factor of 1.8. The necessity for this scaling implies that the approximate forms for the viscosity greatly underestimate the wave damping. A similar, as yet unexplained discrepancy in the

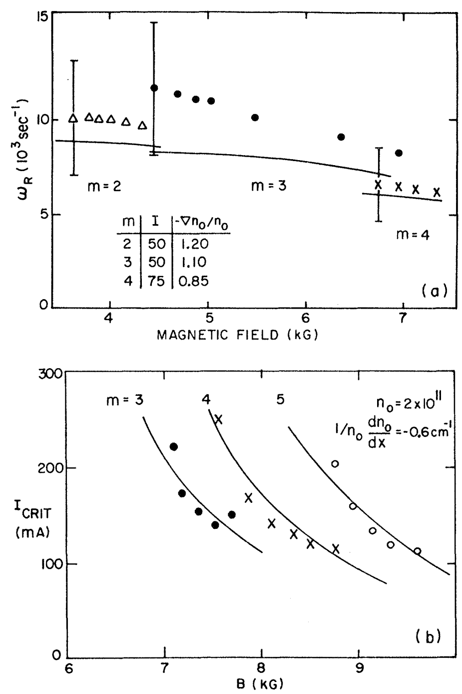

FIG. 3. (a) Measured frequency ω_R in the plasma rest frame versus magnetic field B at $n_0=3\times10^{10}$ cm^{-3}. Typical error bars are indicated for each mode. Theory (solid lines) is from Eq. (3) for $\lambda_z=250$ cm, $T_0=2500$°K. (b) Critical current I_{crit} for mode onset versus magnetic field B for three azimuthal modes. Theory (solid lines) from Eq. (3), $\lambda_z=250$ cm, $T_0=2500$° K, and B scaled by 1.8.

onset magnetic field has been found by Hendel, Chu, and Politzer,[2] and by the authors, for the *current-free* collisional drift mode. A discussion of this effect is beyond the scope of this Letter but the point is made that the scaling is not a feature of the current-driven mode alone; it seems to apply to many experiments with collisional drift modes in Q machines. With this scaling, theory and experiment are in good agreement on the magnitude of the critical current, the shape of the curve I_{crit} vs B, and the variation of azimuthal mode number with magnetic field. In addition, it is worth noting that a wide variation in the plasma density leads to a relatively small variation in the critical current; the critical parameter is the current, not the electron drift velocity. This is in agreement with theory.

For the above comparisons with theory we have used a parallel wavelength equal to twice the machine length and this only approximates the axial structure of the mode. However, neither the stability characteristics (Fig. 1) nor the frequency are sensitive to changes of λ_z in the vicinity of 250 cm. The details of the axial structure will be considered in a future paper.

In summary, we have observed a strong current-driven collisional drift mode. The characteristics of this mode are in reasonable agreement with a linearized two-fluid theory and strongly indicate the importance of electron heat flow and electron temperature oscillations in considering collisional drift instabilities as indicated in the work of Coppi and Mazzucato.[6,7]

The authors would like to thank T. K. Chu, D. L. Jassby, F. W. Perkins, and T. S. Tsai for helpful discussions, and J. Frangipani and C. Harrison for technical assistance.

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*Work supported by U. S. Atomic Energy Commission Contract No. AT(30-1)-1238. Use was made of computer facilities supported in part by National Science Foundation Grant No. NSF-GP579.

†Current address: Institute for Plasma Physics, Garching bei München, Germany.

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